

安徽大学 2022—2023 学年第一学期  
《数学分析（下）》考试试卷 (B 卷)  
参考答案及评分标准

一、填空题 (每空 3 分, 共 12 分).

1.  $2xy(x^2 + 1)^{y-1}dx + (x^2 + 1)^y \ln(x^2 + 1)dy.$

2.  $x + 4y - 6z + 1 = 0.$

3.  $\frac{\sqrt{2}}{3}.$

4.  $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z-1}{-4}.$

二、选择题 (第 5 题 6 分, 第 9 题 12 分, 其余每小题 10 分, 共 58 分)

5. 解:

因为  $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + \sin y) = 0,$  ... (2分)

又  $\left| \sin \frac{1}{x+y} \right| \leq 1,$  ... (4分)

所以  $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^2 + \sin y) \sin \frac{1}{x+y} = 0.$  ... (6分)

6. 解:

$\frac{\partial z}{\partial x} = f(x^2 + y^2) + 2x^2 f'(x^2 + y^2);$  ... (3分)

$\frac{\partial z}{\partial y} = 2xy f'(x^2 + y^2);$  ... (6分)

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial(2xy f'(x^2 + y^2))}{\partial x} = 2y f'(x^2 + y^2) + 4x^2 y f''(x^2 + y^2) \dots \dots (10分)$

7. 解:

令  $u = xy, v = \frac{y}{x}$ , 则  $D_{uv} = \{(u, v) | 1 \leq u \leq 2, 1 \leq v \leq 4\}$  ... (3分)

且  $\left| \frac{\partial x}{\partial u} \frac{\partial x}{\partial v} \right| = \frac{1}{\partial v}$ . ... (6分)

则原式  $= \iint_{D_{uv}} u^2 \frac{1}{\partial v} dx dy = \frac{1}{\partial} \int_1^2 u^2 du \int_1^4 \frac{1}{v} dv = \frac{7}{3} \ln 2$ . ... (10分)

8. 解:

令  $P(x, y) = e^x \sin 2y - y, Q(x, y) = 2e^x \cos 2y - 10$  ... (2分)

$\frac{\partial Q}{\partial x} = 2e^x \cos 2y, \frac{\partial P}{\partial y} = 2e^x \cos 2y - 1$ . ... (4分)

添加线段  $\vec{BA}$ :

$$\begin{aligned} \int_L (e^x \sin 2y - y) dx + (2e^x \cos 2y - 10) dy &= \oint_{L+\vec{BA}} - \oint_{\vec{BA}} \\ &= \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy - 0 \\ &= \iint_D 1 dx dy = \frac{\pi}{2}. \dots \dots (10分) \end{aligned}$$

9. 解:

添加平面  $\sum_1: z = 0$  取下侧, 由 Gauss 公式可得:

$$\begin{aligned} \oiint_{\sum + \sum_1} x dy dz + 2y dz dx + 3z dx dy &= \iiint_V 6 dV \\ &= 6 \cdot \frac{2}{3} \pi a^3 \\ &= 4\pi a^3. \dots \dots (4分) \end{aligned}$$

又因为  $\iint_{\sum_1} x dy dz + 2y dz dx + 3z dx dy = 0$ , ... (8分)

所以  $\iint_{\sum} x dy dz + 2y dz dx + 3z dx dy = 4\pi a^3 - 0 = 4\pi a^3$ . ... (12分)

10. 解: 令  $\begin{cases} f_x = 4x^3 - 4x = 0 \\ f_y = 8y^3 - 24 = 0 \end{cases}$ , 可得驻点:

$$P_1(0, 0), P_2(0, \sqrt{3}), P_3(0, -\sqrt{3}), P_4(1, 0), P_5(1, \sqrt{3})$$

$$P_6(1, -\sqrt{3}), P_7(-1, \sqrt{3}), P_8(-1, 0), P_9(-1, -\sqrt{3}). \quad \dots \dots (2\text{分})$$

$$\text{又 } f_{xx} = 12x^2 - 4 = A, f_{xy} = 0 = B, f_{yy} = 24y^2 - 24 = C$$

$$H = \begin{vmatrix} A & B \\ B & C \end{vmatrix}. \quad \dots \dots (4\text{分})$$

$$\text{① } H(P_1) > 0, H(P_5) > 0, H(P_6) > 0, H(P_7) > 0, H(P_9) > 0.$$

$$\text{所以在 } P_1, P_5, P_6, P_7, P_9 \text{ 不取极值.} \quad \dots \dots (6\text{分})$$

$$\text{② } H(P_2) = \begin{vmatrix} -4 & 0 \\ 0 & 48 \end{vmatrix} = -192 < 0, A < 0$$

$$\text{所以 } f(P_2) = 18 \text{ 为极大值.}$$

$$\text{同理 } f(P_3) = 18 \text{ 为极大值.} \quad \dots \dots (8\text{分})$$

$$\text{③ } H(P_4) = \begin{vmatrix} 8 & 0 \\ 0 & -24 \end{vmatrix} < 0, A = 8 > 0$$

$$\text{所以 } f(P_4) = 5 \text{ 为极小值.}$$

$$H(P_8) = \begin{vmatrix} 8 & 0 \\ 0 & -24 \end{vmatrix} < 0, A = 8 > 0$$

$$\text{所以 } f(P_8) = 5 \text{ 为极小值.} \quad \dots \dots (10\text{分})$$

### 三、证明题 (每小题 10 分, 共 20 分)

11. 解:

$$\text{① 因为 } \left| \int_0^A \sin x dx \right| \leq 2$$

$$\text{且 } \frac{\ln(1+tx)}{x^2} \text{ 对 } \forall t \in [0, +\infty) \text{ 有单调递减趋于零} \quad \dots \dots (2\text{分})$$

$$\text{所以 } \int_0^{+\infty} \frac{\sin x}{x^2} \ln(1+tx) dx \text{ 收敛.} \quad \dots \dots (4\text{分})$$

$$\text{② } \frac{\partial(\frac{\sin x}{x^2} \ln(1+tx))}{\partial t} = \frac{\sin x}{x} \cdot \frac{1}{1+tx} \text{ 且}$$

$$(i) \int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$(ii) \left| \frac{1}{1+tx} \right| \leq 1, \forall (t, x) \in [0, +\infty) \times [0, +\infty) \quad \dots \dots (8\text{分})$$

$$\text{所以 } \int_0^{+\infty} \frac{\partial(\frac{\sin x}{x^2} \ln(1+tx))}{\partial t} dx \text{ 一致收敛.} \quad \dots \dots (10\text{分})$$

12. 解:

$$f(0,0) = 0, f_x(0,0) = \lim_{x \rightarrow 0} \frac{f(x,0) - f(0,0)}{x} = 0.$$

同理, 可证明  $f_y(0,0) = 0$ .

所以  $f(x,y)$  在  $(0,0)$  处存在偏导数.

... (4分)

$$\text{因为 } \Delta z = f(\Delta x, \Delta y) - f(0,0) = \sqrt{|\Delta x \Delta y|},$$

$$f_x(0,0) \Delta x + f_y(0,0) \Delta y = 0.$$

若  $f(x,y) = \sqrt{|xy|}$  在  $(0,0)$  处可微

$$\text{则 } \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{[\Delta z - (f_x(0,0)\Delta x + f_y(0,0)\Delta y)]}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} = 0.$$

... (7分)

$$\text{但 } \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{[\Delta z - (f_x(0,0)\Delta x + f_y(0,0)\Delta y)]}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{\sqrt{|\Delta x \Delta y|}}{\sqrt{(\Delta x)^2 + (\Delta y)^2}} = \frac{\sqrt{|k|}}{\sqrt{1+k^2}},$$

即极限不存在,

所以在  $(0,0)$  处不可微.

... (10分)

#### 四、应用题 (10 分)

13. 解:

设长方体的长, 宽, 高为  $x, y, z$ , 由条件知  $xyz = V$ . ... (2分)

因此题目是求  $S = 2(xy + yz + zx)$  在  $xyz = V$  极值问题. ... (4分)

设  $L(x, y, z, \lambda) = 2(xy + yz + zx) + \lambda(xyz - V)$

$$\text{令 } \begin{cases} L_x = 2(y + z) + \lambda yz = 0 \\ L_y = 2(z + x) + \lambda xz = 0 \\ L_z = 2(x + y) + \lambda xy = 0 \\ L_\lambda = xyz - V = 0 \end{cases}$$

$$\Rightarrow x = y = z = \sqrt[3]{V}$$

因此, 当  $x = y = z = \sqrt[3]{V}$  时, 表面积最小.

... (10分)