EE 120 SIGNALS AND SYSTEMS, Fall 2019
Homework # 5, Due November 14, Thursday

1. Determine the Laplace transform and the associated region of convergence for the following signals:
   a) $x(t) = e^{-t}\cos(2t)u(t)$
   b) $x(t) = \sin(t)u(-t)$
   c) $x(t) = t\cos(t)u(t)$
   d) $x(t) = \delta(t-1)$

2. For each Laplace transform and region of convergence pair below, determine the function of time, $x(t)$:
   a) $X(s) = \frac{s+2}{(s+2)^2+1} \quad Re\{s\} > -2$
   b) $X(s) = \frac{se^{-s}}{s^2+1} \quad Re\{s\} > 0$
   c) $X(s) = \frac{s}{(s^2+1)^2} \quad Re\{s\} > 0$
   d) $X(s) = \frac{s+2}{s^2+7s+12} \quad Re\{s\} > -3$

3. Consider a causal LTI system with the transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad \zeta \in [0, 1].$$

   a) Using the stability test derived in class show that the system is stable if $\zeta \in (0, 1]$.

   b) Show that the system is *unstable* when $\zeta = 0$.

   c) For $\zeta = 0$ and $\omega_n = 1$ construct a bounded input $x(t)$ such that the output grows unbounded. Hint: see Problem 2, part (c) above.

   d) Show that the magnitude of the frequency response $|H(j\omega)|$ is monotonically decreasing in ω (i.e., no resonance peak) when $\zeta \geq 1/\sqrt{2}$.

4. Draw the Bode magnitude and phase plots for each transfer function below:
   a) $H(s) = s$
   b) $H(s) = -\tau s + 1, \ \tau > 0$
   c) $H(s) = \frac{-s+1}{s+1}$
   d) $H(s) = \frac{s+10}{s+1}$

5. Find the transfer function for each system below given in state-space form:

$$a) \qquad \begin{aligned} \frac{dz_1(t)}{dt} &= z_2(t) \\ \frac{dz_2(t)}{dt} &= -a_0 z_1(t) - a_1 z_2(t) + x(t) \\ y(t) &= b_0 z_1(t) + b_1 z_2(t) \end{aligned}$$

b)
$$\begin{aligned}\frac{dz_1(t)}{dt} &= -a_0 z_2(t) + b_0 x(t) \\ \frac{dz_2(t)}{dt} &= z_1(t) - a_1 z_2(t) + b_1 x(t) \\ y(t) &= z_2(t).\end{aligned}$$

6. Consider the system described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t).$$

Use the unilateral Laplace transform to determine the output $y(t)$ when the input is $x(t) = e^{-3t}u(t)$ and the initial conditions are:

$$y(0^-) = \frac{5}{2} \qquad \left.\frac{dy(t)}{dt}\right|_{t=0^-} = \frac{-9}{2}.$$