EE 120 SIGNALS AND SYSTEMS, Fall 2019
Homework # 1, Due September 19, Thursday

1. Prove the following properties of continuous-time convolution:

$$(x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

a) Commutative property: $x * h = h * x$

b) Distributive property: $x * (h_1 + h_2) = x * h_1 + x * h_2$

c) Associative property: $x * (h_1 * h_2) = (x * h_1) * h_2$

2. Below are impulse responses of several discrete-time LTI systems, where u is the unit step function:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}$$

Determine whether each system is: (i) stable or unstable, (ii) causal or non causal, (iii) FIR or IIR. Show your work.
a) $h[n] = u[n] - u[n-10]$
b) $h[n] = 2^n u[n]$
c) $h[n] = 2^n u[-n]$
d) $h[n] = n(0.8)^n u[n]$

3. This problem proves that the constant-coefficient difference equation:

$$y[n] + a_1 y[n-1] + ... + a_N y[n-N] = b_0 x[n] + ... + b_M x[n-M]$$

defines a LTI system if the system is *initially at rest*, that is, if $y[n] = 0$ for $n < n_0$, where $n = n_0$ is the first instant when $x[n] \neq 0$.

a) Show that if signals $x[n], y[n]$ satisfy the difference equation above, the scaled signals $\hat{x}[n] = \alpha x[n], \hat{y}[n] = \alpha y[n]$ also satisfy the equation for any constant α.

b) Show that if $x_1[n], y_1[n]$ and $x_2[n], y_2[n]$ satisfy the difference equation, then so do $\hat{x}[n] = (x_1 + x_2)[n], \hat{y}[n] = (y_1 + y_2)[n]$.

c) Show that if $x[n], y[n]$ satisfy the difference equation, they also satisfy the equation when shifted in time by the same amount. That is, $\hat{x}[n] = x[n-L]$, $\hat{y}[n] = y[n-L]$ satisfy the equation for any L. Discuss why it is critical for this property that the coefficients $a_1, \ldots, a_N, b_1, \ldots, b_M$ are constant, and do not change with time.

d) We will now see why the initial rest condition is essential. Consider the accumulator system,

$$y[n] = y[n-1] + x[n],$$

which is a special case with $N = 1$, $M = 0$, $a_1 = -1$, $b_0 = 1$. Suppose $y[n] = 1$ for $n < 0$ and find the response when $x[n]$ is the unit impulse. Then find the response when the input is the unit impulse scaled by a constant α. Did the response scale by the same constant?

e) Initial rest means $y[n] = 0$ for $n < n_0$, where $n = n_0$ is the first instant when $x[n] \neq 0$. To see why it is important to define n_0 relative to the input, consider again the accumulator system and suppose, instead, we let n_0 be a fixed point in time such as $n_0 = 0$. When we impose the initial condition $y[n] = 0$ for $n < n_0 = 0$, the system equation becomes

$$y[n] = \begin{cases} 0 & n < 0 \\ y[n-1] + x[n] & n \geq 0. \end{cases}$$

Determine if this system is linear and time-invariant.

4. Consider the filter described by the difference equation:

$$y[n] = 0.25x[n-1] + 0.5x[n] + 0.25x[n+1].$$

a) Determine the impulse response.

b) Calculate and sketch the frequency response $H(e^{j\omega})$.

c) Determine the type of filter (low-pass, band-pass, high-pass, etc.).

5. a) Find the frequency response $H(j\omega)$ of the RC circuit below, which is governed by the differential equation

$$RC\frac{dy(t)}{dt} + y(t) = x(t).$$

Note that it is not necessary to find the impulse response first. Instead, you can substitute $x(t) = e^{j\omega t}$ and $y(t) = H(j\omega)e^{j\omega t}$ in the differential equation to obtain an expression for $H(j\omega)$.

b) Calculate and sketch the amplitude $|H(j\omega)|$ as a function of the frequency ω. Your plot should confirm the low-pass behavior of this circuit.