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A development dream of a poor country

Abstract

The premise of a sustainable development assessment system is a good understanding of the concept. The sustainable development in our eyes should have a good current development level, a coordinate inner structure and a bright prospect for development.

To describe the three factors, we use the Pressure-State-Response (PSR) model as a base. This model use P to stands for the pressure human make, S stands for the nature's state and R stands for people's action in sustainable. Choose 30 index from the World Bank Data. Divide them into three parts(subsystem P,S,R), normalize them , and calculate their weight by entropy method and get the value of P,S,R. P,S,R' s weight sum reflects the current development. The balance between P, S and R reflects the coordinate degree. The growth rate of the current development level reflects the prospect. So we set up the measurement indicators of sustainable development, balance, coordinate and growth. Based on this three indicators, we establish a capacity of sustainable development index (CSD). According to the value of CSD, we divide the index into 5 parts and set a 5 class sustainable development system. From this system, we conclude that the CSD of 44 countries in the UN list of the 48 Least Developed Countries are dangerous. Mauritania has the lowest CSD, and be classified as a very serious unsustainable country.

Then we choose Mauritania as a research object. We find the top worst index which contributes most to the Mauritania's terrible rank and analyze the relationship between economy and ecology. The GDP and forest area has a negative correlation Then according to the cooperative-game theory to find a best strategy which can make the ecology and economy develop together. Under the guidance of this strategy, we make our plan that we should focus on the development of new heavy industry, ecological agriculture and education. What the ICM can do is providing direct investment, attract indirect investment, sponsor technique and fund.

In order to evaluate the effect of our plan, we use grey prediction model to predict the future which based on the current development pattern. Then use the Markov - chain prediction method to predict the future which has taken our measures and the random emergency into consideration. Then compare CSD value of this two pattern. We can find that if we didn't take measures, Mauritania will still be a very serious unsustainable country in 2035. If our plan is put into action, it will change into a Critical unsustainable country in 20 years.

Keywords :
Pressure-State-Response ; Entropy weight ; Cooperative-game theory
Grey prediction model ; Markov - chain prediction method

1. INTRODUCTION

With the resource of earth is consuming in a fast speed, more and more people show the concern about the future of human society. Since 1960, there has been many experts study the sustainable development. However most people's research object is the whole world, a country or an area. Almost no one choose the 48 Least Developed Countries (LDC) in the UN list as study object. However, LDC as a country group share many same points. Their development path also has the connotation of law. This paper chooses these countries as research object aiming at finding the regular of their sustainable development path.

The paper is organized as follows. Section 2 gives an introduction of the research's background and the meaning of this research. Section 3 describes our understanding of sustainable development in details and shows the establish process and principle of our assessment system then we estimate every country of LDC and get their capacity of sustainable development and rank them. Section 4 provides a plan of the country Mauritania which has the worst index in section 3. Section 5 demonstrates the rationality and availability of plan in section 4. Finally in section 6 we summarize the main conclusion of the present paper and discuss the strength and potential weakness.

Table 1 Notations

CL	Current development level
CD	Coordinate degree
GR	Growth rate of current development level
CSD	Capacity of sustainable development
SD grade	The grade of CSD

The notation here appears frequently in this paper. There are many other notations just appear once and they will be described when it is used.

2. BACKGROUND

Resources on the earth is limited. The three major sources of energy petroleum, natural gas and coal are not renewable. How to avoid the development of human stopped by resource depletion and fulfill the sustainable development aim is a hot topic now. In the past two centuries, developed countries have gone the road, *pollute first, then control* and reached to a high level of sustainable development. Developing countries want to develop and be rich. However, because of their weak technique strength and low level of economic base, waste and poor efficiency development are normal in these country. So how to help developing countries especially the 48 Least Developed Countries in the UN list to realize the sustainable development is what this paper focuses on.

3. WHAT IS SUSTAINABLE DEVELOPMENT?

The understanding of sustainable development is the key to solve the problem. The definition of the sustainable development has experienced a long development process. Here, the Brundtland Commission's brief definition of sustainable development as the "*ability to make development sustainable – to ensure that it meets the needs of the present without compromising the ability of future generations to meet their own needs*"[1] is surely the most widely accepted one within the various definitions. This definition has played an important role in many countries' policy making process. However, in order to justify whether a country's present situation is sustainable or

unsustainable, a more specific definition is needed.

To make the concept more specific, we deem that if a country's development is a sustainable, it should have a basic current development level, a balanced country structure and a bright future.

- The basic development level reflects the country's base and potential.
- Whether the country's structure (resource, agriculture, light industry, heavy industry and service) is balanced reflects whether its development is healthy and we can be sure that an unbalanced development is dangerous.
- The development is a dynamic process. So the future prediction can reflect this procedure. A sustainable development is likely to have a bright future and an unsustainable development will experience many challenge in the future.

3.1 The Pressure-State-Response (PSR)[2] and current development level (CL) model

Different institutions have established different assessment system in the last few decades, such as *Wellbeing Index (88 factors included)* and *Environmental Sustainability Index (68 factors included)*[3]. These systems are giant and effective which is based on a huge quantity of data. Considering that our research subjects are the 48 Least Developed Countries (LDC), some factors have a lack of data support. What's more, LDC need an individual, dynamic assessment system rather than a global, static one. Here we choose PSR (Pressure State Response) model to solve this problem.

PSR model was made by the organization for economic cooperation and development (OECD) in cooperation with the United Nations Development Program (UNEP) in a proposed 80 at the end of twentieth Century for sustainable development and evaluation of resource utilization pattern[4].The model of PSR is widely applied to evaluate the utilization of resources and sustainable development, in which the pressure index P is used to characterize the result in unsustainable development of human economic activities of production and consumption patterns or economic system. State index S is used to characterize the system state in the sustainable development process. R is used to characterize the response human countermeasures taken to promote the sustainable development. PSR model in accordance with the "cause - effect - reaction" thinking, interpretation of human activities to the natural world pressure, change the environment and resource state, and decision-making, behavior response through the process, promote the benign cycle of ecological system.

3.1.1Determine the evaluation index set

Combine PSR model with the Commission on Sustainable Development's evaluating indicators. We choose 30 indicators of our own from the World Bank Data as the base of our model and divide these indicators into three part.

- **Pressure**

Pressure (P) is a dynamic factor to influence the sustainable development of the whole system, reflects the influence of the traditional mode of development and economic and social activities caused on resources and environment, as well as the restriction of resources and environment on economic and social sustainable development. So here we choose Natural population growth rate, CO_2 emissions, etc.

- **State**

State (S) reflects the ecological background and foundation of development of, namely

resources, environmental capacity, industry foundation and social development. Here we choose GDP per capita, Population density, etc.

- **Response**

Response (R) reflects the action of the government and residents taken to deal with the pressure of ecological environment and local development foundation. Here we choose GDP growth, Total natural resources rents and some other factors to .

$$X=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1,\ n\text{-}1} & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2,\ n\text{-}1} & x_{2,\ n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{m\text{-}1,1} & x_{m\text{-}1,2} & \cdots & x_{m\text{-}1,n\text{-}1} & x_{m\text{-}1,n} \\ x_{m1} & x_{m,2} & \cdots & x_{m,n\text{-}1} & x_{m,n} \end{bmatrix} \tag{1}$$

X_{ij} means the numerical value of Country i , Indicator j

$$i=1,2,3,4\cdots m \tag{2}$$

$$j=1,2,3,4\cdots n \tag{3}$$

$$m=48 \tag{4}$$

$$n=30 \tag{5}$$

3.1.2 Normalization and determine the weight of every indicator by entropy weight method

Because of the dimension, quantity, positive and negative orientation has difference of each index, it is not realistic using the data to calculate directly. What we need to do is making the data normalized.

Use the extremum standardization method to do with the raw data and make the dimension and quantity unified. After the treatment, the score of each index are located in (0, 1).

$$R=\begin{Bmatrix} r_{11} & r_{12} & \cdots & r_{1,n\text{-}1} & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2,n\text{-}1} & r_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{m\text{-}1,1} & r_{m\text{-}1,2} & \cdots & r_{m\text{-}1,n\text{-}1} & r_{m\text{-}1,n} \\ r_{m1} & r_{m,2} & \cdots & r_{m,n\text{-}1} & r_{m,n} \end{Bmatrix} \tag{6}$$

If x_{ij} has a positive effect on the sustainable development level, then the corresponding r_{ij} is

$$r_{ij} = \frac{x_{ij} - \min\{x_{i1}, x_{i2}, ...x_{in}\}}{\max\{x_{i1}, x_{i2}, ...x_{in}\} - \min\{x_{i1}, x_{i2}, ...x_{in}\}} \tag{7}$$

If x_{ij} has a negative effect on the sustainable development level, then the corresponding r_{ij} is

$$r_{ij} = \frac{\max\{x_{i1}, x_{i2}, ...x_{in}\} - x_{ij}}{\max\{x_{i1}, x_{i2}, ...x_{in}\} - \min\{x_{i1}, x_{i2}, ...x_{in}\}} \tag{8}$$

The entropy weight method is a common method to determine the index weight. Based on "difference driven" principle, this method pays much attention on the local difference. It reflects the utility of information entropy and avoid artificial factors. So the weight given is more objective.

The proportion of indicator j in country i

$$y_{ij} = \frac{x_{ij}^{'}}{\sum_{i=1}^{m} x_{ij}^{'}} \tag{9}$$

Entropy of information

$$e_j = -\frac{1}{\ln m} \sum_{i=1}^{m} y_{ij} \ln y_{ij} \tag{10}$$

Redundancy rate

$$d_j = 1 - e_j \tag{11}$$

The weight of indicator j

$$w_j = \frac{d_j}{\sum_{i=1}^{m} d_j} \tag{12}$$

Table 2. The Evaluation index and its weight

Destination Layer	Module level	Elements layer	Index level	Relationship	Weight
Current development level	presure	society	Population growth (annual %)	−	0.0054
			Prevalence of anemia among pregnant women (%)	−	0.0132
		environment	Arable land (hectares per person)	+	0.0185
			Improved water source (% of population with access)	+	0.0074
			CO2 emissions (kt)	−	0.0021
			Permanent cropland (% of land area)	+	0.0884
	state	economy	Arable land (% of land area)	+	0.0332
			GDP per capita (current US$)	+	0.0797
			Agriculture, value added (% of GDP)	+	0.0097
			Services, etc., value added (% of GDP)	+	0.0044
			Agriculture, value added (current US$)	+	0.0504
			Agriculture value added per worker (constant 2005 US$)	+	0.0506
		society	Population density (people per sq. km of land area)	−	0.0020
			Cereal yield (kg per hectare)	+	0.0212
		environment	Average precipitation in depth (mm per year)	+	0.0197
			Terrestrial protected areas (% of total land area)	+	0.0326
			Renewable internal freshwater resources per capita (cubic meters)	+	0.0735
			Forest area (% of land area)	+	0.0259
	response	economy	GDP growth (annual %)	+	0.0026
			Crop production index (2004-2006 = 100)	+	0.0141
			Secure Internet servers (per 1 million people)	+	0.1465
		society	Adjusted net enrollment rate, primary (% of primary school age children)	+	0.0048
			Telephone lines (per 100 people)	+	0.0544
			Labor force, total	+	0.0559
			Motor vehicles (per 1,000 people)	+	0.0355
		environment	Improved sanitation facilities (% of population with access)	+	0.0215
			CO2 emissions from liquid fuel consumption (% of total)	−	0.0533
			Terrestrial and marine protected areas (% of total territorial area)	+	0.0363
			Total natural resources rents (% of GDP)	+	0.0278
			Fuel imports (% of merchandise imports)	+	0.0094

3.1.3 PSR subsystem's value the current development level

$$P_i(pressure\ index) = \sum_{i=1}^{6} r_{ij} \times w_j$$

$$S_i(state\ \ index) = \sum_{i=7}^{18} r_{ij} \times w_j \tag{13}$$

$$R_i(response\ \ index) = \sum_{i=19}^{30} r_{ij} \times w_j$$

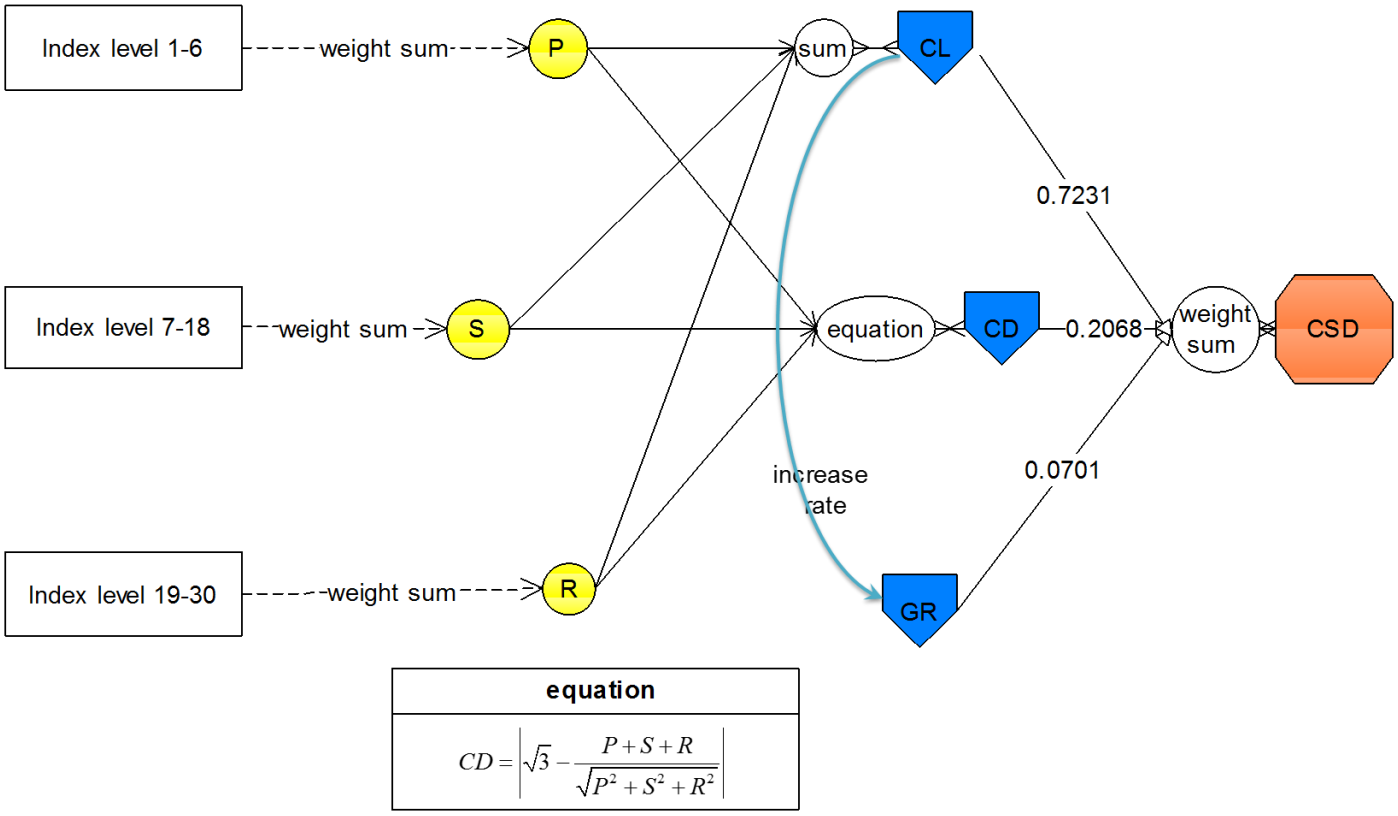


Figure 1 The CSD system

3.2 The current development level

$$CL_i(current\ development\ level) = P_i + S_i + R_i \tag{14}$$

3.3 The calculation of system coordination degree

In PSR model, the influence of regional sustainable development are also affected by the relationship between pressure, state and response. In order to evaluate the interreaction between every subsystem, we add a coordination degree (CD) function. This function is based on the distance between the system size and dispersion degree to judge the coordination degree.
The coordination degree:

$$H = \frac{A + B + C}{\sqrt{A^2 + B^2 + C^2}} \tag{15}$$

$$CD = \left| \sqrt{3} - H \right| \tag{16}$$

At first, we choose the H to describe the coordination. However considering that the $\sqrt{3}$ is the best value and other values are dispersed in it on both sides, it is not convenient to sort the order. So we add CD which reflects the gap between calculated values and the ideal value. The bigger CD was, the coordination degree is smaller.

3.4 Dynamic prediction of sustainable development

The sustainable development assessment system shouldn't be a static one which focuses on a

confirmed figure. It should have a concern about the potential of sustainable development. The country which has a tendency to be more sustainable should be concerned. Here we deem a new indicator growth rate (GR). We calculate the value of (CL) of three years (2007, 2009 and 2011. Data from World Bank). The value of GR is got by least square method.

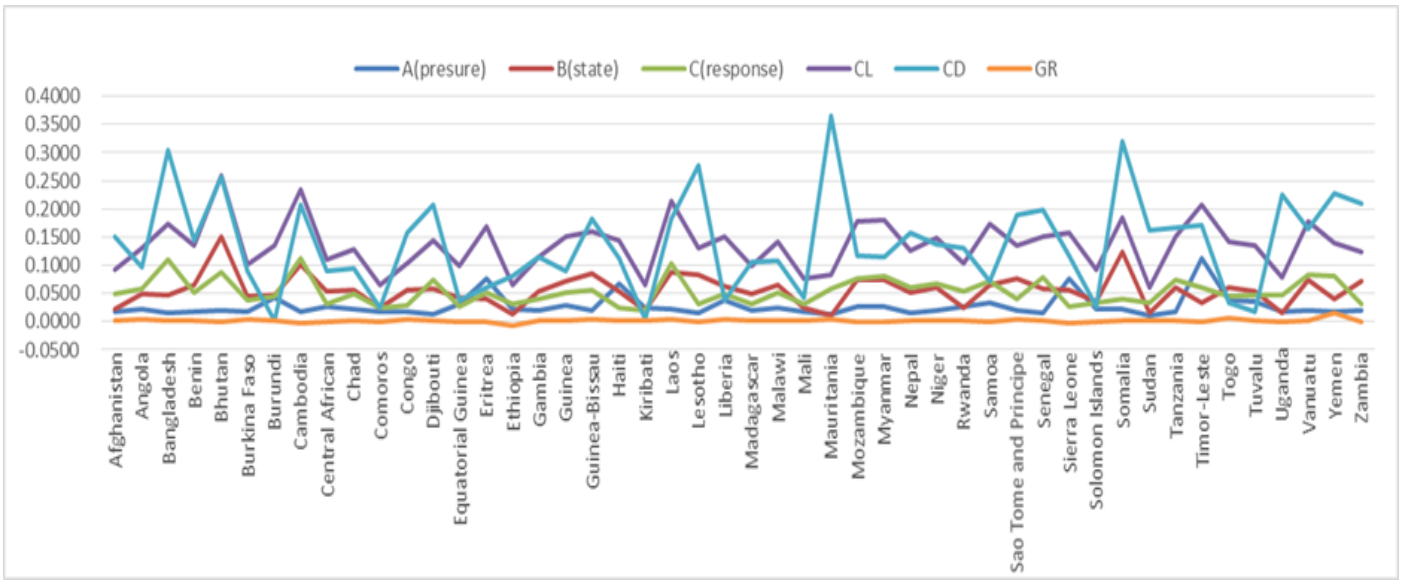


Figure 2 The value of xix factors of every LDC country

Considering that we only use three years data, so growth rate (GR) can't be accurate in a long term, but it can still be useful within several years and that is enough here.

3.5 Capacity of sustainable development index (CSD)

Now we have three indicators (CL,CD,GR). Here we use extremum standardization method again and give a capacity of sustainable development index (CSD) as the final index of every country. What's more, we choose several typical countries (just like Swiss, India, America, etc). Put these countries' data into our model, get their CSD. This measure has two meanings. The first one is test the model. The other one is using the CSD of these developing or middle-income countries as a consult, and make a classification of CSD.

Table 2 The value and rank of CSD

RANK	COUNTRY	CSD	RANK	COUNTRY	CSD
1	Swiss	0.8172	31	Ethiopia	0.3160
2	America	0.7808	32	Tanzania	0.3136
3	Germany	0.7460	33	Togo	0.3128
4	South Korea	0.7117	34	Benin	0.2979
5	France	0.6898	35	Senegal	0.2966
6	England	0.6843	36	Solomon Islands	0.2896
7	Sweden	0.6770	37	Haiti	0.2876
8	Japan	0.6596	38	Sudan	0.2813
9	China	0.5547	39	Timor-Leste	0.2802
10	Russia	0.5067	40	Chad	0.2801
11	Bhutan	0.4542	41	Angola	0.2768
12	Equatorial Guinea	0.4298	42	Guinea	0.2763
13	India	0.4281	43	Sierra Leone	0.2760
14	Laos	0.4194	44	Democratic Republic of the Congo	0.2748
15	Tuvalu	0.4034	45	Mozambique	0.2743
16	Kiribati	0.3996	46	Burkina Faso	0.2708
17	Samoa	0.3994	47	Madagascar	0.2561
18	Myanmar	0.3924	48	Mali	0.2497
19	Rwanda	0.3870	49	Niger	0.2490
20	Bangladesh	0.3866	50	Lesotho	0.2470
21	Vanuatu	0.3803	51	Central African Republic	0.2404
22	Burundi	0.3739	52	Eritrea	0.2373
23	Uganda	0.3678	53	Gambia	0.2365
24	Zambia	0.3634	54	Afghanistan	0.2161
25	Guinea-Bissau	0.3561	55	Liberia	0.2112
26	Sao Tome and Principe	0.3447	56	Djibouti	0.1955
27	Comoros	0.3309	57	Somalia	0.1560
28	Malawi	0.3264	58	Yemen	0.1503
29	Nepal	0.3254	59	Mauritania	0.0872
30	Cambodia	0.3250			

The pink countries in this table are some typical countries in the world.

Table 3 The SD grade partition by CSD

CSD	SD grade
＞0.5	Great sustainable
0.4～0.5	Critical sustainable
0.3～0.4	Critical unsustainable
0.2～0.3	Unsustainable
＜0.2	Very serious unsustainable

4. A 20 YEARS DEVELOPMENT PLAN OF MAURITANIA

From table 2 we can see that Mauritania is the most unstainable country. So we choose this country as our research object.

4.1 Basic situation of Mauritania

Table 4 Some index of Mauritania in 2011

P	S	R	CL	CD	GR	CSD
0.0142	0.0109	0.0588	0.0839	0.3665	0.0034	0.0872

From the model we can find that the value of CL and CD is too small which makes the CSD index very low directly. This means that the country's current development level is very unsatisfactory and the structure is not balanced.

In order to find that which indicator is the most important that makes this country rank last, we deem g_{ij} as indicator j's gap between the actual value and ideal value in country i then

$$g_{ij} = w_j - \frac{r_{ij} \times w_j}{CL_i} \tag{17}$$

After dealing the 30 indicators and soring, we found that Internet usage, Permanent cropland, Renewable internal freshwater resources per capita (cubic meters), GDP per capita (current US$), are the first 4 indicators which are the most terrible one in the 30 indicators.

Table 5 Top 5 index which makes the CSD of Mauritania worst

Index level	g
Secure Internet servers (per 1 million people)	0.1452
Permanent cropland (% of land area)	0.0884
Renewable internal freshwater resources per capita (cubic meters)	0.0725
GDP per capita (current US$)	0.0703
Labor force, total	0.0549

Based on this finding and other information we have, we can have an approximate know about the basic situation of Mauritania.

Area: 1,030,700 km2
Population: 3,537,368(2013 census)[5]
Climate: The tropical desert climate
The poverty rate: 42%
Life expectancy: 60
Adult illiteracy rate: 44.2%
Proportion of desertification: 2/3
Development level is very low and the structure is imbalanced.
Rich in oil, natural gas and iron ore resources.
It became a petroleum production country in Feb.2006

Figure 3 Location of Mauritania (Picture comes from www.geology.com)

4.2 Development strategy for 20 years made by the coordinate game principles

From 4.1, we can get that a task of top priority is increase the current development or to be directly, improve the GDP. What's more the coordination of the whole system need much concern too.

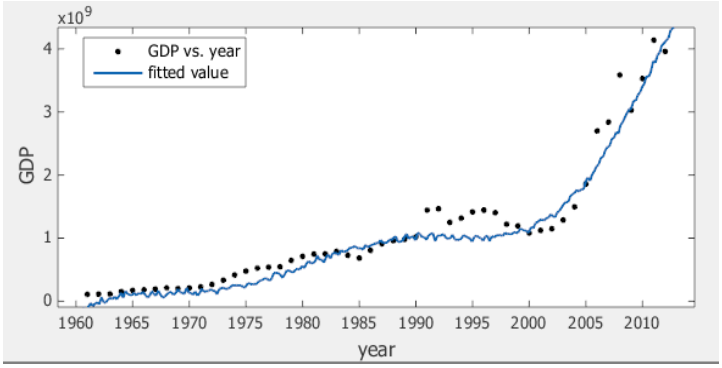
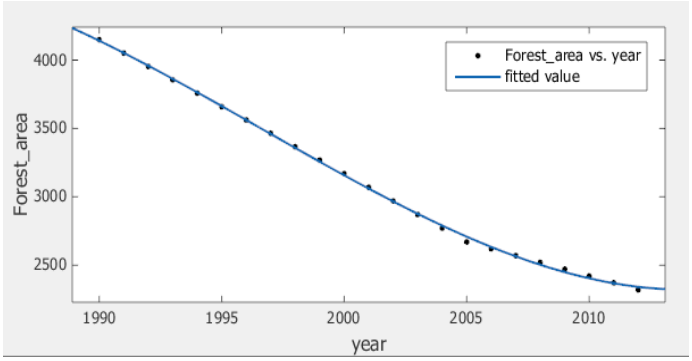


Figure 4 The data of GDP from 1960 to 2010



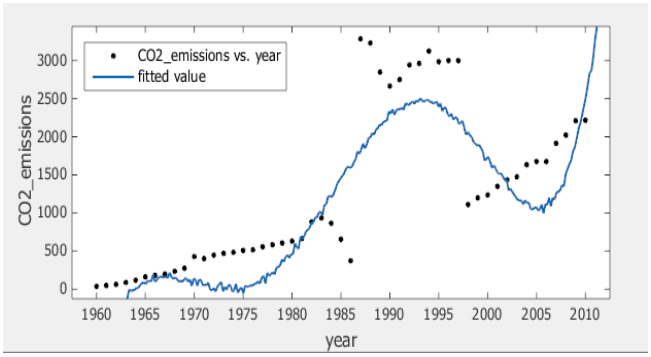


Figure 5 The data of CO_2 footprint from 1960 to 2010 (Left) and forest area from 1990 to 2012 (Right)

In the three figures above, the black point is the actual value and the blue string is the fitted one. Here we can find that with GDP improving, the CO_2 footprint also grow quickly after 2005. And the forest area decreased linearly. It will make the CL index grow and the CD index fall. The best solution to deal with this conflict related to the cooperative game model and the key is to find an equilibrium point.

Here we can use CO_2 emission stands for the economic level. The arable land area stands for the ecological level. Then there will be a table.

CO_2 emissions ↑	P ↓
Arable land ↑	S ↑

From section 3, when the system has the biggest balance degree, S will be equal to P.

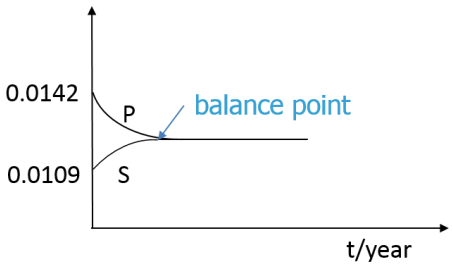


Figure 6 The balance point

Here we make 3 strategies. Make a list of their profit:

Table 6 Profit of every strategy

strategy	period	CL	CD	GR
Chase the GDP's growing speed. Consider the environment later.	short term	++	--	++
	long run	+	0	0
Put both economy and environment into consideration.	short term	+	+	+
	long run	++	+	+
Protect the environment seriously and sacrifice the economy development	short term	0	++	+
	long run	+	+	+

Here the "+" means this index is being better, the "-"means worse. The number of "+"or "-" reflect the degree.

- **Strategy 1** is what a reflection of the country present situation. Develop, then consider the environment. Many developed country walks this way. However, the ecological system in this country is very fragile for 2/3 earth is desert. So if we don't consider the environment at first. The result may be beyond redemption. What's more, the fact shows that the forest area is dropping which is very hard to renew.
- **Strategy 2** demands that we have to develop the economy without hurting the environment. It is a hard goal that only those countries with great technology and enough fund can make it. However if it doesn't have a tough on GR (growth rate), it is still accessible with the help of ICM.
- **Strategy 3** concentrated more on the environment. However, the sustainable development is human's development essentially. Only pay attention to environment without concerning people's feelings in not advisable.

Above all, **strategy 2** has the most beautiful expected revenue. So it's the best strategy in this coordinate game.

4.3 Specific plan and advice to ICM

In 4.1 we have known that the biggest problem of Mauritania is the current development level (CL) too and the coordinate degree (CD) too low. In 4.2 we make the strategy to put both economy and environment into consideration.

- **First** we have a program to improve GDP without hurt the environment too much. Considering that Mauritania is rich in oil, natural gas and iron ore resources. So the country can start from here. The old technique may cause serious pollution and our country doesn't have the best technique or the fund to buy these techniques. So a policy should be made to encourage foreign companies to exploit these resources and make up our own industry system.

- **Second** develop ecological agriculture.

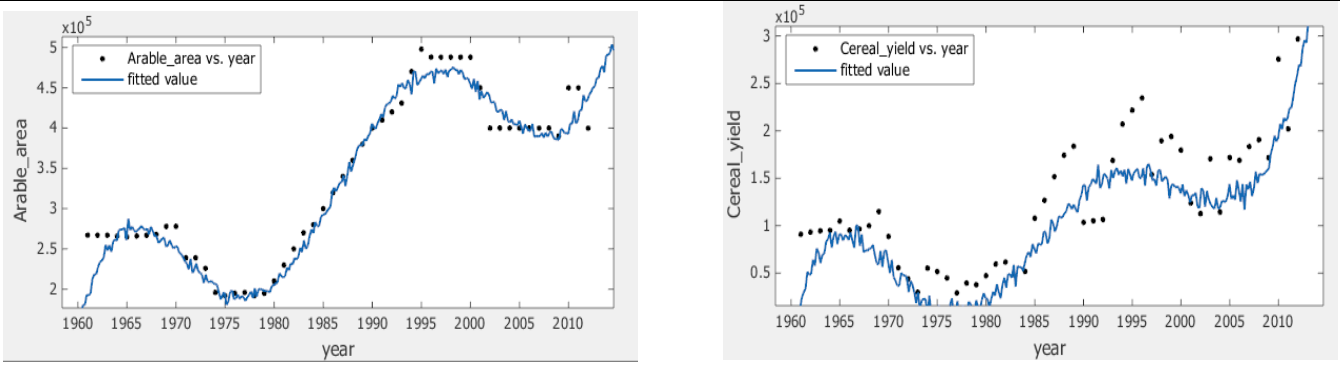


Figure 7 the arable area (Left) and cereal yield (Right) from 1960 to 2010

From figure 7 we can see that the cereal yield changes follow the arable area's change by the large. However, it is just a tendency. From 1980 to 1995 the arable area changed a lot but the cereal yield changed not as large as the arable area. What's more, the cereal yield differs a lot in different years even the arable areas are same. Thus we can conclude that the agriculture is unadvanced. The arable area has the tendency to become larger with the forest area is reducing. It is better to improve the cereal yield by using high technology such as sprinkling irrigation, drip irrigation and rainwater collection system.

- **Third**, promote education. The former two method both have a great demand of technology. In 21st century, knowledge is strength. 44.2% adults of Mauritania are illiterates. This is badly needed to change. The most useful method is making the chance to get educated free. This need a fund support.

Table 7 the effect our plan have on(CL,CD,GR)

	Industry	Agriculture	Education
	develop oil	ecological agriculture	compulsory education
CL	+	+	0(+)
CD	−	+	0(+)
GR	++	+	0(+)

From above form, we can find that industry makes CD decrease and ecological agriculture makes CD increase. They have a coordinate relationship which is just the conclusion of the coordinate game principles.

The value of education "0(+)" means that education may have no effect on the three factors now, but it will benefit them later.

ICM has 3 roles to play:

I. Be an investor, invest their money on the countries' sustainable development establishment.

II. Be an introducer, introduce this country to the world, attract more countries and companies come here to invest.

III. Be a sponsor, help support the technique, fund and food Mauritania wants if necessary.

5. FUTURE PREDICTION OF OUR COUNTRY

In order to demonstrate that our measures are effective. We need to make two predictions. **Prediction A** based on the current situation, it predict what the future will be according to the existing development pattern. **Prediction B** takes our measures and the random emergency into consideration. At last, the effect should be shown by the contrast of CSD and several specific index of **Prediction A, B** and the current situation.

5.1 The grey prediction model[6] for Prediction A based on the current situation

Grey prediction is a method to predict the system containing uncertain factors. The original data of the chosen index. The step of this system is:

Step 1

Find out the system factors' difference in the development trend.

Step 2.

Association analysis.

Step 3

Create a data sequence with strong regularity

Step 4

Write the differential equation model and predict the future development trend.

The original data of the chosen index

$$X^{(0)} = \left\{ x_1^{(0)},\ x_2^{(0)}, \cdots x_{n\text{-}1}^{(0)},\ x_n^{(0)} \right\} \tag{18}$$

$x_j^{(0)}$ means the original data in Year j .

$$X^{(1)} = \left\{ x_1^{(1)},\ x_2^{(1)}, \cdots x_{n\text{-}1}^{(1)},\ x_n^{(1)} \right\} \tag{19}$$

$$x_k^{(1)} = \sum_{i=1}^{k} x_i^{(0)} \tag{20}$$

$$k = 1, 2, 3 \cdots n \tag{21}$$

As close to the mean generation on it

$$z_k^{(1)} = 0.5 x_k^{(1)} + 0.5 x_{k\text{-}1}^{(1)} \tag{22}$$

Construct the sum matrix B and constant Y

$$Y = \begin{bmatrix} x_2^{(0)} \\ x_3^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix} \qquad B = \begin{bmatrix} -z_2^{(1)} & 1 \\ -z_3^{(1)} & 1 \\ \vdots & \vdots \\ -z_n^{(1)} & 1 \end{bmatrix} \tag{23}$$

Make the Grey Differential Equation white

$$x_k^{(0)} + a \times z_k^{(1)} = b \tag{24}$$

$$\frac{dx^{(1)}}{dt} + a \times x^{(1)} = b \tag{25}$$

The answer

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B \cdot B^T)^{-1} B^T Y \tag{26}$$

$$x_{k+1}^{(1)} = \left[x_1^{(0)} - \frac{b}{a} \right] e^{-a \times k} + \frac{b}{a} \tag{27}$$

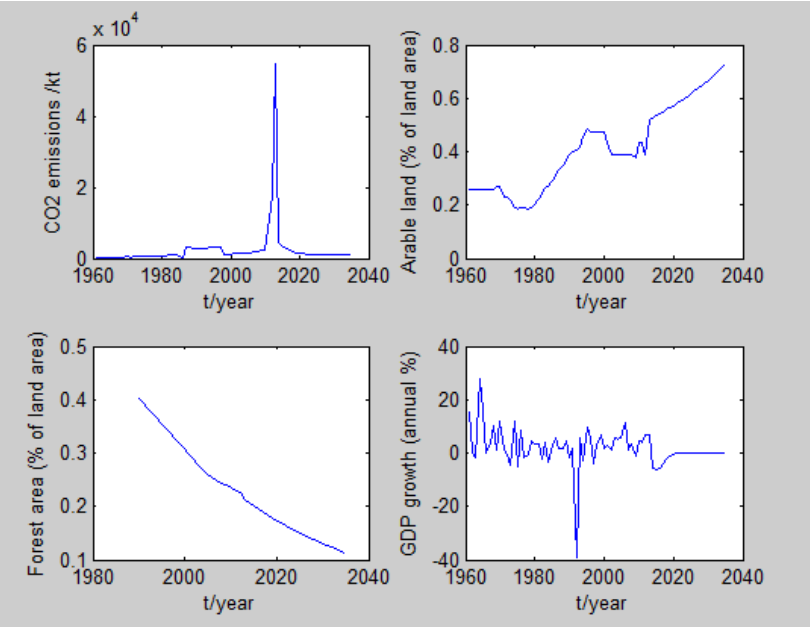


Figure 8 Four major index's change

Table 8 Result of Predication A

Mauritania	P	S	R	CL	CD	GR	CSD
2011	0. 0142	0. 0109	0. 0588	0. 0839	0. 3665	0. 0034	0. 0872
2035	0. 0198 ↑	0. 0180 ↑	0. 0550 ↓	0. 0928 ↑	0. 2149 ↓	0. 0023 ↓	0. 1841 ↑

From the table we can see that Mauritania will still be a very serious unsustainable country if it develop alone its original path.

5.2 Markov - chain prediction method[7] for Prediction B which takes our measures and the random emergency into consideration

As a quantitative description of **a series of random events dynamic relationship**, which has important application in natural science, engineering science and Social Sciences in various fields. With this theory ,we can do the analysis for most process and make predication and decisions.

In the Markov analysis, he introduced the concept of "state transition". One step transition probability represents the current in the state of I, the next step is transferred to the state probability of j, denoted by p_{ij}

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots \\ P_{21} & P_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{28}$$

N step transition probability represents the current in the state of I, step n is transferred to the state probability, J wrote: Pij (n).

$$P = \begin{bmatrix} P_{11}(n) & P_{12}(n) & \cdots \\ P_{21}(n) & P_{22}(n) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{29}$$

Markov chain model can meet the most homogeneous, therefore, we only need to get the one-step transition probability matrix, multiplication, using Matlab software to multiply the matrix, predict the probability of each state in the future.

5.2.1 According to the analysis of the previous data, we divide the speed of the GDP development rate into 4 condition

Table 9 The classify of GDP growth rate

Grade	Recessionary	Stable	Incremental	Rapid growth
Growth rate of GDP(%)	＜-2	[-2, 2]	[2, 10]	＞10
Condition	1	2	3	4

5.2.2 Calculate the transition probability and transition probability matrix

In the systems we studied, the probability distribution is unknown, we need to estimate the frequency of state transfer. The following table is GDP development level of state transfer statistics from 1957 to 2013.

Table 10 The state transmission collection from 1957 to 2013

	Condition 1	Condition 2	Condition 3	Condition 4	Sum
Condition 1	1	1	6	1	9
Condition 2	2	5	7	2	16
Condition 3	4	6	10	2	22
Condition 4	1	5	1	1	8

From table we can get the transmission matrix

$$P=\begin{bmatrix} 0.1111 & 0.1111 & 0.6667 & 0.1111 \\ 0.1250 & 0.3125 & 0.4375 & 0.1250 \\ 0.1818 & 0.2727 & 0.4545 & 0.0910 \\ 0.1250 & 0.6250 & 0.1250 & 0.1250 \end{bmatrix} \tag{30}$$

Positive “state transmission” martix

$$H=\begin{bmatrix} 0.1111 & 0.6667 & 0.1111 \\ & 0.4375 & 0.1250 \\ & & 0.0910 \\ & & \end{bmatrix} \tag{31}$$

Negative “state transmission” martix

$$L=\begin{bmatrix} & & \\ 0.1250 & & \\ 0.1818 & 0.2727 & \\ 0.1250 & 0.6250 & 0.1250 \end{bmatrix} \tag{32}$$

Set a random factor σ (σ may be caused by climate change, development aid, foreign investment, natural disasters, and government instability and other factors, it can be positive or negative)

$$\sigma=\begin{cases} (0,1), & positive \\ 0, & independent \\ (-1,0), & negative \end{cases} \tag{33}$$

$$P^{'}=\begin{cases}\frac{1}{\sigma}H+\sigma L, & \sigma>0\\ H+L, & \sigma=0\\ \sigma H+\frac{1}{\sigma}L, & \sigma<0\end{cases} \tag{34}$$

Normalize the row vectors

$$P_{ij}=\frac{P_{ij}^{'}}{\sum_{j=1}^{n}P_{ij}^{'}} \tag{35}$$

P_{ij} can reflect the effect taken by the random factor.

Table 11 The final result

Mauritania	P	S	R	CL	CD	GR	CSD
2035(A)	0.0198	0.018	0.055	0.0928	0.2149	0.0023	0.1841
2035(B best)	0.0398 ↑	0.0310 ↑	0.0350 ↓	0.1128 ↑	0.0089 ↓	0.0123 ↑	0.3557 ↑
2035(B worst)	0.0401 ↑	0.0290 ↑	0.0330 ↓	0.1021 ↑	0.0155 ↓	0.0121 ↑	0.3127 ↑

A stands for Prediction A and B stands for Prediction B.

From this table we can see, put the random factors into consideration, the predicting CSD will have a fluctuation between 0.3127 and 0.3557. Nevertheless, CSD of Prediction B still much bigger than that of Prediction A. So we can say our plan is successful.

5.3 The best policy and best program

Best policy: the balanced development strategy which takes both economy and ecology into consideration.

Best program: Promote education. This program has no bad effect but all good income.

6. DISCUSSION AND CONCLUSION

6.1 Strengths

✧ **The indexes are comprehensive**

In order to describe the capacity of sustainable development accurately, our model selects 30 indexes from thousands and thousands of the data. And they can reflect the capacity of sustainable development comprehensive. And they could stand for other data or could reflect other data.

✧ **The stability of model is excellent**

Because we select many indexes and the model is advanced, through our test, we find that the model we build is steady; even though a little data is wrong we collect, the comprehensive ranking of country is not changed. So the model could endure a little mistake.

✧ **Standing on the shoulders of giants**

Before we build model, we research a lot of others' achievements about sustainable development. We find that there are many experiences our model could use for reference. And after we analysis others' study, we find that they have done many job on it, and some of them have been high up in the pictures. So the model we build is more scientific.

✧ **The model is global**

The model we build not only could analyze low income country, but also could help developed

country draw up plans. And our model considers many stochastic events, so it is more adaptable.

6.2 Weakness

✧ **The formulas are too many**

Because a country wants to develop, we should consider many factors, and they are complicated. So we have to build many formulas to find the relationship between many factors. But it makes our model complex. And we have to spend much time to calculate.

✧ **A big need of data**

In order to analyze capacity of sustainable development comprehensive, we need much data to calculate. Though when we select data, we consider availability of the data, sometime a litter data cannot find, because of data deficiency.

6.3 Sensitivity

Our model is not very sensitive about the data. On the one hand, we have 30 indicators, according to table 2, every indicator's weigh is very low, so it won't have a big affection on the CSD value.

On the other hand, we set the value of CO_2 emissions (one of index levels) of all countries to zero, then we deal with the new data.

Table 12 The CSD's change rates of LDC and 11 typical countries

Afghanistan	−2. 60%	Gambia	−2. 46%	Rwanda	−1. 04%
America	0. 98%	Germany	0. 56%	Samoa	−0. 79%
Angola	−1. 04%	Guinea	−1. 91%	Sao Tome and Principe	−0. 22%
Bangladesh	−0. 84%	Guinea-Bissau	−1. 10%	Senegal	−1. 43%
Benin	−1. 56%	Haiti	−1. 43%	Sierra Leone	−1. 64%
Bhutan	−0. 15%	India	0. 13%	Solomon Islands	−0. 81%
Burkina Faso	−2. 22%	Japan	0. 37%	Somalia	−4. 88%
Burundi	−0. 90%	Kiribati	−0. 31%	South Korea	0. 09%
Cambodia	−1. 23%	Laos	−0. 64%	Sudan	−1. 80%
Central African Republic	−1. 79%	Lesotho	−1. 55%	Sweden	0. 48%
Chad	−12. 57%	Liberia	−1. 74%	Swiss	0. 45%
China	1. 43%	Madagascar	−2. 19%	Tanzania	−1. 31%
Comoros	−0. 53%	Malawi	−1. 29%	Timor-Leste	−1. 64%
Democratic Republic of the Congo	−1. 55%	Mali	−2. 53%	Togo	−1. 49%
Djibouti	−1. 87%	Mauritania	−6. 56%	Tuvalu	0. 02%
England	0. 59%	Mozambique	−1. 88%	Uganda	−1. 10%
Equatorial Guinea	−0. 26%	Myanmar	−0. 78%	Vanuatu	−1. 18%
Eritrea	−2. 58%	Nepal	−1. 32%	Yemen	−3. 82%
Ethiopia	−1. 56%	Niger	−1. 86%	Zambia	−0. 95%
France	0. 26%	Russia	−0. 62%		

6.3 Conclusion

- Sustainable development is a complex goal, one index's improvement may not have much improve of the CSD index.
- Every index is independent, it's increase may cause another index's decrease.

- Most countries in LDC have a serious sustainable development situation now for only 4 countries are critical sustainable. They need help a lot
- The best development pattern for LDC is the strategy 2 which takes both economy and ecology into consideration.
- Education is a long term investment with a great repay for the whole nation.
- Fund and technique is the direct way to help LDC get rid of unstainable.

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