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Problem Chosen

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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

The Better Traffic Rule

Contents

1 Introduction

Generally, overtaking takes place when a car is driven at a higher speed than the car before it. In that way, the car can't move at the speed it expected, so it will try to exceed the car before which academic term is overtaking. The drivers can drive their cars at a higher speed on the highway in contrast to driving on the normal roads. However the phenomenon of overtaking is ubiquity as a result of the personal factors of different drivers and the wide range of speed limits.

It is dangerous to overtake cars for lots of factors affect the process like the way of overtaking, the reaction time of driver and environment factors, etc. Also overtaking will affect the traffic on the highway because of it need time to adjust when a car exceeds another through which might lead to partial blockage and even traffic jam.

In countries where driving automobile on the right is the rule (that is, USA, China and most other countries except for Great Britain Australia and some former British colonies), multi-lane freeways often employ a rule that requires drivers to driver in the right-most lane unless they are passing another vehicle to ensure the safety of drivers driving on the freeway and the traffic of the freeway. As the tradition rule, it is important to improve safety and traffic.

Things change as time passes by. Nowadays there are more and more cars but the number of roads and highways didn't catch the pace. So it is common to see the traffic jam on the roads. It inspires us start to think about: How does 'The Keep-Right-Except-To-Pass-Rule' influence the traffic on the road? What does the speed limit do? Is there any other improved rule or alternative rules that may cause better influence? What makes difference of driving on the right and driving on the left?

In order to address problems above, we conclude four sub-problems to tackle in our paper:

- Model building of the keep right except to pass rule to simulate the traffic. Find out the relation between the safety and flow, the role of speed limit and other factors that may be important.
- Finding out improved rule or alternative rules that may have better influence.
- Model building and analysis the influence of rules in the situation of driving automobile on the left and find out what change should be make.
- Model building of the intelligent transportation system and analysis what changes it brings to the traffic.

2 Nomenclatures

$\rho_{Vehicle}$	
$\Delta X_{headway}$	
$t_{headway}$	
Δv	
S	
MSS	
Q	
S_1	
S_2	
a	
t_a	
$h_{\min}$	
v_f	
$v_{\max}$	
h_s	
κ	
V	
p_n	
η_{rule1}	
$v_{overtake}$	
v_{change}	
$\vec{a}_a$	
$\vec{a}_e$	
$\vec{a}_r$	
$\vec{a}_C$	
F_C	
G	
v_i	
w_{ij}	
d_{ij}	

3 Model One：Relation of traffic parameters and indexes

3.1 Introduction

In order to find out how the overtaking rules affect the traffic, we study some indexes and parameters of traffic including safety, traffic flow, limiting of velocity and etc. Then, we model and find the relationships between these indexes and parameters. Through the equations that represent the relationships of these indexes and parameters, we can obtain how they affect the traffic. In other words: the effect is based on the relationships of overtaking rules and those parameters and indexes.

3.2 Terminology

After observing the performance of the cars on the highway, we make an analogy with traffic flow and liquid flows through pipes. The flow velocity of liquid is equal to the traffic flow and the quantity of flow is equal to the amount of cars passing through unit length during unit time. Then we build a simple model and obtain the relationship among time, traffic flow and the amount of cars passing through according the formula of flow:

$$Q = \mathrm{v} \cdot t$$

Through further study, we find out that the degree of congestion of road might be influenced by the amount of cars on the road. So we bring in a new variable: traffic density $\rho_{vehicle}$, which represents the amount of cars on the unit length of the road. In this paper, density is averaged according to time. So it can be obtained easily through measuring the average velocity and flow rate:

$$\rho_{vehicle} = \frac{n}{l}$$

In which $\rho_{vehicle}$ represents traffic density (pcu/km)；

n stands for the amount of vehicles (pcu)；

l means length of freeway(or roadway) (km).

In the further research of the factors that influence the degree of traffic congestion, we find that headway is related to the density of traffic flow directly. Headway means the distance of the heads of adjacent cars on the lane.

In order to analyze the relationship between the degree of traffic congestion and time, we bring in a new variable: time headway[1], which means the time required for a car catching up with the car ahead:

$$\Delta X_{headway} = \frac{1000}{\rho_{vehicle}}, \Delta t_{headway} = \frac{3.6 \Delta X_{headway}}{\Delta v}.$$

In which:

$\Delta X_{headway}$ stands for space the space between two adjacent vehicles.(m)

$\Delta t_{headway}$ means the time that the vehicle takes to go through the space headway(s)

Δv represents the difference in the velocity of the front car and back car(km/h).

3.3 Assumptions

- Drivers are not drink-driving or tired-driving.
- Drivers obey the keep right except to pass rule.
- One lane of the road can only contain one car in the cross section.
- The factors of weather, the road surface, the car condition and etc are not considered.

3.4 The tradeoffs between traffic flow and safety

3.3.1 The quantification of safety

To quantitatively analyze the relationship between safety and flow, we quantified the index of safety. The safety means the degree of safe when cars driving on the freeway. It depend on many factors like the reaction time of different people, the dynamic coefficient of friction μ which represents the coarseness of the road surface, the quality of cars and the distant of cars etc. So we define the index of safety S has following some attributes:

$$S = f\left(t, \mu, m, x...\right)$$

In which:

t represents the reaction time of human in emergency situation,

μ represents the dynamic coefficient of friction,

m stands for the total mass of the car and people in the car,

x is the distance of two cars.

3.3.2 Determination of the safety distance

In order to simplify the model, we use the minimum headway MSS [2]to represent safety which the greater MSS is, the more safe the safety will be.

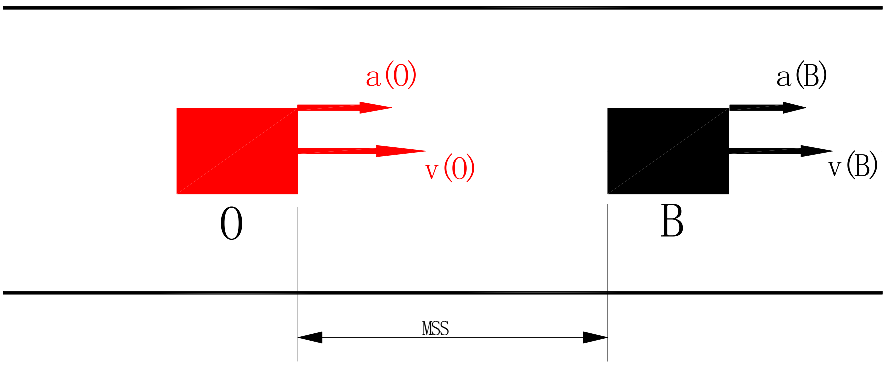


Fig. 1.最小安全距离示意图

Assume the car labeled O is the one to overtake and the car labeled B is the one to be overtaken. Car O will not have enough space for driving freely and achieve the speed it expected when it moves at a higher speed than car B before it. So car O will try to overtake car B. The first step to overtake the car before is changing the lane, and then it accelerates on the other lane, passes the car before. Finally it returns to the initial lane. It seem easy to make overtake come true, but each step of overtake is full of danger because you will never know what the driver in the car before think of. To avoid the danger in the process of overtake, the distance of cars should no less than MSS which we defined above means the minimum headway. According to the lane-changing model proposed by UNIVERSITY OF CALIFORNIA, the minimum headway should be:

$$MSS = Max\left\{\left[\int_0^t\int_0^\lambda a_O(\tau) - a_B dx d\lambda + (v_O(0) - v_B(0))t\right], 0\right\}$$

In which,

a_O, a_B stands for the acceleration of car O and car B,

v_o, v_B represents the velocity of car O and car B.

When car O and car B are moving at a constant speed, the minimum headway has a simple form as:

$$MSS = Max\{[(v_o(0) - v_B(0))t], 0\}.$$

3.3.3 Analysis of flow and safety

In order to analyze the relationship of flow and safety, we establish set of equations. First of all, we collect data of the velocity of cars passing a section of the freeway we select. Then we calculate the average velocity $\bar{v}$. We can obtain average time that the cars spend to pass the section of the freeway through the following equation:

$$\bar{t} = \frac{l}{\bar{v}}$$

In which:

l stands for the length of this section of freeway.

After that, according the relation of time and flow, we calculate the number of cars passing through the section of freeway during time $\bar{t}$ as following:

$$n = \bar{t} \cdot Q$$

Then the average length that each car occupies on the freeway should be:

$$\bar{l} = \frac{l}{n} = \frac{\sum l_{\text{car}} + l_{headway}}{n}$$

In which:

l_{car} represents the length of car itself,

$l_{headway}$ stands for distance between adjoin cars on the highway.

Since the length of car itself (about 2 to 4 meters) is too small compared to the headway of cars(about 100 meters or much), we decide to omit the length of car itself to simplify the model. In this way, we can get the approximate average length each car occupies on the freeway in the following form:

$$\bar{l} \approx \frac{l}{n} = \frac{\bar{v}}{Q}$$

We can draw the conclusion of tradeoffs between traffic flow and safety in the following formula:

$$MSS \le \bar{l} = \frac{\bar{v}}{Q}$$

It means when traffic is low, cars can move on the freeway freely to keep away from each other at a long length which represents high degree of safety. However, when traffic raise, cars have to drive closely to each other that means low degree of safety. In order to obtain high traffic flow and also the safety of drivers, we should limit the traffic flow through:

$$Q \le \frac{\bar{v}}{MSS}$$

3.4 The role of speed limit

In this part, we omit other factors and consider the relationship among limiting of velocity, load and traffic flow. We define the minimum velocity v_l and the maximum velocity v_h. To represent the actual situation of the velocity distribution when define the velocity distribution of each lane obey normal distribution. So when adjacent cars move at the maximum and the minimum velocity, the safety distance of the two cars should equal to MSS. In that way we can draw conclusion that MSS is relative to v_l and v_h in this condition as the formula shown below:

$$MSS_{\min} = f(v_h, v_l)$$

Then considering different degree of safety that required, we can obtain the minimum distance of adjacent cars in the following way:

$$l_{cars} = MSS_{\min} + const\ ,$$

Where constant $const$ is added to control the degree of safety.

Then we take a section of a highway lane into consideration, and we assume that there are n vehicles. Each of vehicle's speed is $\tilde{v}$, which represents value of expectation under the control of v_l , v_h and normal distribution. We define the interval time between the first car passes section P and the car labeled n passes section p as Δt . So L in the picture can calculate as:

$$L = \tilde{v} \cdot \Delta t$$

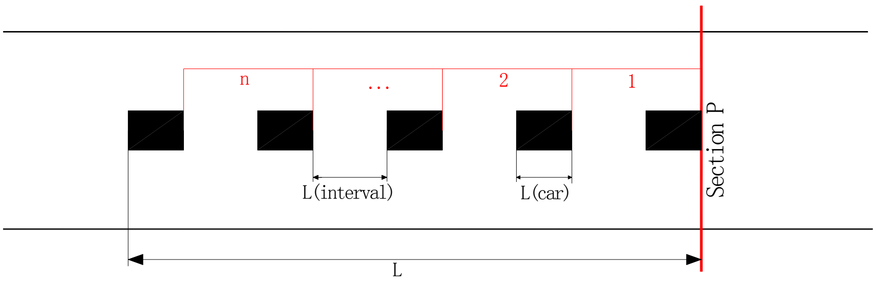


Fig. 2.单侧高速公路车流辆示意图

Firstly we calculate the traffic flow Q:

$$Q = \frac{n}{\Delta t}\ .$$

Next substitute $L = \tilde{v} \cdot \Delta t$ into the equation above, we can get:

$$Q = \frac{n}{\Delta t} = \frac{\tilde{v} \cdot \Delta t}{l_{cars} \cdot \Delta t} = \frac{\tilde{v}}{l_{cars}}\ .$$

Since :

$$\rho_{vehicle} = \frac{n}{l}\ .$$

So :

$$\rho_{vehicle} = \frac{n}{l} = \frac{\tilde{v} \cdot \Delta t}{l_{cars} \cdot \tilde{v} \cdot \Delta t} = \frac{1}{l_{cars}} = \frac{1}{MSS + const}\ .$$

Finally, we can draw the relation that:

$$\rho_{cehicle} \propto \frac{1}{MSS} \propto \frac{1}{f(v_h, v_l)}\ .$$

It means $\rho_{vehicel}$ is inversely proportional to the limiting of velocity. So if we want to ensure safety and traffic flow, theoretically we should set higher limiting of velocity on the freeway.

4 Model Two：How the rule affects traffic

4.1 Introduction

In our simulation model, we successfully emulate the change of the indexes and parameters in different situation of traffic. Now we will study whether the rule is able to promote the traffic flow or not. As we know, a change of a rule will change the operation mode, so it will influent the traffic condition. To Judge whether it can promote flow, we need to choose an object to make comparative analysis. We choose the operation mode which eliminated the rule as the object. That means the vehicle could overtake or change lane if they can promise their safety on two-lane freeway. In this paper we define the rule that keep on right except to pass as Rule1.

4.2 Assumption

- Assuming that there is no toll-gate in the road to affect traffic.
- Assuming that vehicle will accelerate when overtaking.
- Assuming that the freeway is straight, without any curve.
- Assuming that there is no car in front of the car which is overtaken.
- Assuming that the passing lane is absolutely safe, and no accident will happen.
- Assuming that the speed of the vehicle is under the speed limit.

4.3 Analysis

4.3.1 Constraints

There are many regulations when driving on the highway. Through carefully analyzing we determined the problem specifies the following constraints[3]:

1. The width of each lane is 3.5 meters;
2. The freeways are two-lane freeways and employ the keep right except to pass rule;
3. The mass of the car, the bus and the truck are 0.8 tons, 1.1tons and 1.8 tons respectively;
4. The expected velocity of the car is 50~110 km/h and to the bus and the truck it should be 50~90 km/h;
5. The distance of foresight ought to be 0~250 meters;

6. The speed parameters of the car overtake and the car is overtaken.

Table 1.参数限制

	Overtake	Be overtaken
Maximum decelerationm/s^2	-4	-3
-1m/s^2(m)	100	100
可接受的减速度 m/s^2	-1	-1

4.3.2 Analysis of overtaking

The performance of overtaking is set as a model. We simplify the passing way as overtaking in the condition of constant speed and accelerate[4].

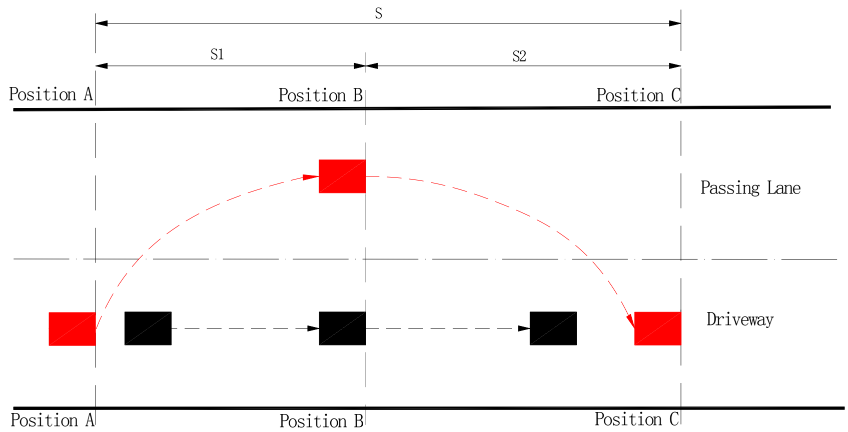


Fig. 3.passing from the left side

The red square stands for vehicle in high velocity. And the black square stands for vehicle in low velocity. The position A is location of the beginning of overtaking. Then red vehicle catch up with black vehicle at position B. Finally red vehicle completes the process of overtaking and return to their former travel lane at position C.
In reality situations,we assume performance of overtaking is sum of two conditions.

1) acceleration

The red vehicle change the lane and accelerate to overtake black vehicle,the process from position A to position B.(a as per acceleration, t_a as per time of acceleration's process)

$$S_1 = \frac{\dot{x}_R(t)}{3.6} \cdot t_a + \frac{1}{2} a t_a^2$$

in which $\dot{x}_R(t)$ velocity of red vehicle at time t [km/h];

a acceleration[m/s^2];

t_a time of acceleration[s] $t_a = \frac{\dot{x}_R(t+t_a) - \dot{x}_R(t)}{3.6 \cdot a}$, if $\dot{x}_R(t+t_a) > \dot{x}_R(\max)$;

then $t_a = \frac{\dot{x}_R(\max) - \dot{x}_R(t)}{3.6 \cdot a}$

At the end of acceleration,the velocity of red vehicle is a constant.

$$\frac{\dot{x}_R(t+t_a)}{3.6} = \frac{\dot{x}_R(t)}{3.6} + a t_a$$

2) constant speed

At time $t+t_a$,the red vehicle drives in constant speed and return to the return to the former travel lane.(t_b as per time of driving in a constant velocity)

$$S_2 = \frac{\dot{x}_R(t+t_a)}{3.6} \cdot t_b,$$

4.3.3 Analysis of time headway

The time headway can be worked out by the function:

$$h_{\min} = t_1 + t_2 + t_3$$

in which $h_{\min}$: minimum time headway[s];

t_1 : time of people judgment (about 1s);

t_2 : time Variable deceleration time[s];

t_3 : time shortest stopped time [s];

L:minimum headway for stopping,

V:velocity before stopping.

Here the function can turn to:

$$h_{\min} = 1 + t_2 + \frac{L}{v}$$

$$t_2 = e^{-a(v_f - v)} \frac{V_f}{g\mu}$$

in which v_f the average velocity of minibus on the roadway;

μ coefficient of friction(value in this section is about 0.35);

v velocity of vehicle [m/s]（米/秒;

a unknown coefficient,

$$a = \sigma / v_f$$

σ average deviation of velocity's free stream velocity[m/s]

in a word ,the function of headway:

$$h_{\min} = 1 + e^{-a(v_f - v)} \frac{v_f}{g\mu} + \frac{L}{v}.$$

4.3.4 Vehicle following model

In the process of overtaking,the lateral clearance width produced by different transverse location of front vehicle would generate the different nerve stimulation of overtaking and influence the driving behaviors. Base on this theory,we develop a model that talk about the relation of expectation of overtaking and lateral separation.we search the model's condition of linear stability by the linear stability theory[5]. In the periodic boundary condition,we simulate the model by the data,analyze the key property of model and verify the model's rationality.

The OV model's differential equation is raised by Bando in 1995:

$$\frac{dv_n(t)}{dt} = \alpha[V(\Delta x_n(t)) - v_n(t)]$$
$$V((\Delta x_n(t))) = 0.5v_{\max}[\tanh \Delta x_n(t) - h_s] + \tanh(h_s)$$
$$\Delta x_n(t) = x_{n+1}(t) - x_n(t),$$

In which

α ---sensitivity coefficient

$V(\Delta x_n(t))$ ---function of optimized velocity

$v_n(t)$ ---velocity of No.n vehicle in time t

$\Delta x_n(t)$ ---No.n vehicle's time headway

$x_n(t)$ ---location of No.n vehicle in time

$v_{\max}$ ----maximum optimized velocity of vehicle

h_s ----safe interval

The performance of car-following at present is affected by front vehicle. Due to the lateral separation that is influenced by the second front vehicle. And the two kinds of influence's distribution is decided by the lateral separation parameters.

$$\begin{cases} \frac{dv_n(t)}{dt} = a\{V[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)] - v_n(t)\} + \kappa G(\Delta v_{n,n+1}(t), \Delta v_{n,n+2}(t)) \\ V[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)] = V[(1-p_n)\Delta x_{n,n+1}(t) + p_n \Delta x_{n,n+2}(t)] \\ G(\Delta v_{n,n+1}(t), \Delta v_{n,n+2}(t)) = (1-p_n)\Delta v_{n,n+1}(t) + p_{n,n+2}(t) \end{cases}.$$

In which κ ---sensitivity coefficient of velocity difference

$\Delta x_{n,n+1}(t)$ ---headway between No.n vehicle and front vehicle in time t

$\Delta x_{n,n+2}(t)$ ---headway between No.n vehicle and second front vehicle in time t

V ---function of optimized velocity

p_n ---horizontal separation parameters: $p_n = \frac{LS_n}{LS_{\max}}$

LS_n ---horizontal separation to the front vehicle

$LS_{\max}$ ---maximum horizontal separation

When vehicle's horizontal separation is maximum,the vehicle has been the different lanes. In this condition, the front vehicle do not influences the vehicle. Therefore, the value of $LS_{\max}$ is the wide of normal lane---3.5m.

4.4 Model Building

This process can be described by the following flow chart:

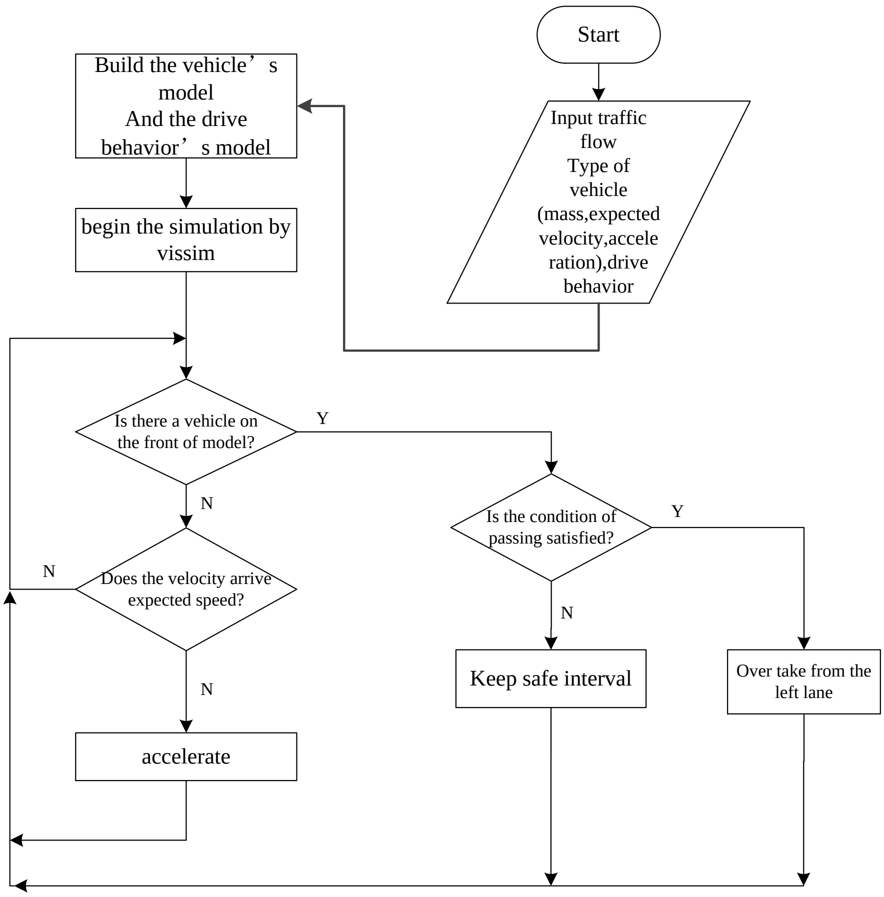


Fig. 4. Algorithm Flowchart for

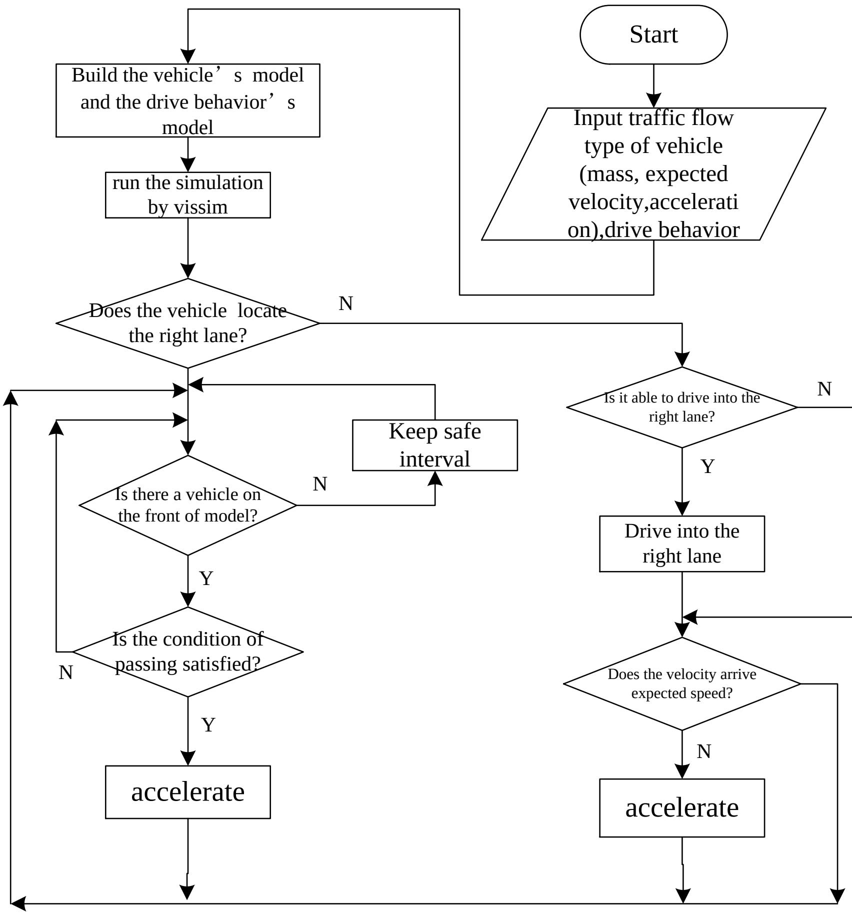


Fig. 5.

4.5 Simulation and conclusion

Through the analysis above, we selected some main properties and parameters to describe the phenomenon of overtake and obtain the relationships between the properties and parameters through model building. In this section we use traffic simulation software VISSIM to simulate the process of overtake with the method of control variable and verify whether or not our model is effective enough in evaluate the traffic.

4.5.1 The setting of parameters

To ensure the authenticity of simulation, we firstly set the characteristic of velocity and acceleration.

- **velocity characteristic:**

Table 2.

	Minimum velocity	Maximum velocity
car	50	110
HGV	50	90
BUS	50	90

The desired speed of car is set to 100[km/h] and the desired speed of HGV and BUS is set to 80[km/h]. The velocity characteristic has been shown as Fig. 5 and Fig. 6.

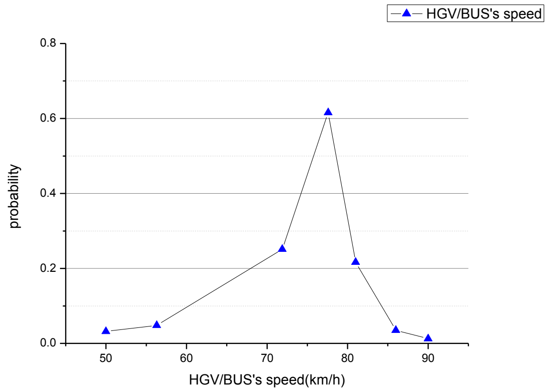


Fig. 6.大货车速度特性曲线

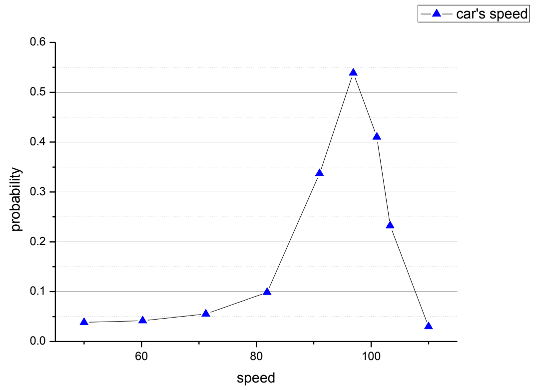


Fig. 7.小汽车速度特性曲线

- **acceleration characteristics:**

In consideration of the difference of vehicle type and vehicle mass, also the interaction of mass and acceleration, we set the vehicle type and the mass of vehicle as the following table:

Table 3.

Vehicle type	Mass of vehicle [t]
car	1.1~1.5
HGV	6~10
BUS	6~8

Through simulation we can obtain the performance of acceleration in the different velocity as shown in the following figure:

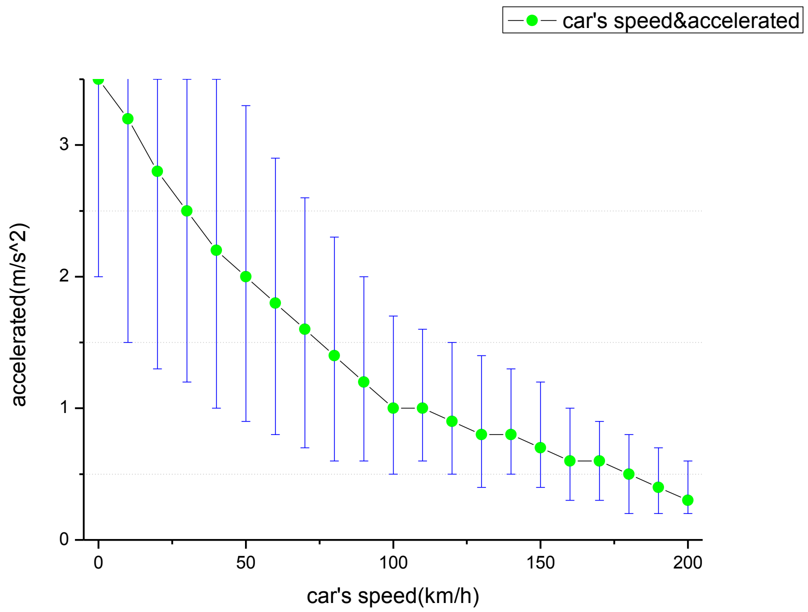
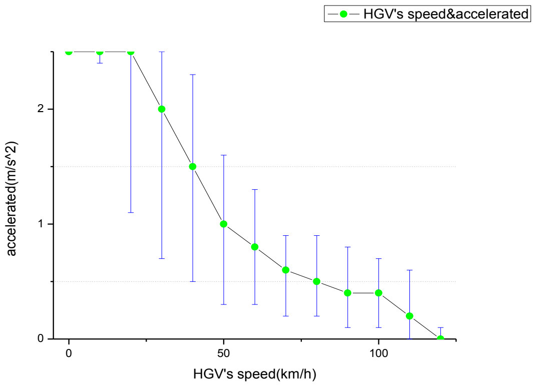


Fig. 8.小汽车加速表现图



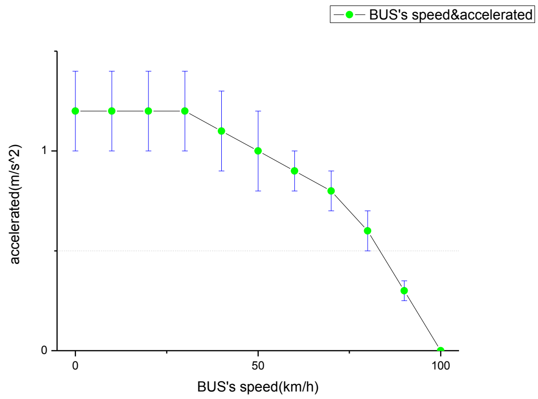


Fig. 9.货车加速表现图 **Fig. 10.**客车加速表现图

4.5.2 Analysis and Conclusion

- **simulation of velocity and traffic flow(simulation 1)**

The vehicle model is put into the freeway model, and starts the simulation. Considering about the relation of velocity and lane, we put the velocity collection on the left lane to get obvious difference of velocity. We randomly select 150 vehicles as the sample vehicles. And velocity collection collects the velocity of sample vehicles in the same location. We analyze the relation of velocity and traffic flow. The result of simulation has been shown as figure:

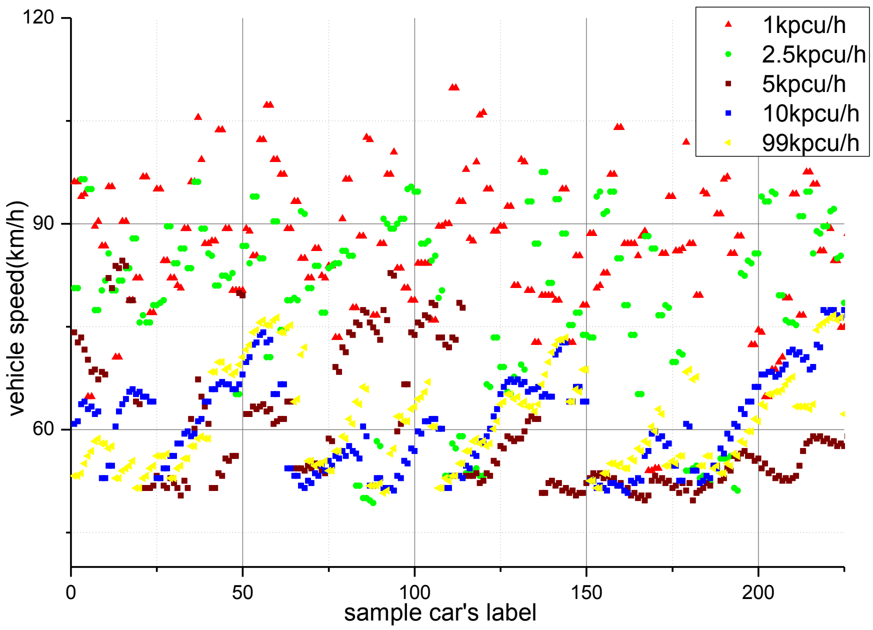


Fig. 11.车流量与速度关系图

In five different conditions of traffic flow, we average the velocity of sample vehicles and summarize in the following table:

Table 4.

Traffic flow [pcu/h]	Average velocity of sample vehicles [km/h]
1000pcu/h	86.95578947km/h
2500pcu/h	78.65684211km/h
5000pcu/h	60.18157895km/h
10000pcu/h	61.09736842km/h
99000pcu/h	61.17947368km/h

Through the setting of other different values of traffic flow and the process of simulation, we obtain the relation between traffic flow and velocity as shown in the following picture:

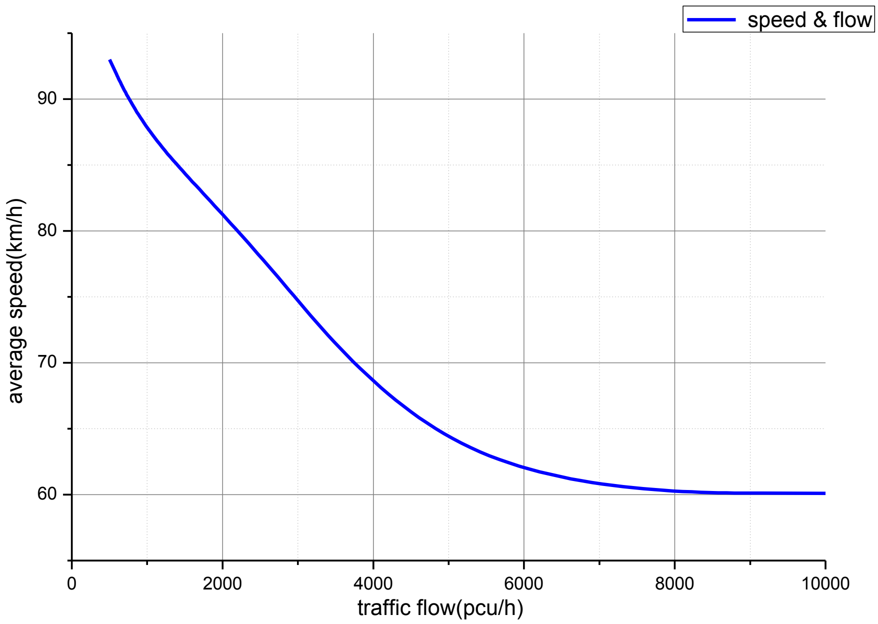


Fig. 12.车流量与速度关系图

Conclusion:

According to the relation of the velocity and traffic flow, it is not difficult to discover the velocity of vehicle declines with the raise of traffic flow. In other words, if we promote the average velocity by the different rules, the traffic flow in that condition will decline. And the method of promote the average velocity will promote the better traffic flow.

- **Simulation model of freeway rules(simulation 2)**

In this section, we analyze how the traffic rules affect the parameters above. The normal rule is named rule1. And the method of passing without rules is named free rule. In the free rule, the fast vehicle on the left or right lane can passing the slow vehicle like the picture below:

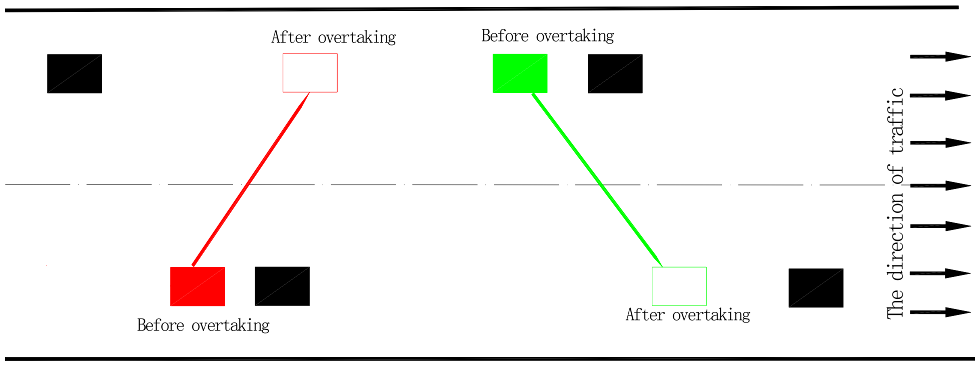


Fig. 13.自由超车示意图

After building the simulation model of freeway, we analyze the simulation result in the different rules. To exclude the influence of different traffic flow and different type of vehicle we establish the simple simulation model 2. The traffic flow has been set as 5000puc/h and consists of the cars only. The simulation model 2 emulates the traffic situation in the condition of rule1and the free rule.

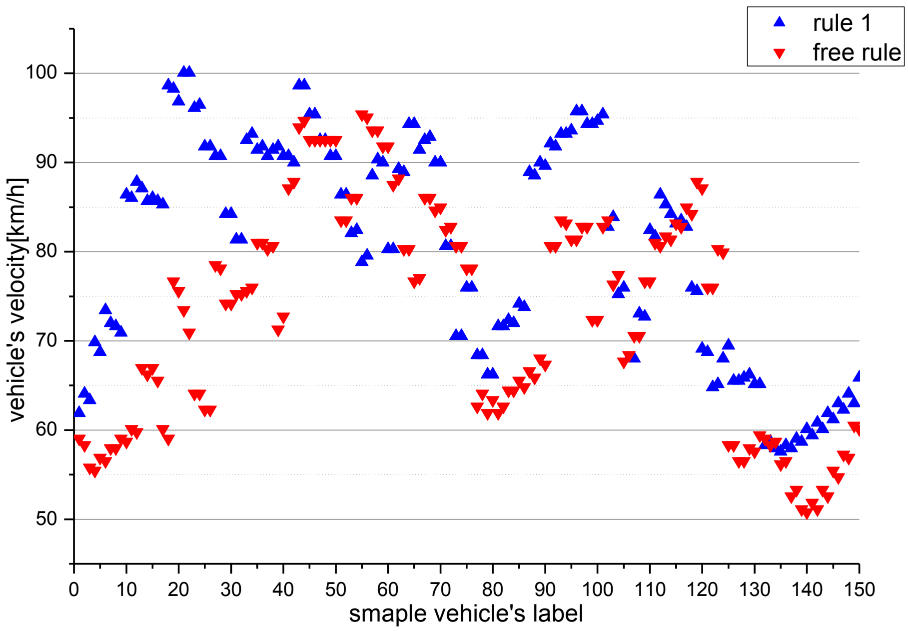


Fig. 14.不同规定下车辆速度仿真图

The average velocity is worked out by the sample vehicle’s velocity.

$$\begin{cases} v_{rule1} = 80.1696 km/h \\ v_{free} = 72.6264 km/h \end{cases}.$$

Conclusion:

On the basis analysis of the result, the average velocity is promoted. And the better traffic flow is promoted as well. Therefore, the normal rule is effective in the condition of 5000pcu/h and without the influence of vehicle's type.

- **Simulation model in complex traffic flow(simulation 3)**

Considering of the influence of HGV/BUS, we complicate the simulation model and establish the simulation model 3. Simulation model emulate the traffic situation in the condition of that the HGV/BUS drive on the freeway. The simulation result has been shown as shown below:

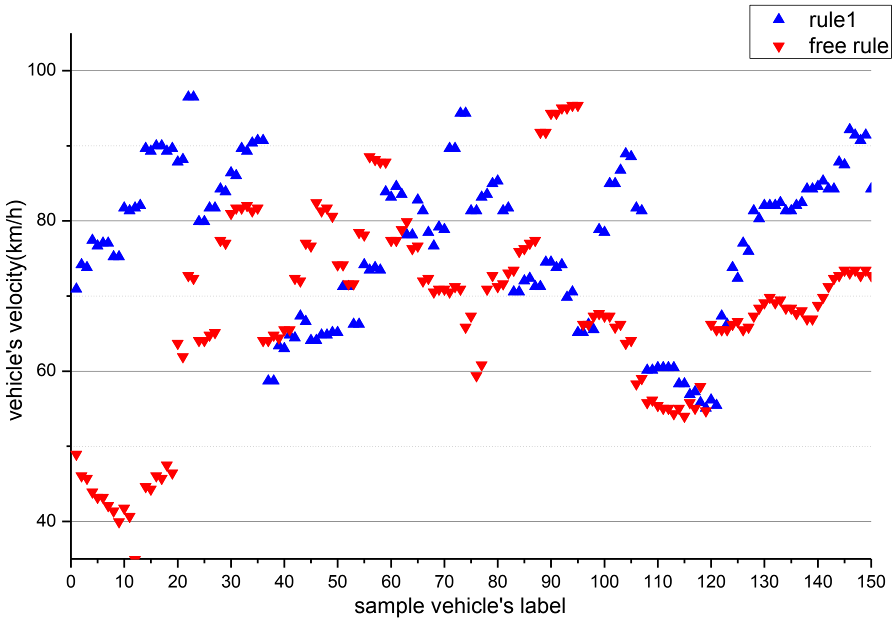


Fig. 15.不同规定下多种车辆速度仿真图

We average the velocity of sample vehicles:

$$\begin{cases} v_{rule1} = 77.1336 km/h \\ v_{free} = 67.8264 km/h \end{cases}.$$

Conclusion: According to the analysis of result, the average velocity is promoted. And the better traffic flow also is promoted. Therefore, rule1 is effective in the condition of 5000pcu/h complex traffic flow.

Then, we define a new variable η_{rule1} to judge optimization degree of rule1：

$$\begin{cases} \Delta v_{effect} = v_{rule1} - v_{free} \\ \eta_{rule1} = \dfrac{\Delta v_{effect}}{v_{free}} \end{cases}$$

Where,

η_{rule1} represents the proportion of Δv_{effect} in v_{free}.

In the next, we emulate the traffic situation in the condition of different scale of HGV/BUS. The data of simulations has been shown as table below:

Table 5.

Ratio of HGV/BUS	v_{rule1}	v_{free}	η_{rule1}
0	80.1696	72.6264	0.1039
0.2593	77.1336	67.8264	0.1372
0.3548	73.1452	63.8256	0.1460
0.4286	70.3873	59.6456	0.1801
0.4872	65.3456	54.6554	0.1956
0.5349	62.1364	52.7645	0.1776

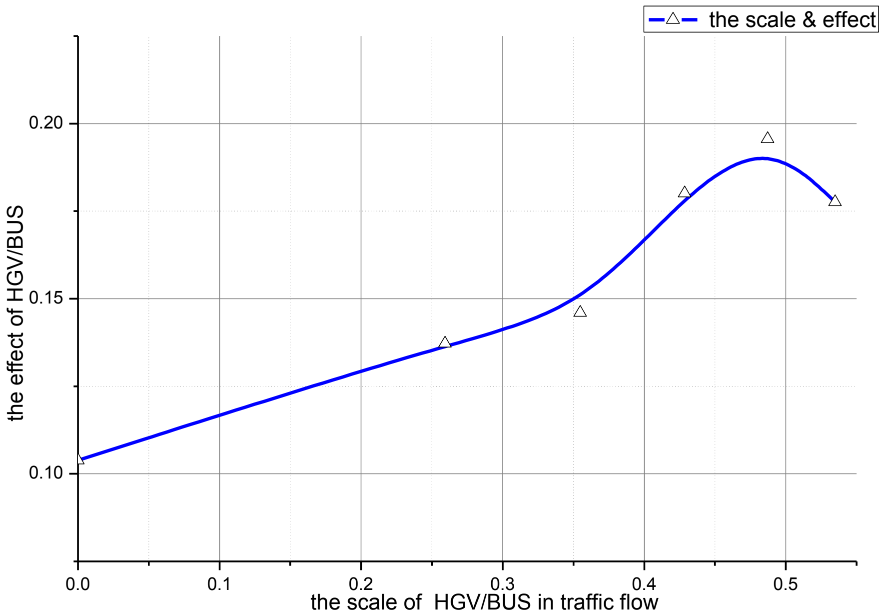


Fig. 16.

Conclusion: According to the result of simulation, it is not difficult to find out the effect of rule1 promote with the scale of HGV/BUS in the traffic flow growing.

- **Complex simulation model in different traffic flow(simulation 4)**

Considering of the traffic flow is not a constant in the real freeway, we sequentially complicate the simulation model and establish the simulation model 4. We simulate the traffic situation in the condition of different traffic flow, complex traffic flow and different rules.

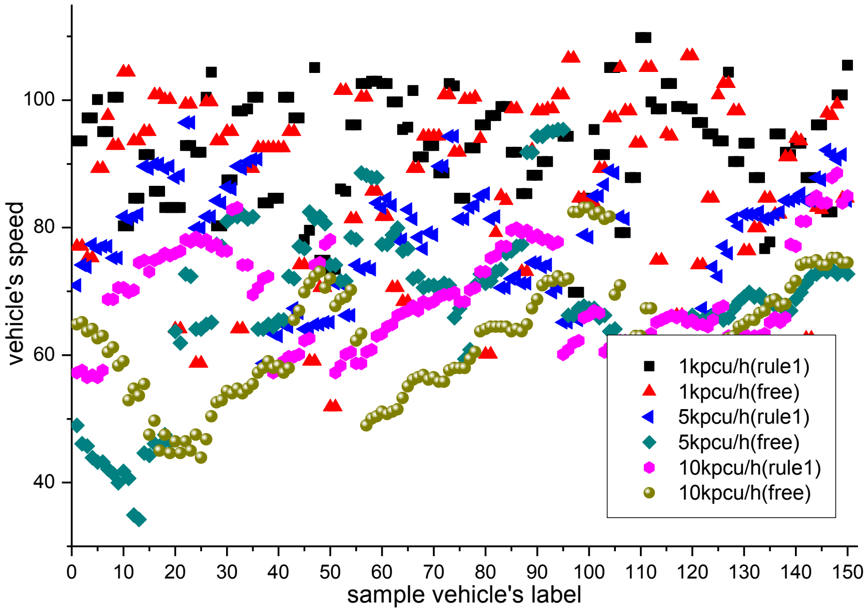


Fig. 17.不同规则与不同车辆的速度仿真图

According to the figure above, we calculate the average of each traffic flow as shown in the following table:

Table 6.

Traffic flow [pcu/h]	v_{rule1} [km/h]	v_{free} [km/h]	η_{rule1}
1000pcu/h	92.3808	87.6721	0.0537
5000pcu/h	77.1336	67.8264	0.1372
10000pcu/h	69.3024	62.3856	0.1109

The effect of rule1 in the different traffic flow is variation. We sequentially simulate the traffic situation in the different traffic flow and get more data for drawing the curve that fit the relation of traffic flow and optimization degree as shown below:

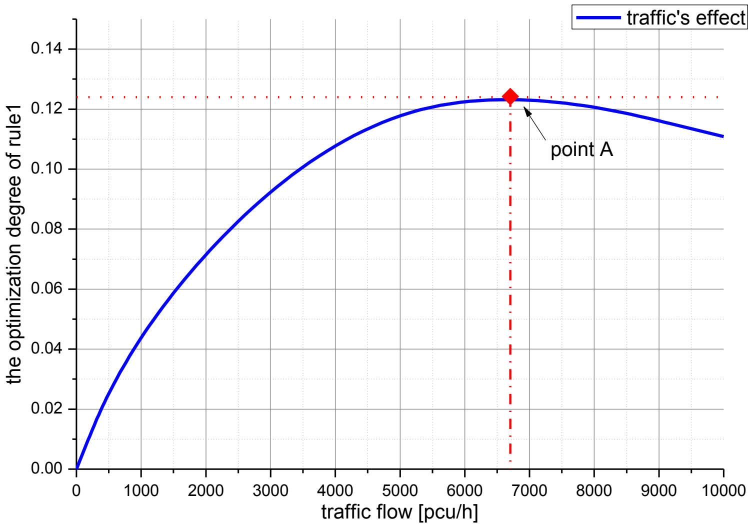


Fig. 18.

Conclusion: According to the curve, we can find out the optimization degree change with the traffic flow. And the best optimization degree is about 6703pcu/h base on the intersection point of tangent line and curve.

- **Simulation model in the different rules and limits of vehicle's velocity(simulation 5)**

Considering of the influence of velocity limits, we sequentially calculate the optimization degree of rule1 and establish the simulation model 5. The result of simulation has been shown as follows.

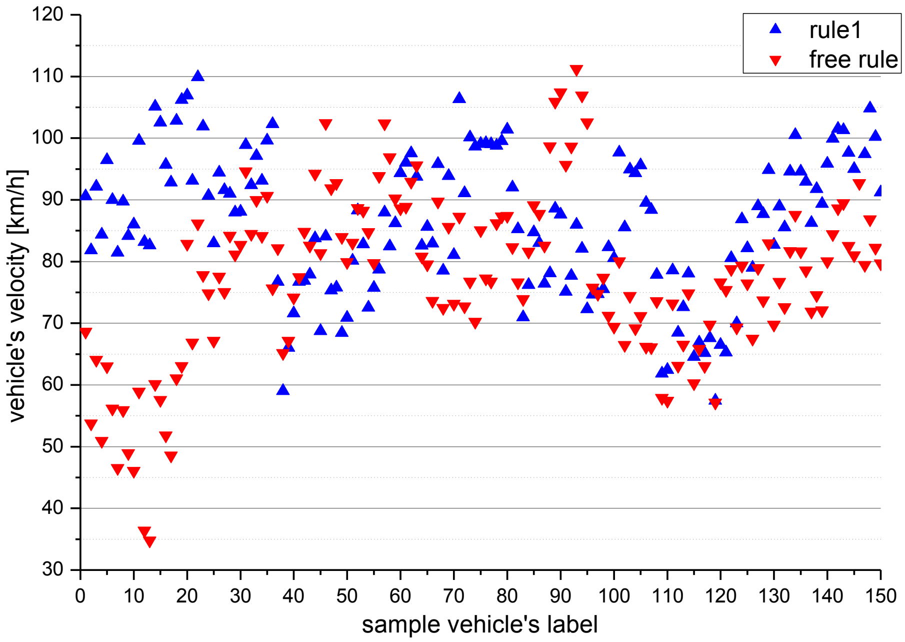


Fig. 19. The relation of velocity and rules in the new limits of velocity

Through the simple calculate, we work out the η_{rule1}.

$$\eta_{rule1} = 0.1204 .$$

Conclusion:

According to the result, rule1 has some optimization degree in the condition of different limits of velocity. Therefore, rule1 is effective in the different limits of velocity.

4.6 New rules to raise traffic flow

Comparing the result of rule1 and free rule, we can draw the conclusion that the traffic flow of keep right except to pass rule is less than that of the free rule. It means rule can not raise the traffic flow. In order to obtain higher traffic flow, we develop a new rule.

We analyze the difference of rule1. It is not difficult to find out that in rule1 cars should drive back to the initial lane while in free rule cars just need to change lane. So the total distance traveled of rule1 is larger than that of free rule and that's what causes the difference of traffic flow. Also, we can analyze through observing speed change in the picture below：

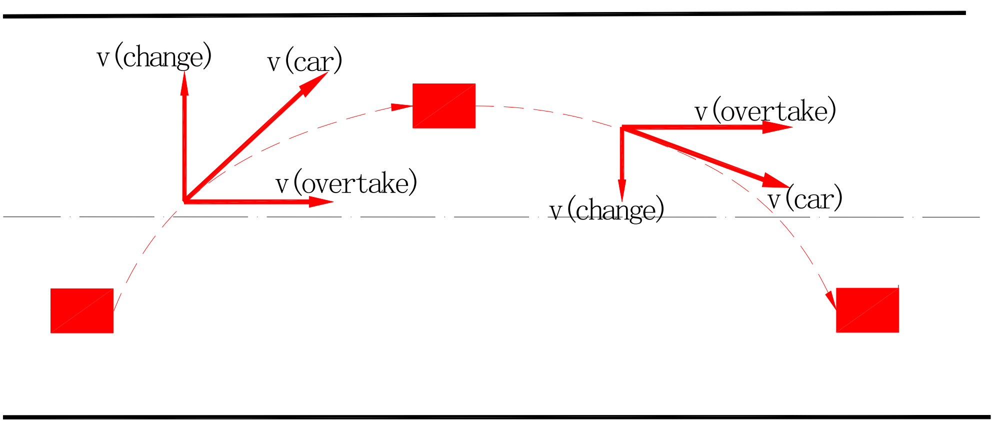


Fig. 20.

Assuming the car's speed don't change, we can find from the picture that when it tries to overtake another one, it's velocity along the freeway direction has reduced. It means the velocity of the car reduce and cause the limit of traffic flow.

In order to solve this problem we develop a new rule: The slower car changes lane but the faster one go straight. Shown in the picture below, the car in red moves faster than the car in black, usually black should take the initiative to change lane and pass the black one in rule1. However in our new rule, the white car should take the initiative to change lane to give the path to the red one.

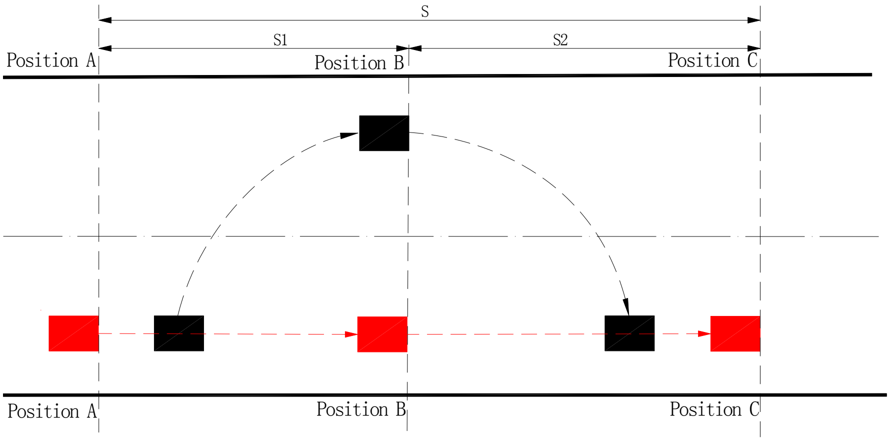


Fig. 21.

Now let's take a close look at the new rule. Suppose that the velocity of R and B is v_R and v_B. The value of v_R is larger than v_B and they will not change during our observation. In our new rule, the total distance traveled of R is s as show in the picture above. It is common sense that the straight line is shorter than the curve. Also, velocity of the red car along the freeway direction will not reduce because of it needn't to change lane. According to the kinematics formula:

$$t = \frac{s}{v}$$

The time t which R spends to achieve overtaking is less than that in rule1. It means car R can pass the section of free way in a short time and it can raise the traffic flow.

In a world, the new rule we develop is effective in raise traffic flow.

5 Model Three：Traveling on the left

5.1 Introduction

In most countries driving on the right is the rule. However there are some exception like England, Japan, Indian, etc in which they are usually driving on the left. About three quarter of the world population is driving on the right and the rest are driving on the left.

In this section, we will analyze the reason of driving on the left and right and search for some improve ways. To describe easily, we name the rule we develop new rule.

5.2 The Coriolis force

When we study the motion state of object on the earth surface, we usually simplify the earth surface a plane as the reference system. It just a simplified model and the result will not be accurate. To obtain accurate result of motion and the force it bear, we should considerate the earth as a ball and its rotation is constant. In this way objects' movement on the earth will be shown as following picture:

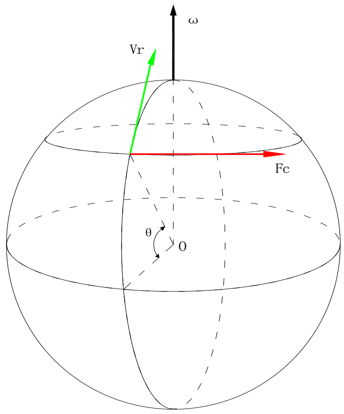


Fig. 22.

According to the knowledge of theoretical mechanics: when the object has the relative motion to the rotation reference system, its force analyze will add an extra force named Coriolis force. The direction of Coriolis force is different when the direction of relative motion and the location changes. For example, in the northern hemisphere when the relative motion is point to the north, the direction of Coriolis force is shown in the picture above.

Assuming the angular velocity of the earth rotation is ω, the velocity of relative motion is v_r, then we can obtain the Coriolis force through formula:

$$F_C = 2m\omega v_r$$

Where,

m represents for the mass of object.

5.3 The direction of the force

The direction of the Coriolis force[6] obeys the right hand law. Since it's a relation in the three-dimensional space and might make mistakes through we have a strong ability of space imagination. To make the process of analysis simple, we conclude relationship of the direction of Coriolis force, the direction of relative motion and the location as shown in the following picture:

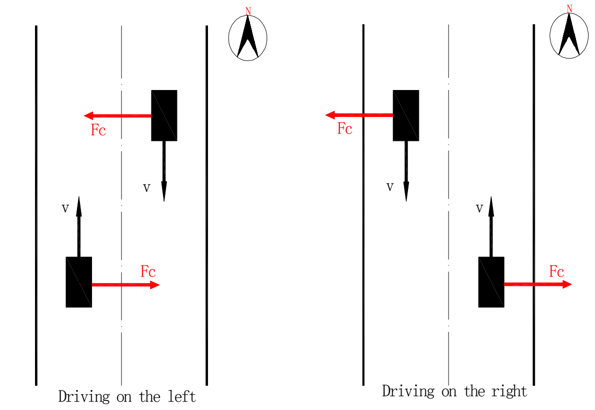


Fig. 23.北半球受科里奥利力分析

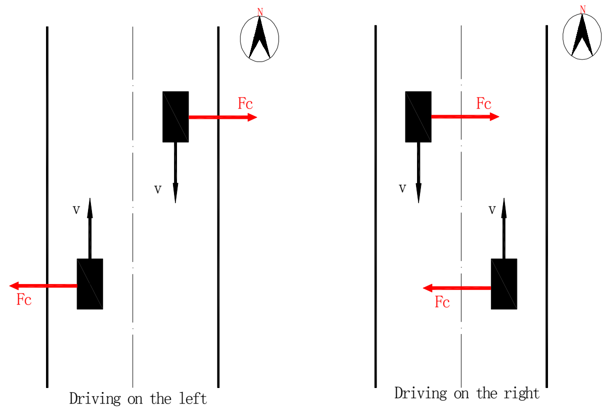


Fig. 24.南半球受科里奥利力分析

It easy to draw the conclusion that in the northern hemisphere, the direction of Coriolis force is on the right when you face to the direction you drive. While it's on the left when you are in the southern hemisphere. However when you are not drive directly to the north or the south, it make difference of the direction of the Coriolis force so we do not talk about it here to save time.

Then we draw the conclusion that driving on the right in the northern hemisphere is safer than driving on the left, while driving in the southern hemisphere is opposite. So to ensure the safety it's better to drive on the right in the northern hemisphere and left in the southern hemisphere.

5.4 Additional requirements

Through the analysis above, we know the direction of Coriolis force and details when the cars are driving on the right or left and in the northern hemisphere or southern hemisphere. In this section, we will discuss about the difference of rule1 and new rule when considering about the effect of Coriolis force.

The process of overtake can be divided into three steps. In the first step, the car changes its lane to another. Then it drives on the other lane and pass the car above to some distance. Finally it returns to its initial lane. But it makes no difference between the car drives on this lane or another. So we take step1 and step 3 into consideration as show in the following pictures:

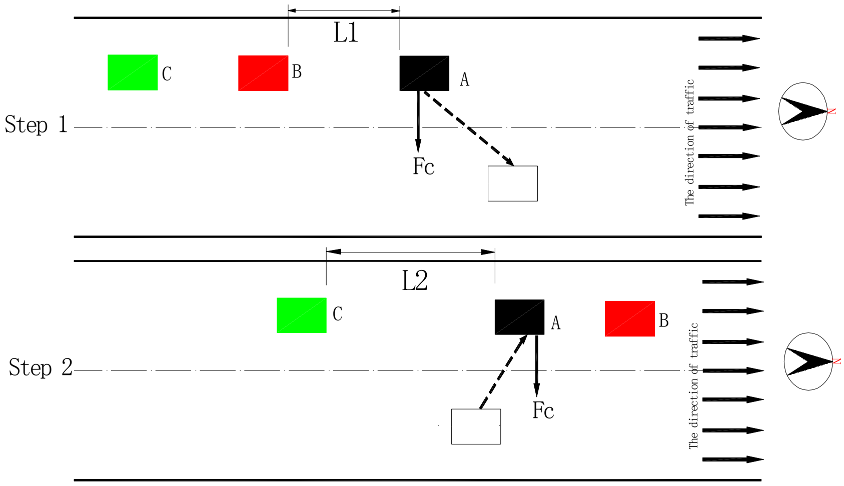


Fig. 25.北半球 左行 分析图

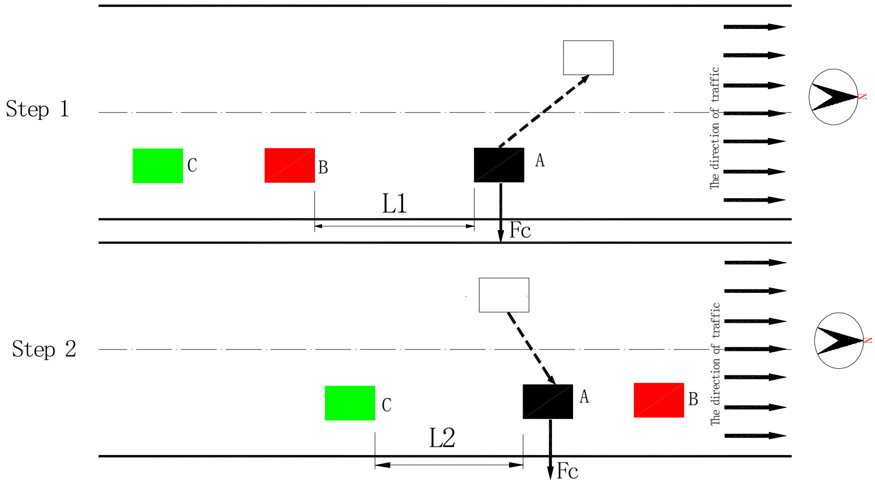


Fig. 26.北半球 右行 分析图

The analysis of overtaking in the northern hemisphere and the southern hemisphere is the same. So we analyze the case when it takes place in the northern hemisphere for example. The result of case in the southern hemisphere can be obtained through analogy.

In Fig. 23, we can find out that when car A tries to achieve step 1 in the northern hemisphere when driving on the left, because of the Coriolis force pulls it away from the initial lane so that accelerate of changing lane is larger. It means that the time it spends to change lane is less. In that way, the distance L_1 can be shorter. However, when A tries to achieve step 2, the Coriolis force pulls it back to the passing lane so that accelerate of changing lane becomes smaller. It increases the time of changing lane. So the distance for safety L_2 ought to be longer.

In Fig. 24, the cars are in the northern hemisphere and drive on the right. During the step 1, the Coriolis force pulls its back to the initial lane that makes the acceleration less. So the time spent on the changing lane is longer. It means the longer L_1 to ensure safety. While step 2 is on the contrary that L_2 can be shorter.

5.5 Conclusion

To obtain the accurate result of motion on the earth, we consider about the effect of Coriolis force. In the motion analysis considering the effect of Coriolis force, we divide the question into two part of in the northern hemisphere and southern hemisphere. After analyzing the performance of car A as shown in the Fig.23 and Fig.24 and comparing the result, we find out that our new rule can be carried over with a little requirement. As talked and defined above, when carrying our new rule to the location in the northern hemisphere where travels on the left, the L_1 can be shorter to travel on the right, however L_2 ought to be longer than traveling on the right to ensure. When it is in the southern hemisphere, it is opposite.

6 Model Four：Intelligent system model

6.1 Introduction

The vehicles on the normal freeway are manual controlled. In this section, we discuss about the influence of the intelligent system – either part of the road network or imbedded in the design of all vehicles using the roadway – without changing of the rules. We analyze the traffic situation of two conditions: manual control system and intelligent system.

6.2 Analysis

Drivers would be limited by some subjective and objective conditions including the emotion, sight and some external factors while driving alone. Therefore, drivers can only observe the traffic situation nearby the cars. The manual control system has low error as well. But if we use the intelligent system to control the traffic, computer will collect the information of traffic situations and process the digital signs. Then computers can summarize the information and select the shortest route for the drivers. The computer’s computation speed and reaction speed are much faster then human being[7].

In other word，the intelligent system is really a macroscopic and microcosmic refinements of manual control. In the next section we will analyze this point by simulating of the system and analyzing of data.

6.3 Intelligent global optimal system

6.3.1 Model Building

Because of the extraordinary ability in solving global optimal of the intelligent system, to full use of this ability, we consider a more complex condition: the three-lane freeway. When it comes to three-lane freeway, cars have more choices to change lane than the two-lane freeway as shown in the following picture:

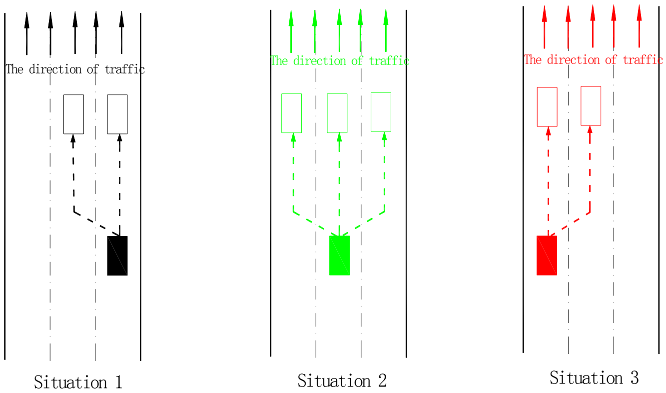


Fig. 27.三车道换车的可能情况

In the situation of three-lane freeway, when obey the keep right except to pass rule, cars can change lane in the example way shown in Fig.26. According to the rule, we build the model of three-lane traffic.

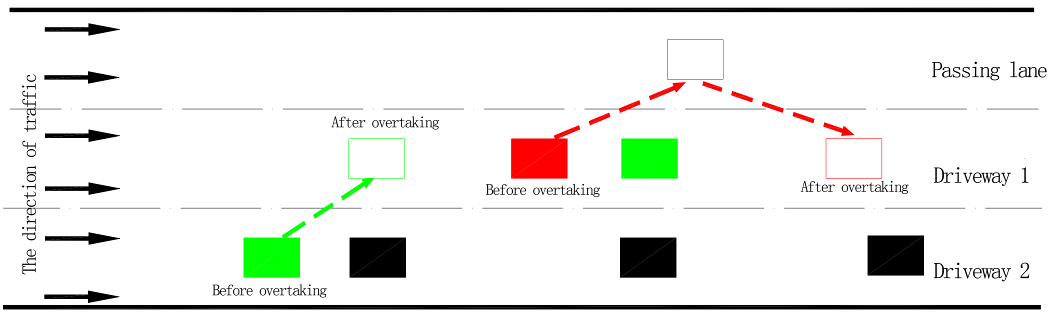


Fig. 28.三车道超车示意图

6.3.2 Model Solution

To find out the quickest way of overtaking with the help of the intelligent system, firstly we make an analogy with the freeway and cells[8]. We divided the freeway into different part and cars can only move between the cells according to the regulation we set.

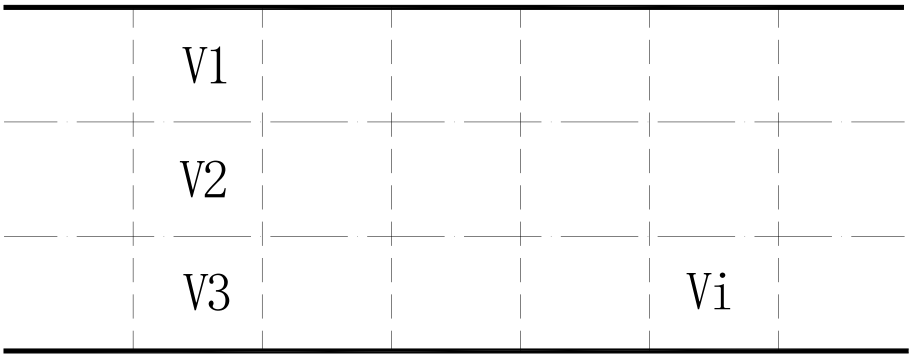


Fig. 29.

Then we turn the freeway into a graph model:the roadway is stand for a matrix. Every diamond in the graph model is replaced by element in matrix. v_i is the coordinate of the element in the matrix.

$$v_i = (x_i, y_i) \qquad i = 1,2,3,\cdots,n$$
$$V = (v_1, v_2, \cdots, v_n)$$
$$E = (v_1v_2, v_1v_3, v_1v_4, \cdots, v_1v_n, v_2v_3, \cdots, v_2v_n, \cdots, v_{n-1}v_n)$$
$$G = (V, E)$$

To simplify the model, we ignore the errors that changing lanes brings. Then we define the new variable ω_{ij} which stands for the weight of $\left|\overrightarrow{v_iv_j}\right|$. If the v_i and v_j are not adjacent, the value of ω_{ij} is infinitely great.

Next, we calculate the shortest way to overtake:

We define distances from v_i to v_j as d_{ij} and calculate the shortest route by Floyd algorithm [9]

Step1: import the weight matrix d . $d_{ij} = \omega_{ij}$

Step2: update d_{ij} , to all of i , j ,if $d_{ik} + d_{kj} < d_{ij}$,then $d_{ij} = d_{ik} + d_{kj}$

Step3: if $d_{ij} < 0$, then consist a negative loop with top v_i and stop, or when $k = n$ stop else jump to step2.

6.3.3 Model Testing

We use example as shown in the Fig.28 to test feasibility of model. In the picture, the red vehicle wants to drive from point 2 to point 14.

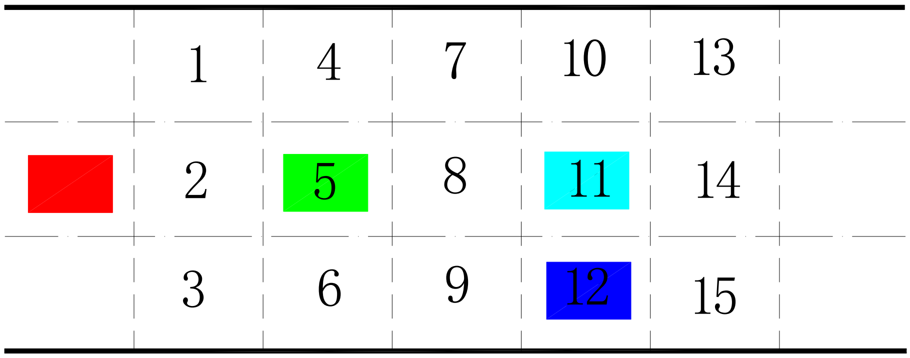


Fig. 30.

We calculate the shortest route by Floyd algorithm and the matrix of weight is:

$$d = \begin{bmatrix} 0 & d_{1,2} & \cdots & d_{1,31} \\ d_{2,1} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ d_{15,1} & d_{15,2} & \cdots & 0 \end{bmatrix},$$

Finally we work out the shortest route $2 \to 1 \to 4 \to 7 \to 10 \to 13 \to 14$. Due to the limit of foresight, the red vehicle could not see the sky-blue vehicle that locates at the point 11. The driver in the red vehicle would select the right lane to pass. The route that is selected by human being might be $2 \to 3 \to 6 \to 9 \to 8 \to 7 \to 10 \to 13 \to 14$.

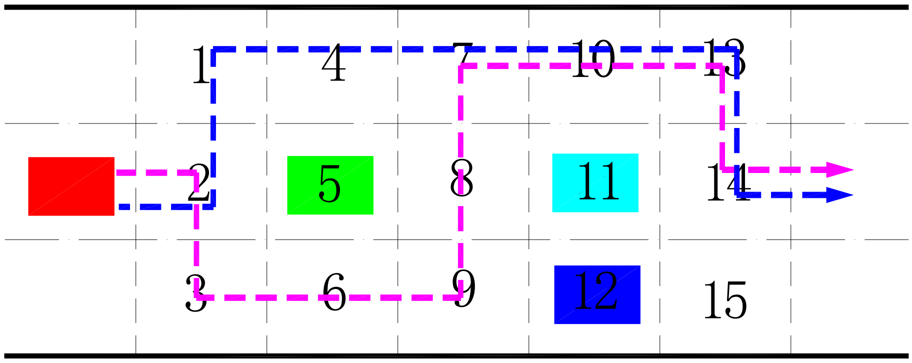


Fig. 31.

Comparing to the route that selected by human, the route obtained through calculate is obviously the shorter one. So we can draw the conclusion that intelligent system is better to control traffic.

6.4 The Intelligent system's promotion in part of the road network

The intelligent system spends less time on analyzing the traffic, dealing with the mass of data and controlling of vehicle's velocity compared to manual control system and the result tend to be more accurate.

6.4.1 Reduce the response time

The average response time of human being is about 1s. But millisecond is the measure of the time that intelligent system spends in dealing with the data.

Comparing with the response time of human being, we can ignore the time that intelligent system spends.

6.4.2 Increase the data size

Intelligent system collects the data from all roadways. And the size of data is vast. The specific analysis has been told about in the last section. However the size of data that people can deal with can be ignored compared to the intelligent system.

6.4.3 Control the velocity

When the vehicle's velocity is high, the driver has emotional variation:speed illusion. The speed illusion is produced by driver himself. It means driver's ability to feel velocity is weaken in a high velocity for a long time as shown in the following table:

Table 7. The subjectivity velocity and deviation rate in the test

The condition of test	The actual velocity at subjectivity velocity of 60km/h	Deviation rate
Keep 5s at speed of 100km/h	66.7 km/h	11%
Keep 30km at speed of 100km/h	75.7 km/h	26%
Keep 60km at speed of 100km/h	80.1 km/h	32%

Through the data of intelligent system measured by the velocity collection on the roadway of vehicles, the error produced by intelligent system can be ignored.

6.5 conclusion

We talk about the situations of global optimum and local optimum in our three-lane freeway model. Through modeling, we find that intelligent system can find the shortest route by the calculation of global optimum after ignore the reaction time of human being.

The intelligent system will change rule through method that route with out passing calculated by local optimization solving and global optimization solving.

7 The Evaluation of Model

7.1 Strengths

- In Model 1, we use equations to express the relationship between the indexes and attributes precisely and also verify it through simulation.

- In Model 2, after analyzing the attributes and parameters, we simulate the relationship of different parameters and discuss the question fully. Also, we get satisfied result.

- In Model 3，we cleverly make an analogy with traffic flow and river and think of the question through a new sight. Through our analysis, we make a new sense of driving on the right and left.

- In Model 4, considering of intelligent system, we simplify the complex overtaking problem to the problem to the shortest distance using the method of graph theory and obtain the best way to overtaking.

7.2 Weaknesses

- In Model 1, though we use the equations to show relationships among parameters, but we don’t have enough to draw the curve of the equation to demonstrate the validity of the relationship.

- In Model 2, we only simulate the traffic with theoretical knowledge, rather than according to the actual situation. So we cannot verify the practicability of our model.

- In Model 3, the effect of Coriolis force might be an major reason, but we should extend our mind to think about other possibilities.

- In Model 4, we just verify our model in a small scale of traffic, what performance will it be when facing large scale of traffic needs further study.

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