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2013 Mathematical Contest in Modeling (MCM) Summary Sheet
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2025, Water War

Summary

Water resources shortage is a new challenge that the whole world confront and it's especially significant in China. This article establishes a mathematical model to solve the water resources crisis that faces China.

We primarily use the Gray Model and the Linear Regression Model to forecast total water resources and total water needs with the average error of 15% and 2%.Because of the big fluctuation of total water resources, the predication model proves ineffective with significant error. Based on the average error, we work out the floor limit for the predication of total water resources to deal with the problem.

With the predication statistics, we get the water deficit and water shortage degree of each province in 2015、2020、2025. The results of water shortage risk assessment shows we must pay close attention to Shanghai and Ningxia which are in great water shortage.

To meet the projected water needs, we formulate four different strategies that consist of water save and conservation, movement of water resources among adjacent regions, de-salinization and water storage. Afterwards, we analyze each strategy by feasibility, water-saving and economic benefits. The application of the Analytical Hierarchy Process (AHP) gives the strategy prioritization of the comprehensive implications. The results shows that the best strategy is water save and conservation, the second is movement of water resources among adjacent regions, the third is de-salinization, and the worst strategy is water storage.

Finally, this paper gives the specific solution to water shortage as well as the expenditure predication. By applying our plans, the water shortage can be reduced from 42.8 billion cubic meters to 36.8 billion cubic meters, and the costs also reduce by 45.1% in 10 years. It is obvious that our plan is effective and feasible.

2025, Water War

Team: 17578

Abstract

Water is an important natural resource that is used every day, not only do we humans use it just about every day, but every living thing needs it to live. Environmental problems, water waste and pollution have created many challenges for water conservancy in China, causing serious problems. To meet the projected water needs of China in 2025, we establish a mathematical model to determining an effective, feasible, and cost-efficient water strategy for 2013.

By using the Gray Model and the Linear Regression Model, we primarily forecast the total water resources and total water needs. By comparing the average error of the two models, we choose the Gray Model for its better results. With total water resources and total water needs, we obtain the water shortage degree and corresponding risk assessment of different provinces and regions. To meet the projected water needs, we formulate four different strategies that consist of water save and conservation, movement of water resources among adjacent regions, de-salinization and water storage. Afterwards, we analyze the four strategies by feasibility, water-saving and economic benefits. In order to evaluate the economic, physical, and environmental implications of the four strategies, we apply the Analytical Hierarchy Process (AHP) to give the comprehensive implications and strategy prioritization. Finally, we discuss the solution to water shortage in 2025 and its cost. The relative low and decreasing expenditure of proves the effectiveness and feasibility of our plans.

The application of computer programming enables the fast process of mass data, which solves the management of water resources quickly and effectively. The application of model is not limited in handling water resources management, it is also powerful to deal with the other management problems of similar kinds.

Key Words: Water crisis; Water resources management; Gray Model; AHP

Contents

1 Introduction

As we all know, water resources are vital to human production and life. Many countries are now facing a shortage and pollution of freshwater , and China is not an exception. Water resources deficiency has restricted economic development and people's life enormously. Thus, Close attention has been paid to water storage, movement, de-salinization and conservation, as well as the prevention and disposal of water pollution.

China is an arid and hydropenic countries. Although the fourth country in total freshwater amount, China is one of the poorest countries in per capita water. At the end of the 20th century, a scant supply of water had occured in about 400 cities in all the more than 600 cities of China, 100 of which are facing serious water scarcity[1].

The facing problems of China's water resources are as followings: shortage and pollution of water resources, serious soil erosion, serious waste of water, and uneven distribution. Freshwater is renewable resource, supplied by atmospheric precipitation. The distribution of atmospheric precipitation in space and time is extremely uneven in China, which exacerbates the supply and demand contradiction of freshwater.

Freshwater is one of the few resources that can not be imported. We can only rely on water conservation and reasonable utilization and regulation. Surface water and groundwater are both the components of freshwater. The conductivity of surface water is high, while the volume of water storage is small. Groundwater is on the contrary. Therefore, the water resources can be regulated more sufficiently and efficiently when the two combined. Moreover, the exploitation potential of deep aquifers is low, and overusing deep aquifers will lead to surface subsidence and other serious consequences. Water resources in the north of China are less than those in the south, so the movement of people to water-deficient areas should be avoided. South-to-North water diversion project can relieve the shortage of water in the north, while the cost is high, so this project can only be used in extreme arid regions. In the south of China, atmospheric precipitation is sufficient, but water needs of agricultural, domestic and industrial is huge, so water shortage is caused largely by unreasonable utilization and pollution[2].

2 Assumptions

- There will be no wars or serious natural disasters from now to the year 2025.
- The total water resources in our models are all available.
- Underground water will not decrease because of leaking.
- Risk of water shortage can be measured by water shortage degree.

3 Total Water Resources Prediction Model

3.1 Calculation of Total Water Resources from 2003 to 2011

We apply the following equation to calculate total water resources:

$$W = R + Q - D$$

where

W is the total water resources,

R is the surface water resources,

Q is the groundwater resources, and

D is the transform amount of surface water and groundwater.

Based on the statistic data from 2003 to 2011[3], we can calculate the total water resources of China and each province. See **Table 1**.

Table 1.
Total water resources of China and each province
from 2003 to 2011(100 million cubic meters).

Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	27460.2	24129.6	28053.1	25330.1	25255.2	27434.3	24180.2	30906.4	23258.5
Beijing	18.4	21.3	23.2	22.1	23.8	34.2	21.8	23.1	26.8
Tianjin	10.6	14.3	10.6	10.1	11.3	18.3	15.2	9.2	15.4
Hebei	153.1	154.2	134.6	107.3	119.8	161.0	141.2	138.9	157.2
Shanxi	134.9	92.5	84.1	88.5	103.4	87.4	85.8	91.5	124.3

Inner Mongolia	495.6	437.6	456.2	411.3	295.9	412.1	378.1	388.5	419
Liaoning	220.0	285.7	377.2	261.4	261.7	266.0	171.0	606.7	294.8
Jilin	326.5	323.7	559.7	353.6	346.0	332.0	298.0	686.7	315.9
Heilongjiang	826.8	652.1	744.3	727.9	491.8	462.0	989.6	853.5	629.5
Shanghai	15.1	25.0	24.5	27.6	34.5	37.0	41.6	36.8	20.7
Jiangsu	619.1	204.0	467.0	404.4	495.7	378.0	400.3	383.5	492.4
Zhejiang	574.5	675.7	1014.4	903.6	892.1	855.2	931.3	1398.6	745
Anhui	1083.0	500.7	719.3	580.5	712.5	699.3	733.1	922.8	602.3
Fujian	806.6	712.2	1401.1	1623.5	1072.9	1036.9	800.8	1652.7	774.9
Jiangxi	1362.7	1034.6	1510.1	1630.0	1113.0	1356.2	1166.9	2275.5	1037.9
Shandong	489.7	349.5	415.9	199.3	387.1	328.7	285.0	309.1	347.6
Henan	697.7	406.6	558.5	321.8	465.2	371.3	328.8	534.9	328
Hubei	1234.1	926.4	934.0	639.7	1015.1	1033.9	825.3	1268.7	757.5
Hunan	1799.2	1641.3	1671.0	1770.3	1426.5	1600.0	1400.5	1906.6	1126.9
Guangdong	1458.4	1187.7	1747.5	2216.2	1581.2	2206.8	1613.7	1998.8	1471.3
Guangxi	1901.0	1604.5	1720.8	1881.1	1386.3	2282.5	1484.3	1823.6	1350
Hainan	291.8	171.1	307.3	227.6	283.5	419.1	480.7	479.8	484.1
Chongqing	590.7	558.8	509.8	380.3	663.0	576.9	455.9	464.3	514.6
Sichuan	2589.8	2434.2	2922.6	1865.8	2299.8	2489.9	2332.2	2575.3	2239.5
Guizhou	915.5	991.0	834.6	814.6	1054.6	1140.7	910.0	956.5	626
Yunnan	1699.4	2106.3	1846.4	1711.7	2255.5	2314.5	1576.6	1941.4	1480.2
Tibet	4757.1	4665.2	4451.1	4157.1	4321.4	4560.2	4029.2	4593.0	4402.7
Shanxi	574.6	309.4	490.6	275.5	377.0	304.0	416.5	507.5	604.4
Gansu	247.2	171.9	269.6	184.6	228.7	187.5	209.0	215.2	242.2
Qinghai	634.7	606.8	876.1	569.0	661.6	658.1	895.1	741.1	733.1
Ningxia	12.3	9.9	8.5	10.6	10.4	9.2	8.4	9.3	8.8
Xinjiang	920.1	855.4	962.8	953.1	863.8	815.6	754.3	1113.1	885.7

3.2 Prediction of Total Water Resources

We draw scatter plots according to data from **Table 1**, and combining statistic data and analyses, we establish the grey prediction model and linear regression model for predicting the total water resources

3.2.1 An Introduction to Grey Prediction Model

The grey theory can deal with systems that are characterized by poor information and for which information is lacking. The grey system theory is a useful method for short-term prediction. A relatively small number is sufficient for prediction in the grey model, and its margin of error is less than that of other models[4]. Therefore, the grey prediction model is suitable to predict the total water resources.

This paper uses the first-order linear model of grey prediction, or GM 1,1 , to predict the total water resources. The establishment of the GM 1,1 model consists of five steps[4]:

Step 1: Organize the total water resources data into a nonnegative original sequence $X^{(0)}$.

$$X^{(0)} = \left(X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(n) \right) \tag{1}$$

Step 2: Apply the original sequence to establish an accumulated-generation sequence $X^{(1)}$.

$$\begin{aligned} X^{(1)} &= \left(X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(n) \right) \\ &= \left(\sum_{t=1}^{1} X^{(0)}(t), \sum_{t=1}^{2} X^{(0)}(t), \cdots, \sum_{t=1}^{n} X^{(0)}(t) \right) \end{aligned} \tag{2}$$

Step 3: Establish the GM 1,1 model.

$$\begin{gathered} X^{(0)}(t) + aZ^{(1)}(t) = b, \\ t = 2, 3, \cdots, n, \\ Z^{(1)}(t) = \alpha X^{(1)}(t) + (1-\alpha) X^{(1)}(t-1). \end{gathered} \tag{3}$$

Here, a and b are parameters to be estimated, α is the horizontal-adjusting factor, and $0 < \alpha < 1$. Because the data structure of the total water resources is not special, this study sets α at a general value of 0.5. The solution of (3) can be expressed as

$$\hat{X}^{(1)}(t+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-at} + \frac{b}{a} \tag{4}$$

Step 4: Use the GM 1,1 model to create a matrix and the estimates of a and b.

where

$$Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix}, \ Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix}, \ \theta = \begin{bmatrix} a \\ b \end{bmatrix}.$$

Using the ordinary least-square method, the estimation of parameters $\hat{a}$ and $\hat{b}$ is given by

$$\hat{\theta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(B^T B\right)^{-1} B^T Y \tag{5}$$

Step 5: Make predictions through inverse accumulated-generation operation.

Substitute the parameter values of $\hat{a}$ and $\hat{b}$ obtained from step 4 into equation (4) in order to obtain $\hat{X}^{(1)}(t+1)$. Series $X^{(1)}$ is the original series $X^{(0)}$ produced through the one-time accumulated generation. Therefore, before the prediction begins, $X^{(1)}(t+1)$, obtained through prediction, must undergo an inverse accumulated-generation restoration to become $\hat{X}^{(0)}(t+1)$.

The inverse generated series is

$$\hat{X}^{(0)}(t+1) = \hat{X}^{(1)}(t+1) - \hat{X}^{(1)}(t). \tag{6}$$

3.2.2 An Introduction to Linear Regression Model

Suppose a model of the form $y = Ax + B$ is expected and it has been decided to use the m data points (x_i, y_i), $i = 1, 2, \cdots, m$, to estimate A and B. Denote the least-squares estimate of $y = Ax + B$ by $y = ax + b$. Applying the least-squares criterion to this situation requires the minimization of

$$S = \sum_{i=1}^{m} \left[y_i - f(x_i)\right]^2 = \sum_{i=1}^{m} \left(y_i - ax_i - b\right)^2$$

A necessary condition for optimality is that the two partial derivatives $\partial S / \partial a$ and $\partial S / \partial b$ equal zero, yielding the equations

$$\frac{\partial S}{\partial a} = -2 \sum_{i=1}^{m} \left(y_i - ax_i - b\right) x_i = 0$$

$$\frac{\partial S}{\partial b} = -2 \sum_{i=1}^{m} \left(y_i - ax_i - b\right) = 0$$

These equations can be rewritten to give

$$\left.\begin{aligned} a\sum_{i=1}^{m} x_i^2 + b\sum_{i=1}^{m} x_i &= \sum_{i=1}^{m} x_i y_i \\ a\sum_{i=1}^{m} x_i + mb &= \sum_{i=1}^{m} y_i \end{aligned}\right\}$$

The preceding equations can be solved for a and b once all the values for

x_i and x_i are substituted into them. The solutions for the parameters a and b are easily obtained by elimination and are found to be

$$a=\frac{m\sum x_i y_i-\sum x_i \sum y_i}{m\sum x_i^2-\sum x_i{}^{2}}, \qquad \text{the slope}$$

and

$$b=\frac{\sum x_i{}^{2} \sum y_i-\sum x_i y_i \sum x_i}{m\sum x_i^2-\sum x_i{}^{2}}, \qquad \text{the intercept}$$

Computer codes are easily written to compute these values for a and b for any collection of data points[5].

3.2.3 Fitted Value and Predicted Value of Total Water Resources

Applying the two models, we obtain the fitted value from 2003 to 2011(see **Appendix 1**) and predicted value of 2015, 2020 and 2025(see **Table 2)**.

Table 2.
Predicted value of total water resources in 2015, 2020 and 2025

Model	Gray Model			Linear Regression Model		
Province	2015	2020	2025	2015	2020	2025
China	26678.9	27094.2	27516.0	25940.7	25764.2	25587.7
Beijing	28.9	32.3	36.1	30.3	34.3	38.3
Tianjin	15.3	17.0	18.8	15.6	17.4	19.2
Hebei	157.7	171.4	186.3	145.8	148.9	152.0
Shanxi	119.4	139.6	163.2	93.4	89.8	86.2
Inner Mongolia	359.2	334.5	311.5	329.3	278.5	227.7
Liaoning	419.2	508.1	615.9	418.9	490.1	561.3
Jilin	439.8	467.1	496.1	460.4	502.2	544.0
Heilongjiang	805.0	889.4	982.8	713.9	717.2	720.5
Shanghai	37.4	42.4	48.2	42.7	51.2	59.6
Jiangsu	545.5	669.0	820.5	410.1	399.4	388.7
Zhejiang	1158.0	1345.1	1562.4	1239.3	1459.0	1678.7
Anhui	891.0	1065.0	1273.0	660.2	617.6	575.1
Fujian	1070.4	1029.9	990.9	1219.0	1294.6	1370.2
Jiangxi	1640.0	1832.2	2046.8	1582.5	1704.5	1826.4
Shandong	296.7	277.8	260.1	236.2	167.7	99.2
Henan	352.5	316.8	284.6	245.4	120.0	0.0
Hubei	1023.0	1094.3	1170.5	865.7	807.2	748.6

Hunan	1299.2	1147.4	1013.3	1246.3	1029.3	812.2
Guangdong	1925.5	2050.3	2183.3	2014.6	2198.6	2382.5
Guangxi	1564.7	1485.7	1410.7	1499.1	1364.3	1229.4
Hainan	890.3	1678.0	3162.7	647.3	833.4	1019.5
Chongqing	481.5	460.2	439.9	457.3	415.7	374.1
Sichuan	2261.4	2176.7	2095.2	2212.0	2084.1	1956.2
Guizhou	794.3	722.7	657.6	811.3	745.9	680.6
Yunnan	1572.0	1384.9	1220.1	1706.9	1597.9	1488.8
Tibet	4283.0	4208.4	4135.1	4160.8	3987.9	3815.0
Shanxi	735.5	1101.2	1648.7	508.0	557.5	607.0
Gansu	237.5	255.1	273.9	216.2	215.5	214.8
Qinghai	830.5	916.1	1010.5	831.5	908.5	985.5
Ningxia	8.4	7.8	7.3	7.4	5.9	4.5
Xinjiang	929.9	950.1	970.8	913.5	920.2	927.0

3.3 Error Analysis and Model Evaluation

After establishing the model, we must go through another step to review the difference between the predicted and actual values. The equation below shows the residual and mean residual errors for the grey prediction model:

$$\text{Error} = \left|\frac{x(t)-\hat{x}(t)}{x(t)}\right|$$

$$Ae\ (\text{Average error}) = \frac{1}{n}\sum_{t=1}^{n}\frac{x(t)-\hat{x}(t)}{x(t)}$$

This average error has been defined as the mean absolute percentage error (MAPE), and it represents the strengths and weaknesses of the prediction model[4]. The MAPE assessment criteria according to Lewis's standard[6] are shown in **Table 3**.

Table 3.
MAPE assessment criteria.

MAPE(%)	Prediction Effectiveness
Less than 10	Very good
10-20	Good
20-50	Ideal
Over 50	Inaccurate

Figure 1 shows the result of error analysis.

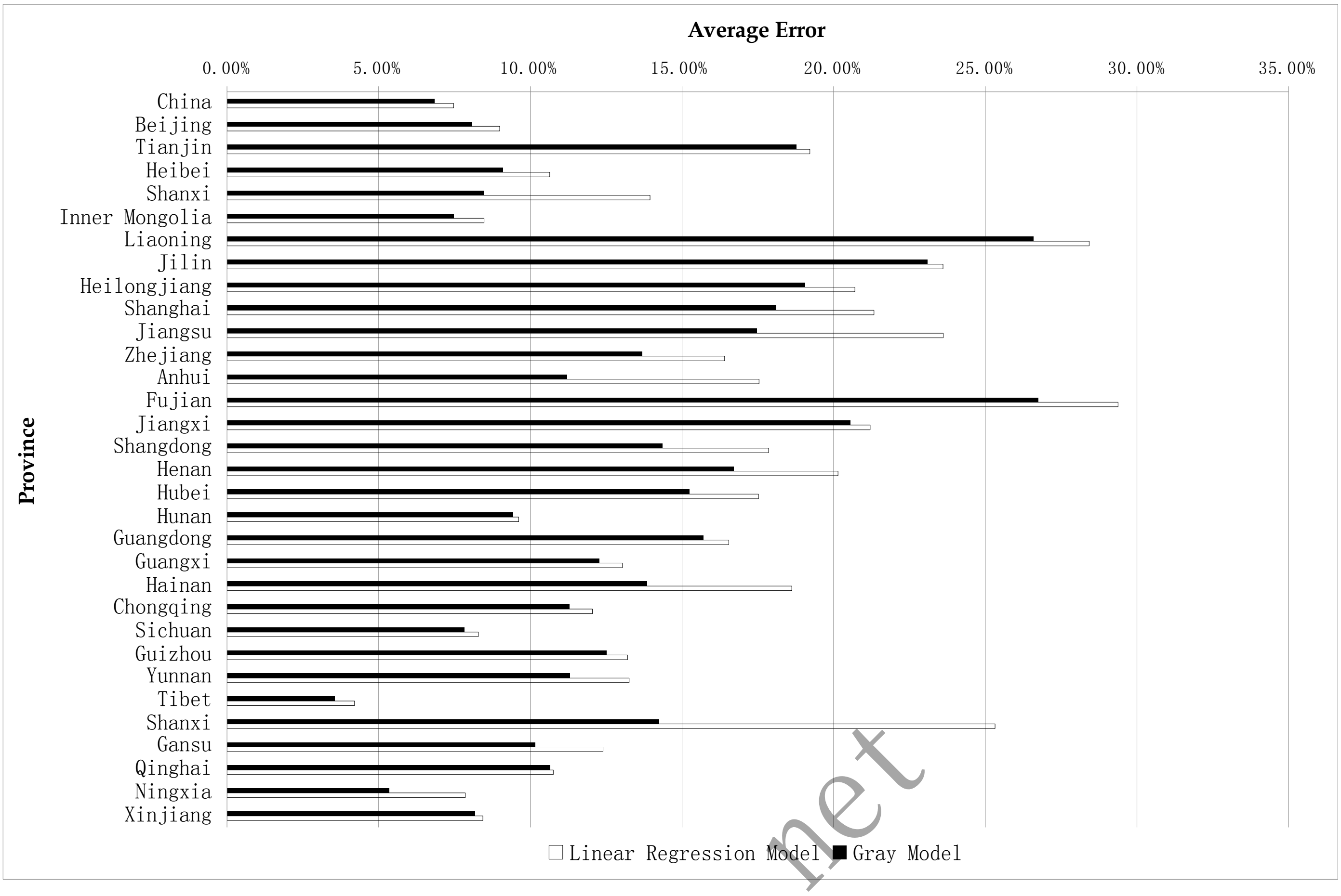

Figure 1.

As we can see, average errors of the two models are both large. On one hand, the models may not be able to perfectly reflect the changes of the total water resources, and the accuracy should be improved. On the other hand, annual precipitation varies from year to year and climate change is erratic, which will lead to high fluctuation and difficulty to simulate its trend.

Eventually, we determine the grey prediction model to predict total water resources. Allowing for the prediction errors, we must count in the fluctuations. Total water resources will vary between $Wp(1-Ae)$ and $Wp(1+Ae)$, where Wp is predicted value, and Ae is average error. We adopt the relatively conservative total water resources $Wp(1-Ae)$, see **Table 4**.

Table 4.
Adjusted predicted value of total water resources in 2015, 2020 and 2025.

Province	2015	2020	2025	Province	2015	2020	2025
China	24852.9	25239.8	25632.7	Henan	293.6	263.9	237.1
Beijing	26.6	29.7	33.2	Hubei	867.1	927.6	992.2
Tianjin	12.4	13.8	15.3	Hunan	1176.6	1039.1	917.7
Hebei	143.4	155.8	169.4	Guangdong	1623.2	1728.5	1840.5
Shanxi	109.3	127.8	149.4	Guangxi	1372.8	1303.5	1237.7
Inner Mongolia	332.3	309.5	288.3	Hainan	767.0	1445.6	2724.6
Liaoning	307.7	373.0	452.2	Chongqing	427.2	408.3	390.2

Jilin	338.3	359.3	381.6	Sichuan	2084.4	2006.4	1931.3
Heilongjiang	651.6	720.0	795.5	Guizhou	694.9	632.3	575.3
Shanghai	30.6	34.8	39.4	Yunnan	1394.4	1228.5	1082.3
Jiangsu	450.2	552.2	677.2	Tibet	4131.0	4059.1	3988.4
Zhejiang	999.5	1160.9	1348.5	Shanxi	630.9	944.5	1414.0
Anhui	791.1	945.6	1130.2	Gansu	213.4	229.2	246.1
Fujian	784.2	754.5	725.9	Qinghai	742.0	818.5	902.8
Jiangxi	1303.1	1455.8	1626.4	Ningxia	8.0	7.4	6.9
Shandong	254.2	237.9	222.7	Xinjiang	853.9	872.5	891.5

4 Total Water Needs Prediction Model

4.1 Calculation of Total Water Needs from 2003 to 2011

We apply the following equation to calculate total water needs:

$$U = A + I + L + E$$

where

U is the total water needs,

A is agricultural water needs,

I is industrial water needs,

L is domestic water needs, and

E is ecological water needs.

Based on the statistic data from 2003 to 2011[7], we can calculate the total water needs of China and each province. See **Table 5**.

Table 5.
Total water needs of China and each province from 2003 to 2011.

Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	5320.4	5547.8	5633.0	5795.0	5818.7	5910.0	5965.2	6022.0	6107.2
Beijing	35.0	34.6	34.5	34.3	34.8	35.1	35.5	35.2	36.0
Tianjin	20.5	22.1	23.1	23.0	23.4	22.3	23.4	22.5	23.1
Hebei	199.8	195.9	201.8	204.0	202.5	195.0	193.7	193.7	196.0
Shanxi	56.2	55.9	55.7	59.3	58.7	56.9	56.3	63.8	74.2
Inner Mongolia	166.9	171.5	174.8	178.7	180.0	175.8	181.3	181.9	184.7
Liaoning	128.3	130.2	133.3	141.2	142.9	142.8	142.8	143.7	144.5
Jilin	104.0	99.2	98.4	102.9	100.8	104.1	111.1	120.0	131.2

Heilongjiang	245.8	259.4	271.5	286.2	291.4	297.0	316.3	325.0	352.4
Shanghai	109.0	118.1	121.3	118.6	120.2	119.8	125.2	126.3	124.5
Jiangsu	433.5	525.6	519.7	546.4	558.3	558.3	549.2	552.2	556.2
Zhejiang	206.0	207.8	209.9	208.3	211.0	216.6	197.8	203.0	198.5
Anhui	178.6	209.7	208.0	241.9	232.1	266.4	291.9	293.1	294.6
Fujian	182.8	184.9	186.9	187.3	196.3	198.0	201.4	202.5	208.8
Jiangxi	172.5	203.5	208.1	205.7	234.9	234.2	241.3	239.8	262.9
Shandong	219.4	214.9	211.0	225.8	219.5	219.9	220.0	222.5	224.0
Henan	187.6	200.7	197.8	227.0	209.3	227.5	233.7	224.6	229.1
Hubei	245.1	242.7	253.4	258.8	258.7	270.7	281.4	288.0	296.7
Hunan	318.8	323.6	328.4	327.7	324.3	323.6	322.3	325.2	326.5
Guangdong	457.5	464.8	459.0	459.4	462.5	461.5	463.4	469.0	464.2
Guangxi	278.4	290.8	312.9	314.4	310.4	310.1	303.4	301.6	301.8
Hainan	46.3	46.3	44.1	46.5	46.7	46.9	44.5	44.4	44.5
Chongqing	63.2	67.5	71.2	73.2	77.4	82.8	85.3	86.4	86.8
Sichuan	209.9	210.4	212.3	215.1	214.0	207.6	223.5	230.3	233.5
Guizhou	93.7	94.3	97.2	100.0	98.0	101.9	100.4	101.5	95.9
Yunnan	146.1	146.9	146.8	144.8	150.0	153.1	152.6	147.5	146.8
Tibet	25.3	28.0	33.2	35.0	36.7	37.5	30.9	35.2	31.0
Shanxi	75.1	75.5	78.8	84.1	81.5	85.5	84.3	83.4	87.8
Gansu	121.6	121.8	123.0	122.3	122.5	122.2	120.6	121.8	122.9
Qinghai	29.0	30.2	30.7	32.2	31.1	34.4	28.8	30.8	31.2
Ningxia	64.0	74.0	78.1	77.6	71.0	74.2	72.2	72.4	73.6
Xinjiang	500.7	497.1	508.5	513.4	517.7	528.2	530.9	535.1	523.5

4.2 Prediction of Total Water Needs

Likewise, we also establish the grey prediction model and linear regression model for predicting the total water needs and obtain the fitted value from 2003 to 2011(see **Appendix 2**) and predicted value of 2015, 2020 and 2025.(see **Table 6**).

Table 6.
Predicted value of total water needs in 2015, 2020 and 2025.

Model	Gray Model			Linear Regression Model		
Province	2015	2020	2025	2015	2020	2025
China	6451.4	6888.5	7355.3	6504.2	6950.0	7395.7
Beijing	36.5	37.6	38.7	36.1	36.8	37.5
Tianjin	23.2	23.5	23.7	24.1	25.1	26.0
Hebei	191.0	186.5	182.2	191.8	187.8	183.9
Shanxi	76.5	90.0	105.8	72.2	80.1	88.0

Inner Mongolia	190.6	199.2	208.1	192.3	201.6	211.0
Liaoning	154.6	165.1	176.3	155.6	166.1	176.6
Jilin	146.8	180.1	221.0	134.4	150.9	167.4
Heilongjiang	404.8	496.0	607.6	390.3	450.6	510.9
Shanghai	130.0	135.8	141.9	133.1	141.0	149.0
Jiangsu	580.7	605.4	631.1	618.8	672.2	725.7
Zhejiang	195.8	189.0	182.4	198.5	193.5	188.5
Anhui	383.7	507.1	670.1	367.1	442.7	518.2
Fujian	223.2	243.8	266.2	220.5	236.9	253.3
Jiangxi	297.4	355.0	423.8	297.9	344.9	392.0
Shandong	229.2	235.7	242.4	226.8	231.2	235.7
Henan	253.6	280.1	309.4	256.6	282.4	308.2
Hubei	331.0	380.8	438.1	320.9	355.0	389.2
Hunan	324.0	323.3	322.5	327.0	328.6	330.2
Guangdong	468.1	471.6	475.0	469.1	473.3	477.5
Guangxi	304.5	303.8	303.0	316.3	324.9	333.4
Hainan	44.0	43.0	42.0	44.0	43.0	42.0
Chongqing	104.2	125.8	151.9	102.3	118.0	133.8
Sichuan	244.0	262.9	283.3	239.9	253.9	268.0
Guizhou	102.0	104.3	106.7	103.2	106.5	109.7
Yunnan	151.2	152.9	154.7	151.6	153.6	155.6
Tibet	35.1	36.2	37.4	38.1	41.6	45.1
Shanxi	93.2	101.1	109.6	93.3	100.6	107.8
Gansu	121.8	121.6	121.4	122.1	122.2	122.2
Qinghai	31.1	31.1	31.1	32.1	32.8	33.5
Ningxia	70.3	67.8	65.4	75.5	77.0	78.5
Xinjiang	554.0	578.5	604.1	552.5	574.6	596.7

4.3 Error Analysis and Model Evaluation

Figure 2 shows the result of error analysis.

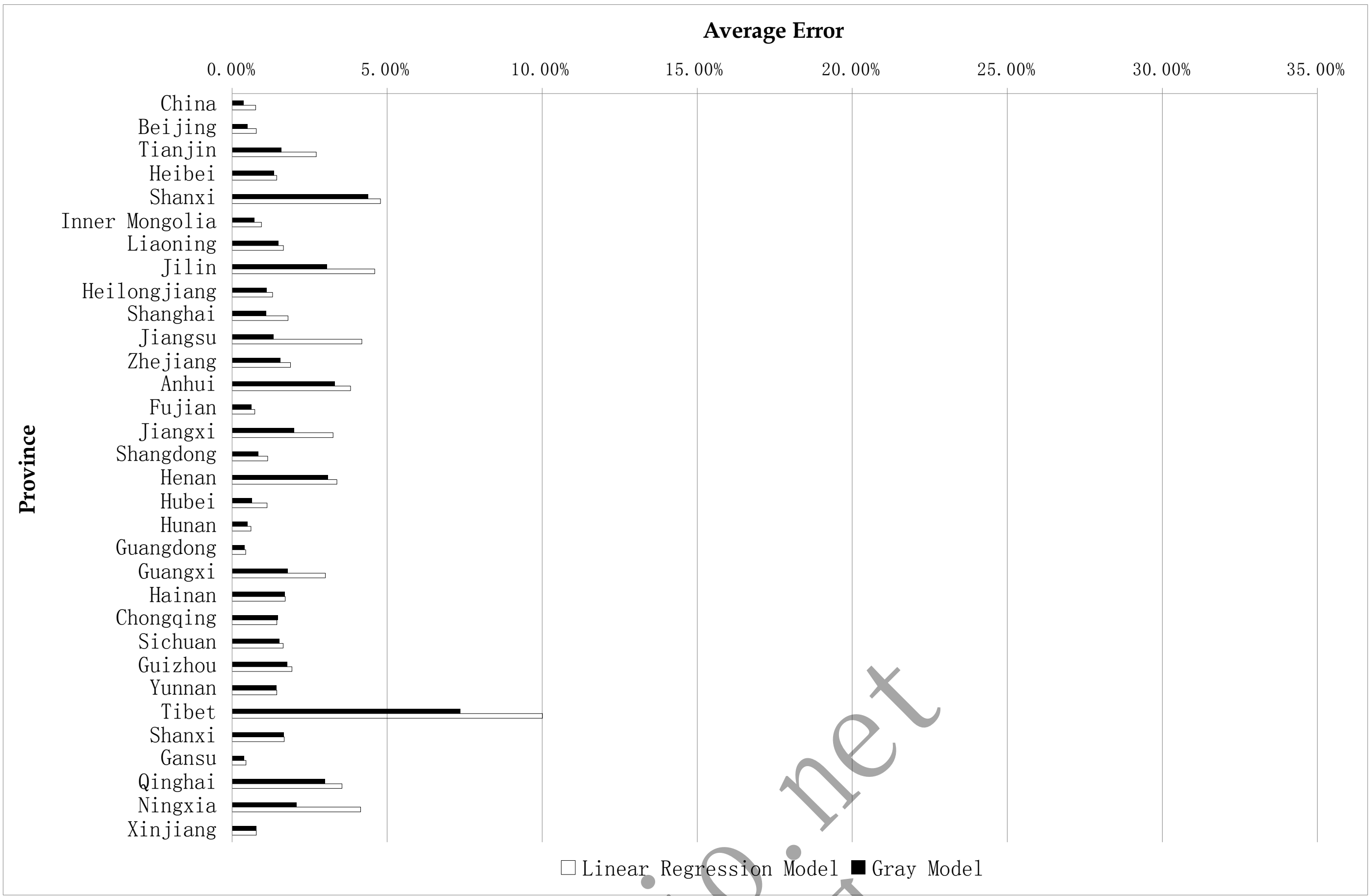

Figure 2.

As we can see, average errors of the two models are both small. Relatively, average error of grey prediction model is smaller, so we adopt the predicted value of grey prediction model.

5 Analyses of Water Supply and Demand

5.1 Prediction of Water Supply and Demand in 2015, 2020 and 2025

We apply the following equation to analyze water supply and demand:

$$S = W - U$$

where

S is spare water resources,

W is total water resources, and

U is total water needs.

Based on the predictive data of 2015, 2020 and 2025, we can calculate the spare water resources of China and each province. See **Table 7**.

Table 7.

Spare water resources of China and each province in 2015, 2020 and 2025.

Province	2015	2020	2025	Province	2015	2020	2025
China	18401.5	18351.3	18277.4	**Henan**	40.0	**-16.3**	**-72.3**
Beijing	**-9.9**	**-7.9**	**-5.5**	Hubei	536.1	546.7	554.1
Tianjin	**-10.8**	**-9.7**	**-8.4**	Hunan	852.6	715.9	595.2
Hebei	**-47.6**	**-30.7**	**-12.8**	Guangdong	1155.1	1256.9	1365.5
Shanxi	32.8	37.8	43.5	Guangxi	1068.3	999.7	934.7
Inner Mongolia	141.7	110.4	80.2	Hainan	723.0	1402.6	2682.5
Liaoning	153.1	207.9	275.9	Chongqing	323.0	282.5	238.4
Jilin	191.5	179.2	160.6	Sichuan	1840.3	1743.4	1648.0
Heilongjiang	246.8	224.0	187.9	Guizhou	592.9	528.0	468.6
Shanghai	**-99.3**	**-101.0**	**-102.4**	Yunnan	1243.2	1075.5	927.5
Jiangsu	**-130.5**	**-53.2**	46.1	Tibet	4096.0	4022.9	3951.1
Zhejiang	803.6	972.0	1166.1	Shanxi	537.6	843.4	1304.4
Anhui	407.3	438.5	460.2	Gansu	91.6	107.6	124.7
Fujian	561.0	510.7	459.7	Qinghai	710.9	787.3	871.7
Jiangxi	1005.7	1100.8	1202.6	**Ningxia**	**-62.3**	**-60.4**	**-58.5**
Shandong	25.0	2.2	**-19.7**	Xinjiang	299.9	294.0	287.3

5.2 Analyses of Water Supply and Demand in 2015, 2020 and 2025

5.2.1 Water Shortage Degree

Water shortage degree is determined by total water resources W and total water needs U, so we define a variable Su as the evaluation standard of water shortage degree, and

$$Su = \frac{U-W}{U} = \frac{-S}{U},$$

when $Su<0$, $Su=0$

According to the definition, we can calculate the water shortage degree of China and each province. See **Table 8**.

Table 8.
Water shortage degree of China and each province in 2015, 2020 and 2025.

Province	2015	2020	2025	Province	2015	2020	2025
China	0	0	0	**Henan**	0	**5.8%**	**23.5%**
Beijing	**27.5%**	**21.4%**	**14.7%**	Hubei	0	0	0
Tianjin	**44.9%**	**38.6%**	**32.3%**	Hunan	0	0	0
Hebei	**24.8%**	**16.4%**	**7.0%**	Guangdong	0	0	0
Shanxi	0	0	0	Guangxi	0	0	0
Inner Mongolia	0	0	0	Hainan	0	0	0
Liaoning	0	0	0	Chongqing	0	0	0
Jilin	0	0	0	Sichuan	0	0	0
Heilongjiang	0	0	0	Guizhou	0	0	0
Shanghai	**74.7%**	**71.6%**	**68.7%**	Yunnan	0	0	0
Jiangsu	**21.1%**	**7.9%**	0	Tibet	0	0	0
Zhejiang	0	0	0	Shanxi	0	0	0
Anhui	0	0	0	Gansu	0	0	0
Fujian	0	0	0	Qinghai	0	0	0
Jiangxi	0	0	0	**Ningxia**	**82.5%**	**78.4%**	**74.5%**
Shandong	0	0	**8.4%**	Xinjiang	0	0	0

5.2.2 Risk Assessment

Water shortage degree can reflect the risk of water shortage to some extent, so we can assess the risk of water shortage according to **Table 9**.

Table 9.
Risk assessment of water shortage.

Su	⩽0.2	0.2—0.3	0.3—0.4	0.4—0.6	⩾0.6
Risk assessment	Very low	Low	Medium	High	Extreme

We assess the risk of provinces suffering from water shortage, see **Table 10**.

Table 10.
Risk assessment of provinces suffering from water shortage.

Province	Beijing	Tianjin	Hebei	Shanghai	Jiangsu	Shandong	Henan	Ningxia
2015	low	high	low	extreme	low	very low	very low	extreme
2020	low	medium	very low	extreme	very low	very low	very low	extreme
2025	very low	medium	very low	extreme	very low	very low	low	extreme

6 Water Strategies for China

6.1 Save and Conservation

"The Outline of the Eleventh Five-Year Plan of China" put forward that water needs of per unit of industrial added value should decrease by 30% per year, effective utilization coefficient of agricultural irrigation water should increase from 0.45 to 0.5, and total quantity of pollutant discharge should decrease by 10% per year[8]. The following discussions are all aimed at the three objectives.

6.1.1 Decrease in Industrial Water Needs

We obtain the predicted value of industrial water needs in 2015, 2020 and 2025 according to statistic data from 2003 to 2011(see **Appendix 3**). Allowing for the objective of "water needs of per unit of industrial added value should decrease by 30% per year", we obtain the decrease of industrial water needs. See **Table 11**.

Table 11.
Predicted value and decrease of industrial water needs in 2015, 2020 and 2025(100 million cubic meters).

Province	Predicted value			Decrease		
	2015	2020	2025	2015	2020	2025
China	1615.0	1804.0	2015.1	484.5	541.2	604.5
Beijing	3.5	2.5	1.8	1.1	0.8	0.5
Tianjin	4.6	4.6	4.6	1.4	1.4	1.4
Hebei	23.5	22.6	21.7	7.1	6.8	6.5
Shanxi	11.9	10.9	10.0	3.6	3.3	3.0
Inner Mongolia	36.8	59.7	97.0	11.0	17.9	29.1
Liaoning	28.0	31.6	35.8	8.4	9.5	10.7
Jilin	35.1	49.6	70.0	10.5	14.9	21.0
Heilongjiang	55.7	55.6	55.5	16.7	16.7	16.7
Shanghai	87.5	91.9	96.6	26.2	27.6	29.0
Jiangsu	194.6	189.2	183.9	58.4	56.7	55.2
Zhejiang	62.1	63.6	65.2	18.6	19.1	19.5
Anhui	119.8	154.6	199.5	35.9	46.4	59.8
Fujian	102.7	130.2	164.9	30.8	39.1	49.5
Jiangxi	64.9	72.2	80.3	19.5	21.7	24.1

Shandong	29.6	32.9	36.5	8.9	9.9	11.0
Henan	68.6	84.6	104.2	20.6	25.4	31.3
Hubei	151.5	203.9	274.5	45.4	61.2	82.3
Hunan	102.3	116.9	133.5	30.7	35.1	40.0
Guangdong	136.7	136.8	136.8	41.0	41.0	41.0
Guangxi	69.3	86.5	108.1	20.8	26.0	32.4
Hainan	4.8	5.5	6.2	1.4	1.6	1.9
Chongqing	60.1	77.9	101.1	18.0	23.4	30.3
Sichuan	69.0	76.2	84.1	20.7	22.9	25.2
Guizhou	38.5	44.7	51.9	11.6	13.4	15.6
Yunnan	32.0	41.9	54.8	9.6	12.6	16.5
Tibet	3.6	8.3	19.1	1.1	2.5	5.7
Shanxi	12.2	12.1	11.9	3.7	3.6	3.6
Gansu	12.6	11.5	10.4	3.8	3.4	3.1
Qinghai	3.1	2.2	1.5	0.9	0.7	0.5
Ningxia	5.2	6.5	8.2	1.6	2.0	2.5
Xinjiang	15.7	21.7	30.1	4.7	6.5	9.0

6.1.2 Increase in Effective Utilization Coefficient of Agricultural Water

Likewise, we can obtain the predicted value of agricultural water needs in 2015, 2020 and 2025 according to statistic data from 2003 to 2011(see **Appendix** 4). Allowing for the objective of "effective utilization coefficient of agricultural irrigation water should increase from 0.45 to 0.5", we obtain the decrease of agricultural water needs. See **Table 12**.

Table 12.
Predicted value and decrease of agricultural water needs in 2015, 2020 and 2025(100 million cubic meters).

Province	Predicted value			Decrease		
	2015	2020	2025	2015	2020	2025
China	3828.6	3948.4	4071.9	191.4	197.4	203.6
Beijing	9.1	7.8	6.6	0.5	0.4	0.3
Tianjin	11.1	10.2	9.4	0.6	0.5	0.5
Hebei	136.9	130.9	125.1	6.8	6.5	6.3
Shanxi	45.7	54.4	64.8	2.3	2.7	3.2
Inner Mongolia	126.4	118.0	110.2	6.3	5.9	5.5
Liaoning	93.2	95.7	98.2	4.7	4.8	4.9
Jilin	85.6	97.2	110.4	4.3	4.9	5.5

Heilongjiang	328.7	428.9	559.5	16.4	21.4	28.0
Shanghai	14.9	13.4	12.1	0.7	0.7	0.6
Jiangsu	329.4	361.8	397.4	16.5	18.1	19.9
Zhejiang	84.8	76.2	68.4	4.2	3.8	3.4
Anhui	221.9	298.8	402.4	11.1	14.9	20.1
Fujian	95.4	92.3	89.4	4.8	4.6	4.5
Jiangxi	193.2	232.3	279.4	9.7	11.6	14.0
Shandong	149.8	145.0	140.5	7.5	7.3	7.0
Henan	133.3	137.2	141.2	6.7	6.9	7.1
Hubei	147.9	153.3	158.8	7.4	7.7	7.9
Hunan	173.1	160.8	149.3	8.7	8.0	7.5
Guangdong	218.2	211.4	204.8	10.9	10.6	10.2
Guangxi	176.9	159.6	144.1	8.8	8.0	7.2
Hainan	31.7	29.5	27.5	1.6	1.5	1.4
Chongqing	21.8	23.1	24.5	1.1	1.2	1.2
Sichuan	129.5	134.8	140.3	6.5	6.7	7.0
Guizhou	48.7	47.3	45.9	2.4	2.4	2.3
Yunnan	89.8	81.6	74.1	4.5	4.1	3.7
Tibet	30.7	31.1	31.5	1.5	1.6	1.6
Shanxi	61.1	65.4	70.1	3.1	3.3	3.5
Gansu	93.0	91.6	90.2	4.6	4.6	4.5
Qinghai	24.2	25.8	27.6	1.2	1.3	1.4
Ningxia	61.7	58.1	54.6	3.1	2.9	2.7
Xinjiang	512.7	538.0	564.6	25.6	26.9	28.2

6.1.3 Decrease in Water Pollution

Water pollution is an extremely serious problem in China. We have known the objective "total quantity of pollutant discharge should decrease by 10%" put forward in "The Outline of the Eleventh Five-Year Plan of China", so we can similarly think that sewage discharge should decrease by 10% per year.

Based on statistics from 2006 to 2011(see **Appendix 5**), we can approximately calculate the volume of sewage discharge, and obtain predicted value of sewage discharge in 2015, 2020 and 2025. Allowing for the objective of "sewage discharge should decrease by 10% per year", we obtain the decrease of sewage discharge in 2015, 2020 and 2025. See **Table 13**.

Table 13.
Predicted value and decrease of sewage discharge in 2015, 2020 and 2025(100 million cubic meters).

Province	Predicted value			Decrease		
	2015	2020	2025	2015	2020	2025
China	751.2	904.9	1090.0	75.1	90.5	109.0
Beijing	19.3	26.9	37.5	1.9	2.7	3.7
Tianjin	7.6	8.8	10.2	0.8	0.9	1.0
Hebei	33.2	41.9	52.9	3.3	4.2	5.3
Shanxi	13.0	14.9	17.1	1.3	1.5	1.7
Inner Mongolia	15.0	25.4	42.8	1.5	2.5	4.3
Liaoning	24.8	27.7	30.8	2.5	2.8	3.1
Jilin	12.6	14.0	15.6	1.3	1.4	1.6
Heilongjiang	18.9	27.8	40.8	1.9	2.8	4.1
Shanghai	23.1	23.4	23.7	2.3	2.3	2.4
Jiangsu	68.3	83.5	102.0	6.8	8.3	10.2
Zhejiang	50.5	64.8	83.1	5.1	6.5	8.3
Anhui	32.6	51.8	82.2	3.3	5.2	8.2
Fujian	37.9	53.0	74.2	3.8	5.3	7.4
Jiangxi	26.4	41.1	64.2	2.6	4.1	6.4
Shandong	57.5	78.6	107.4	5.7	7.9	10.7
Henan	47.2	62.6	83.1	4.7	6.3	8.3
Hubei	32.3	37.6	43.7	3.2	3.8	4.4
Hunan	31.1	35.9	41.4	3.1	3.6	4.1
Guangdong	88.9	107.3	129.5	8.9	10.7	13.0
Guangxi	18.0	11.7	7.6	1.8	1.2	0.8
Hainan	3.6	3.6	3.6	0.4	0.4	0.4
Chongqing	11.5	9.8	8.4	1.2	1.0	0.8
Sichuan	28.3	29.9	31.6	2.8	3.0	3.2
Guizhou	10.1	15.3	23.2	1.0	1.5	2.3
Yunnan	23.9	51.4	110.2	2.4	5.1	11.0
Tibet	0.6	0.9	1.3	0.1	0.1	0.1
Shanxi	14.2	17.4	21.2	1.4	1.7	2.1
Gansu	7.1	9.3	12.2	0.7	0.9	1.2
Qinghai	2.4	2.8	3.1	0.2	0.3	0.3
Ningxia	4.3	4.6	5.0	0.4	0.5	0.5
Xinjiang	9.6	11.4	13.4	1.0	1.1	1.3

6.1.4 Summary

Finally, we obtain the decrease in water needs after implementing the three strategies (see **Table 14**).

Table 14.
Decrease in water needs in 2015, 2020 and 2025

(100 million cubic meters).

Province	2015	2020	2025	Province	2015	2020	2025
China	879.5	829.1	917.1	Henan	34.3	38.5	46.6
Beijing	1.8	3.8	4.6	Hubei	60.8	72.6	94.7
Tianjin	2.4	2.8	2.9	Hunan	46.8	46.7	51.6
Hebei	20.1	17.5	18.0	Guangdong	62.2	62.3	64.2
Shanxi	9.1	7.5	7.9	Guangxi	36.8	35.1	40.4
Inner Mongolia	22.9	26.4	38.9	Hainan	4.4	3.5	3.6
Liaoning	18.0	17.0	18.7	Chongqing	20.3	25.5	32.4
Jilin	20.3	21.1	28.1	Sichuan	34.2	32.6	35.4
Heilongjiang	61.1	40.9	48.7	Guizhou	16.3	17.3	20.2
Shanghai	27.6	30.6	31.9	Yunnan	17.8	21.8	31.2
Jiangsu	94.7	83.2	85.2	Tibet	4.2	4.1	7.4
Zhejiang	26.3	29.4	31.3	Shanxi	10.2	8.6	9.2
Anhui	67.2	66.5	88.2	Gansu	12.9	9.0	8.9
Fujian	40.1	49.0	61.4	Qinghai	3.5	2.2	2.2
Jiangxi	43.1	37.4	44.5	Ningxia	7.4	5.3	5.7
Shandong	23.4	25.0	28.7	Xinjiang	58.6	34.6	38.6

Combining the predicted value of spare water resources, we can obtain the spare water resources in 2015, 2020 and 2025 after implementing strategies of save and conservation (see **Table 15**).

Table 15.

Spare water resources in 2015, 2020 and 2025 after taking strategies of save and conservation(100 million cubic meters).

Province	2015	2020	2025	Province	2015	2020	2025
China	19281.1	19180.4	19194.5	**Henan**	74.3	22.2	**-25.7**
Beijing	**-8.1**	**-4**	**-0.9**	Hubei	596.9	619.3	648.7
Tianjin	**-8.4**	**-6.9**	**-5.5**	Hunan	899.4	762.6	646.9
Hebei	**-27.5**	**-13.2**	5.2	Guangdong	1217.3	1319.2	1429.7
Shanxi	41.9	45.3	51.5	Guangxi	1105.1	1034.8	975.1
Inner Mongolia	164.6	136.7	119.1	Hainan	727.4	1406.1	2686.1
Liaoning	171.1	225	294.6	Chongqing	343.4	308	270.8
Jilin	211.9	200.3	188.7	Sichuan	1874.5	1776	1683.4
Heilongjiang	307.9	264.9	236.6	Guizhou	609.2	545.3	488.8
Shanghai	**-71.8**	**-70.4**	**-70.5**	Yunnan	1261	1097.3	958.7
Jiangsu	**-35.8**	29.9	131.3	Tibet	4100.2	4027	3958.5
Zhejiang	829.9	1001.3	1197.4	Shanxi	547.9	852	1313.6
Anhui	474.5	505	548.3	Gansu	104.5	116.5	133.5
Fujian	601	559.7	521	Qinghai	714.4	789.6	873.9
Jiangxi	1048.8	1138.2	1247	**Ningxia**	**-54.9**	**-55.1**	**-52.8**
Shandong	48.4	27.2	9	Xinjiang	358.5	328.5	326

Obviously, water shortage problem is greatly relieved after taking strategies of save and conservation. We assess the risk of provinces suffering from water shortage again (see **Table 16**), and find that water shortage problem of Shandong Province in 2025 has been solved.

Table 16.
Risk assessment of provinces suffering from water shortage.

Province	Beijing	Tianjin	Hebei	Shanghai	Jiangsu	Shandong	Henan	Ningxia
2015	low	medium	very low	extreme	very low	very low	very low	extreme
2020	very low	medium	very low	extreme	very low	very low	very low	extreme
2025	very low	low	very low	extreme	very low	very low	very low	extreme

6.2 Movement

6.2.1 South-to-North Water Transfer Project

South-to-North Water Transfer Project of China is a huge project, which consists of three routes: western route, middle route and eastern route. We find that this project has covered most of the provinces suffering from water shortage in 2025 we have predicted, so we can make full use of this project to transfer water from provinces rich in water resources to provinces suffering from water shortage.

6.2.2 Analyses of Water Price

Comprehensive cost of supporting projects (see **Table 17**) is determined by distance of water delivery, water price of provinces that supply the water and cost of purification[9].

Table 17.
Comprehensive cost of supporting projects.

Province	Beijing	Tianjin	Hebei	Henan
Comprehensive cost of supporting projects($Yuan/m^3$)	1.51	1.59	0.92	0.42

6.2.3 Cost Estimation

We find the provinces suffering from water shortage in 2025 that this project has covered, and their water deficit is shown in the following table.

Table 18.
Water deficit in 2015, 2020 and 2025(100 million cubic meters).

Province	2015	2020	2025
Beijing	8.1	4.0	0.9
Tianjin	8.4	6.9	5.5
Hebei	27.5	13.2	0.0
Heinan	0.0	0.0	25.7
Total	44.0	24.2	32.1

Table 19 shows the cost of water movement.

Table 19.
Cost of water movement in 2015, 2020 and 2025 (100 million Yuan).

Province	2015	2020	2025
Beijing	12.2	6.1	1.3
Tianjin	13.4	11.0	8.8
Hebei	24.8	11.9	0.0
Heinan	0.0	0.0	10.7
Total	50.4	29.1	20.8

6.3 De-salinization

6.3.1 Introduction

Although up-front investment of de-salinization is relatively high, its influence on environment and ecology is much smaller, and de-salinization will not be influenced by weather and climate. Moreover, the cost of de-salinization has always been decreasing.

6.3.2 Cost Estimation

The current cost of de-salinization methods, distillation and reverse osmosis, are between 5 to 6 yuan per cubic meter[10], and will decrease by 5% per year with the development of technology and implement of preferential policy[11].

We apply the following equation to estimate the cost of de-salinization:

$$F(n) = a_1 \times (1-r)^{n-2012}$$

Where

n is the year,

$F(n)$ is the cost of de-salinization,

a_1 is the cost of de-salinization in 2012($a_1 = 5.5$), and

r is the rate of decrease($r = 5\%$).

Costs of de-salinization in 2015, 2020 and 2025 are as follows.

Table 20.
Costs of de-salinization in 2015, 2020 and 2025.

Year	2015	2020	2025
Cost (Yuan/m3)	4.72	3.65	2.82

According to our prediction, Ningxia Hui Autonomous Region will suffer from serious water shortage in 2025, but "South-to-North Water Transfer Project" does not cover it. Meanwhile, the cost of construction of ditch will be very large. However, we find that there is plenty of brackish water which can be used for de-salinization. Cost of de-salinization in Ningxia is as follows.

Table 21.
Costs of de-salinization in Ningxia.

Year	2015	2020	2025
Water shortage（100 million cubic meters）	54.9	55.1	52.8
Total cost(100 million Yuan)	259.0	200.9	149.1

6.3.3 Comparative Analyses between De-salinization and Movement

Next, we will give a brief analysis of the two strategies: de-salinization and movement. See **Table 22**.

Table 22.

Comparison of costs between de-salinization and movement(Yuan/m^3).

Strategy	2015	2020	2025
South-to-North Water Transfer Project	0.42~1.54		
De-salinization	4.72	3.65	2.82

We find that the cost of de-salinization will decrease year by year, and will surely be lower than the cost of movement in the future. But before 2025, the cost of de-salinization is still higher than that of movement.

6.4 Storage

Whether to build a reservoir or not depends on the economic influence, building environment and transport situation[12]. Until 2008, there had been more than 86353 reservoirs, and the total reservoir storage has reached 692.4 billion cubic meter, ranking the fourth in the world[13].

Allowing for the facts that there have been so many reservoirs, reservoir capacity may not be able to satisfy the requirement, and building environment is not suitable, we determine not to build more reservoirs.

6.5 The economic, physical, and environmental implications of the strategy

6.5.1 Introduction and Analysis

In order to evaluate the economic, physical, and environmental implications of the four strategies, we apply the Analytical Hierarchy Process (AHP) to give the comprehensive implications and strategy Prioritization.

The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions. It provides a comprehensive and rational framework for evaluating alternative solutions by putting a set of alternatives in order from most to least desirable[14].

6.5.2 The Analytic Hierarchy Process Model

Step1: The establishment of a hierarchy

The problem of case can be divided into three layers in order

- Objective layer (O) : rank the strategies by comprehensive implications level
- Criteria layer (C): the three implications, marked with $C_k(k=1,2,3)$
- Strategy layer (S) : 4 strategies, marked with $S_n(n=1,2,3,4)$

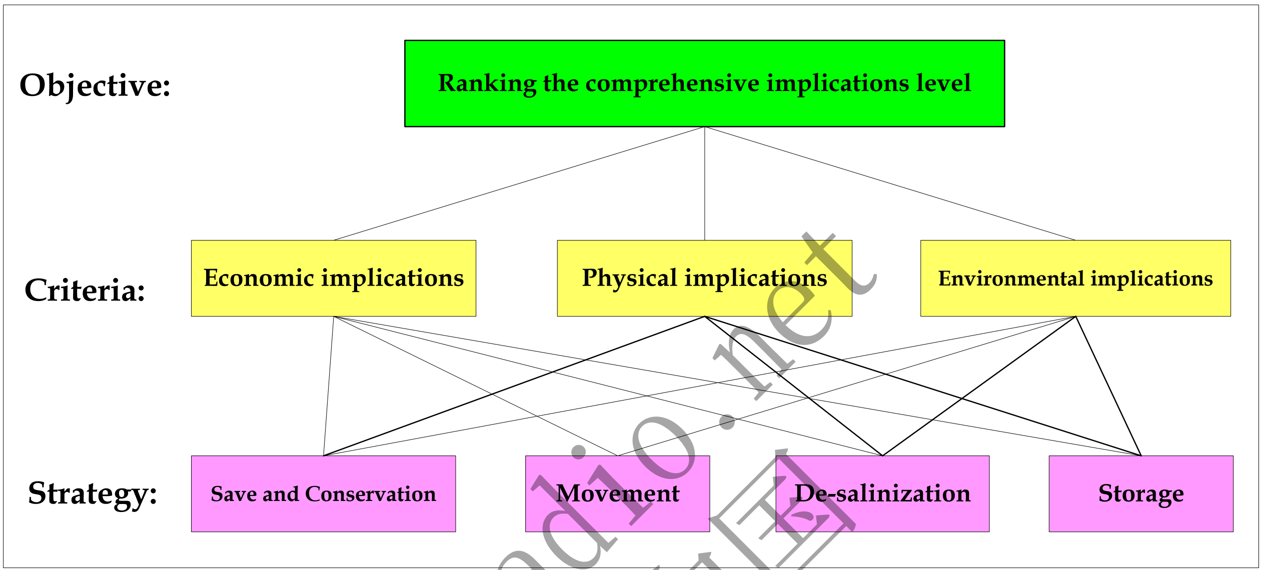

Figure 3. AHP hierarchy for ranking the comprehensive implications level

Step2: The weight of layer C in layer O

We use the grad $a_{i,j}(i,j=1,2,3)$ to represent the relative importance of C_i and C_j to objective layer O.

The table below shows the relative importance of different grade.

Table 23.

Relative importance of different grade[15].

Grade	Relative importance
1	Equally Important
3	Generally more Important
5	Far more Important
7	More Important at the second highest degree
9	More Important at the highest
2,4,6,8	Importance level is in between
$\frac{1}{2},\cdots,\frac{1}{9}$	Less important

Considering the relative importance of C1 compared with C2 and C3, we might arrive at the following pairwise comparison matrix

$$A=\begin{pmatrix} 1 & 9/7 & 9/5 \\ 7/9 & 1 & 7/5 \\ 5/9 & 5/7 & 1 \end{pmatrix}$$

The max eigenvalue $\lambda_{\max}=3$, and we can get the corresponding normalized eigenvector.

$$W_1=\begin{pmatrix} 0.43 \\ 0.33 \\ 0.24 \end{pmatrix}$$

Step3: Consistent test

$$CI=\frac{\lambda_{\max}-n}{n-1}$$

When $n=3$, $\lambda_{\max}=3$, $CI=0$

Random Consistency Index $RI=0$, it can be get from the table below.

Table 24.

Random consistency index.

Number of order	1	2	3	4	5	6	7	8	9
	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45

Consistency Ratios

$$CR=\frac{CI}{RI}$$

When $CI=0$, $RI=0$, $CR=0$ meets the inspection.

Step 4: The weights of layer S in layer C

Using the value table below, we can make up three comparison matrices separately.

Table 25.

Strategy Value of Different Implications.

Strategy \ Implications	Economical	Physical	Environmental
Save and conservation	9	9	9
Movement	8	3.5	3

De-salinization	2	2	5
Storage	1	1	7

$$B_k = (b_{(i,j)}^k)_{N\times N}$$

$$b_{(i,j)}^{(k)} = \frac{T_i^{(k)}}{T_j^{(k)}} (i, j = 1,2,3,4; k = 1,2,3)$$

Obviously, all $B_k (k = 1,2,3)$ are consistent matrices. So the maximum eigenvalue of $B^{(k)}$, $\lambda_{\max}^{(k)} = N = 4$, $CR_2^{(k)} = 0$, and any column vector of $B^{(k)}$ is the eigenvector of $\lambda_{\max}^{(k)}$. After normalization, we can get weight vector

$$W^{(k)} = (w_1^{(k)}, w_2^{(k)}, \ldots, w_N^{(k)})^T (k = 1,2,3)$$

$$W_2 = [W^{(1)}, W^{(2)}]_{N\times 2}$$

$$CR_2 = \sum_{k=1}^{3} CR_2^{(k)} = 0$$

Step 5: The weights of layer S in layer O

$$W = W_2 \bullet W_1 = [W^{(1)}, W^{(2)}] \bullet W_1 = (w_1, w_2, \ldots w_N)$$

$$CR = CR_1 + CR_2 = 0 < 0.1$$

Step 6: Comprehensive rank

$W_n (n = 1,2,3,4)$ represents everyone's weight in the case. Thus the rank of W_n can be used to prioritize the 4 strategies by comprehensive implications level.

Table 26.
Rank of strategy.

Strategy	Weights	Rank
Save and conservation	0.48	1
Movement	0.28	2
De-salinization	0.14	3
Storage	0.11	4

6.6 Analyses of Water Strategies for China

From the table above we can conclude that water-saving and conservation is the most important things that we should do to deal with the water shortage. And we should also make the most of the South-to-North Water Transfer Project to adjust the distribution of water resources. For the provinces that are not covered by the water transfer project， we can choose de-salinization to meets the water needs. While construction of reservoirs still remains a method in many places, we should notice that there are serious security problems with it, as the 40% of reservoirs in China have potential security liability. For such security consideration, we exclude the plan of constructing reservoirs.

Our detailed water strategies are as follows:

Objectives put forward by China government in "the Eleventh Five-Year Plan" is practical and crucial, which will lead to decrease in industrial water needs, increase in effective utilization coefficient of agricultural water and decrease in water pollution. This strategy will solve the water shortage problem of Shandong Province in 2025.

We make full use of the "South-to-North Water Transfer Project" to solve the water shortage problems of Beijing, Tianjin, Hebei Province and Henan Province.

"South-to-North Water Transfer Project" does not cover Ningxia, so we make use of its brackish water for de-salinization and solve its water shortage problem.

As for Shanghai and Jiangsu Province, we just need to transfer water from neighbor provinces, such as Zhejiang Province.

Our final water strategies are shown in a visual and clear way. See **Figure 4**.

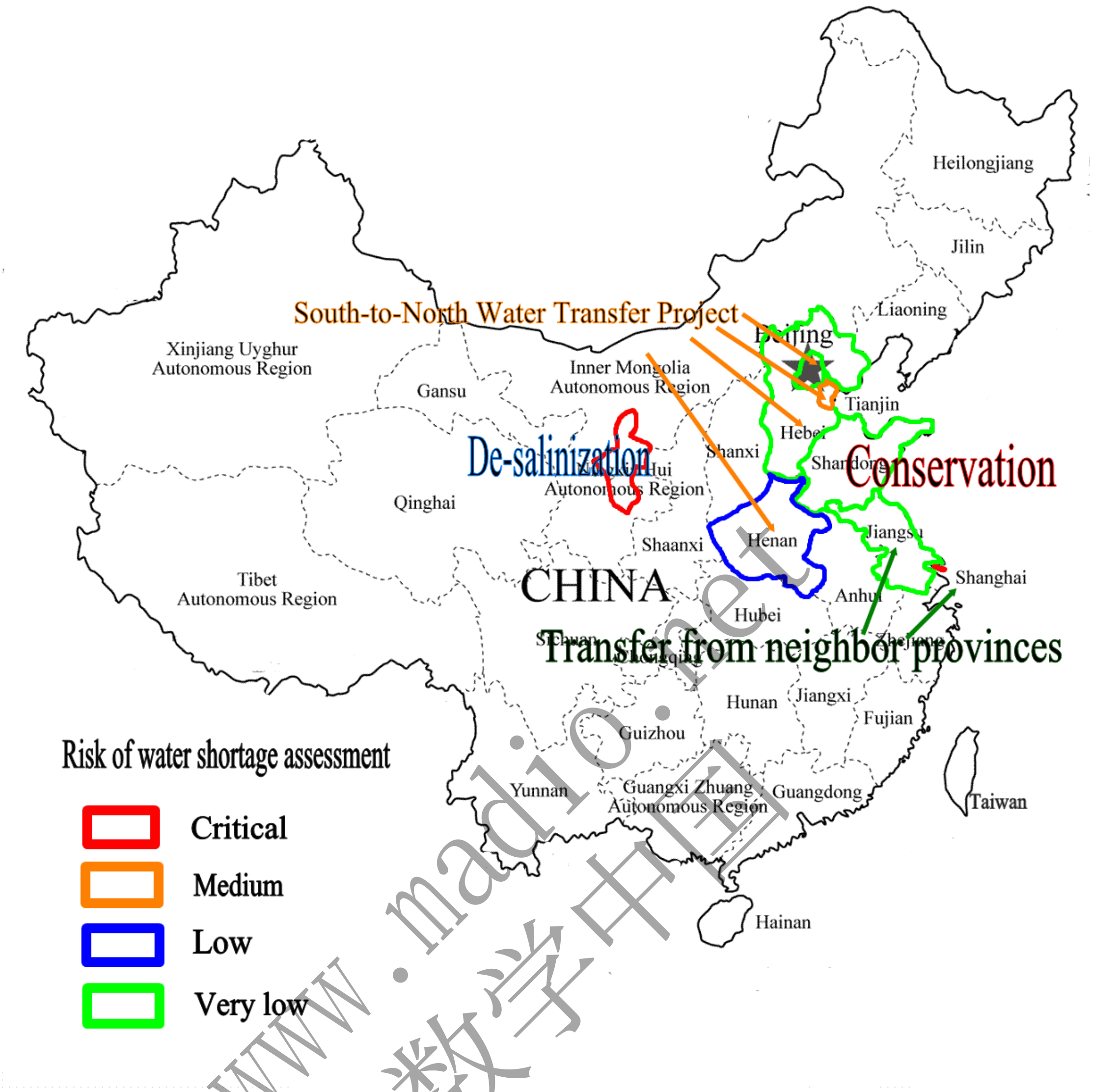

Figure 4.

Costs of our final water strategies are as follows:

Table 27.
Costs of different water strategies.

strategy	Province	2015		2020		2025	
		Water supply	Cost	Water supply	Cost	Water supply	Cost
Save and conservation	All provinces	211.8		206.7		223.8	
South-to-North Water Transfer Project	Beijing, Tianjin, Hebei, Henan	50.4	50.4	29.1	29.1	20.8	20.8
De-salinization	Ningxia	54.9	259.0	55.1	200.9	52.8	149.1
Transfer from neighbor provinces	Shanghai，Jiangshu	107.6		70.4		70.5	
Total		424.7	309.4	361.2	229.9	367.9	170.0

As we can see from **Table 27**, save and conservation strategy is the most effective one. At the same time, the cost of our strategies are decreasing year by year, which indicates that our strategies are practical and economical.

7 Evaluation and Generalization

7.1 Strengths

- Our models cover plenty of information, and have an overall analysis of practical problems.
- We apply the grey prediction model and linear regression model for predicting, and give an error analysis and adjustment of the fitted and predicted value, which make it more convinced and credible.
- We define the water shortage degree to assess the risk of water shortage.

7.2 Weaknesses

- We eventually adopt the predicted value of grey prediction model because its error is relatively small, but a model whose error is small is not necessarily a good one.
- Our "Total water resources" consist of some unavailable water resources, such as the deep aquifers.
- We only define a few variables to do risk assessment, which is obviously insufficient.

7.3 Generalization

- Computer programing can deal with plenty of data all at once, which is convenient and efficient. Therefore, our models can also be applied to other countries and fields.
- The grey model can deal with systems that are characterized by poor information and for which information is lacking. It is a useful method for short-term prediction and can be applied to other problems, such as prediction of disease and population.

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A position paper to governmental leadership

To the government officials:

As we all know, Water is an important natural resource that is used every day, not only do we humans use it just about every day, but every living thing needs it to live. Environmental problems, water waste and pollution have created many challenges for water conservancy in China, causing serious problems. To meet the projected water needs of China in 2025, we establish a mathematical model to determining an effective, feasible, and cost-efficient water strategy for 2013.

To forecast the water shortage problem in the future, we establish two models to predict the total water resources and total water needs from 2013 to 2015, based on the documents we get from the Ministry of Water Resources.

Then we work out the water deficit and water shortage degree of each province in the future. The tables below shows water shortage risk assessment of different provinces.

Table 1.
Risk assessment of provinces suffering from water shortage.

Province	Beijing	Tianjin	Hebei	Shanghai	Jiangsu	Shandong	Henan	Ningxia
2015	low	high	low	extreme	low	very low	very low	extreme
2020	low	medium	very low	extreme	very low	very low	very low	extreme
2025	very low	medium	very low	extreme	very low	very low	low	extreme

The results shows that we must pay close attention to Shanghai and Ningxia which are in great water shortage.

To meet the projected water needs, we formulate four different strategies that consist of water save and conservation, movement of water resources among adjacent regions, de-salinization and water storage. Afterwards, we analyze each strategy by feasibility, water-saving and economic benefits. Considering the economical, physical, environmental implications of the four strategies, we give a table below to show strategy prioritization by comprehensive implications.

Table 2.
Rank of strategy.

Strategy	Weights	Rank
Save and conservation	0.48	1
Movement	0.28	2
De-salinization	0.14	3
Storage	0.11	4

From the table above we can conclude that water-saving and conservation is the most important things that we should do to deal with the water shortage. And we should also make the most of the South-to-North Water Transfer Project to adjust the distribution of water resources. For the provinces that are not covered by the water transfer project，we can choose de-salinization to meets the water needs. While construction of reservoirs still remains a method in many places, we should notice that there are serious security problems with it, as the 40% of reservoirs in China have potential security liability. For such security consideration, we exclude the plan of constructing reservoirs.

Our detailed water strategies are as follows:

Objectives put forward by China government in “the Eleventh Five-Year Plan” is practical and crucial, which will lead to decrease in industrial water needs, increase in effective utilization coefficient of agricultural water and decrease in water pollution. This strategy will solve the water shortage problem of Shandong Province in 2025.

We make full use of the “South-to-North Water Transfer Project” to solve the water shortage problems of Beijing, Tianjin, Hebei Province and Henan Province.

“South-to-North Water Transfer Project” does not cover Ningxia, so we make use of its brackish water for de-salinization and solve its water shortage problem.

As for Shanghai and Jiangsu Province, we just need to transfer water from neighbor provinces, such as Zhejiang Province.

Our final water strategies are shown in a visual and clear way. See **Figure 1**.

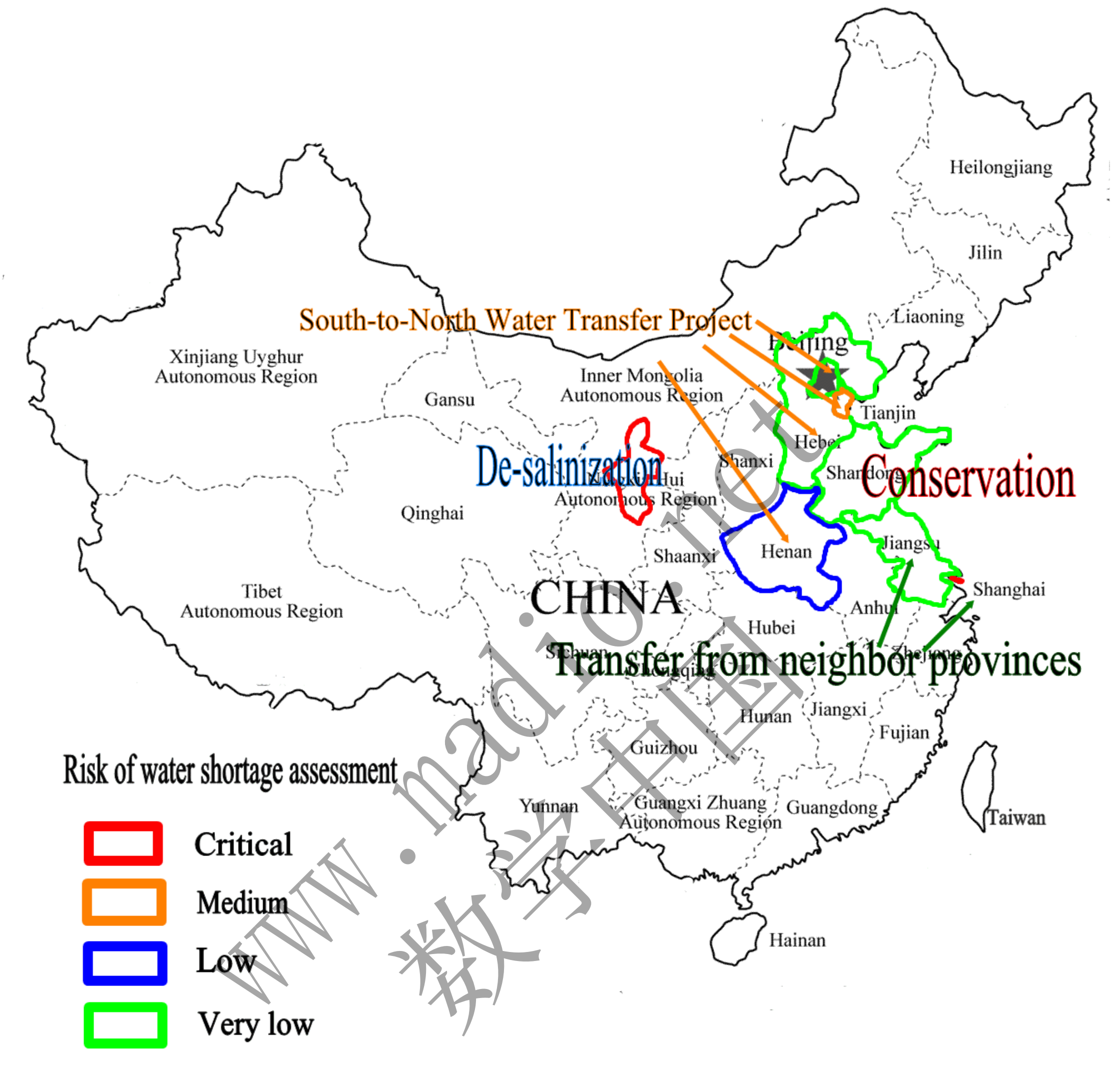

Figure 1.

Costs of our final water strategies are as follows:

Table 3.
Costs of different water strategies.

strategy	Province	2015		2020		2025	
		Water supply	Cost	Water supply	Cost	Water supply	Cost
Save and conservation	All provinces	211.8		206.7		223.8	
South-to-North Water Transfer Project	Beijing, Tianjin, Hebei, Henan	50.4	50.4	29.1	29.1	20.8	20.8
De-salinization	Ningxia	54.9	259.0	55.1	200.9	52.8	149.1

Transfer from neighbor provinces	Shanghai，Jiangshu	107.6		70.4		70.5	
Total		424.7	309.4	361.2	229.9	367.9	170.0

As we can see from **Table 3**, save and conservation strategy is the most effective one. At the same time, the cost of our strategies are decreasing year by year, which indicates that our strategies are practical and economical.

The analysis above shows that our plans are feasible and effective, and it is the best choice as well.

Yours Sincerely

2013-2-4

Appendices

Appendix 1

Fitted value of total water resources from 2003 to 2011

Model	Gray Model								
Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	27460.2	25787.4	25867.2	25947.3	26027.5	26108.1	26188.9	26269.9	26351.2
Beijing	18.4	22.7	23.2	23.7	24.2	24.8	25.3	25.9	26.5
Tianjin	10.6	12.1	12.4	12.6	12.9	13.2	13.5	13.7	14.0
Heibei	153.1	131.3	133.5	135.7	138.0	140.3	142.7	145.1	147.5
Shanxi	134.9	84.6	87.3	90.1	92.9	95.9	98.9	102.1	105.3
Inner Mongolia	495.6	420.0	414.1	408.2	402.4	396.8	391.2	385.6	380.2
Liaoning	220.0	274.5	285.3	296.5	308.1	320.2	332.8	345.8	359.4
Jilin	326.5	385.2	389.9	394.6	399.4	404.2	409.1	414.1	419.1
Heilongjiang	826.8	646.3	659.3	672.6	686.2	700.0	714.1	728.5	743.2
Shanghai	15.1	28.3	29.0	29.8	30.5	31.3	32.1	32.9	33.8
Jiangsu	619.1	348.1	362.6	377.8	393.5	409.9	427.0	444.8	463.3
Zhejiang	574.5	832.9	858.2	884.3	911.2	938.9	967.5	996.9	1027.2
Anhui	1083.0	601.7	623.6	646.2	669.7	694.0	719.3	745.4	772.5
Fujian	806.6	1165.3	1156.3	1147.4	1138.6	1129.9	1121.2	1112.5	1104.0
Jiangxi	1362.7	1285.2	1314.0	1343.4	1373.5	1404.3	1435.8	1468.0	1500.9
Shangdong	489.7	343.1	338.6	334.2	329.8	325.5	321.2	317.0	312.8
Henan	697.7	446.0	436.6	427.4	418.3	409.5	400.8	392.3	384.0
Hubei	1234.1	882.0	894.0	906.1	918.4	930.9	943.5	956.3	969.3
Hunan	1799.2	1707.6	1665.7	1624.9	1585.0	1546.1	1508.1	1471.1	1435.0
Guangdong	1458.4	1676.9	1698.1	1719.6	1741.3	1763.4	1785.7	1808.2	1831.1
Guangxi	1901.0	1753.7	1735.6	1717.7	1700.0	1682.5	1665.1	1648.0	1631.0
Hainan	291.8	220.8	250.6	284.5	323.0	366.6	416.1	472.4	536.2
Chongqing	590.7	531.9	527.1	522.4	517.7	513.0	508.4	503.8	499.3
Sichuan	2589.8	2459.3	2440.6	2422.1	2403.7	2385.4	2367.3	2349.3	2331.4
Guizhou	915.5	977.8	959.5	941.6	923.9	906.6	889.7	873.0	856.7
Yunnan	1699.4	2077.4	2025.4	1974.7	1925.3	1877.1	1830.2	1784.4	1739.7
Tibet	4757.1	4451.7	4436.1	4420.6	4405.1	4389.6	4374.2	4358.9	4343.6
Shanxi	574.6	302.7	328.2	355.7	385.6	418.1	453.2	491.3	532.6
Gansu	247.2	203.1	206.0	209.0	212.0	215.0	218.1	221.2	224.4
Qinghai	634.7	669.3	682.6	696.1	709.9	723.9	738.3	752.9	767.8
Ningxia	12.3	9.9	9.7	9.6	9.4	9.3	9.2	9.1	8.9
Xinjiang	920.1	887.0	890.8	894.6	898.5	902.4	906.3	910.2	914.1

Model	Linear Regression Model								
Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	26364.3	26329.0	26293.7	26258.4	26223.1	26187.8	26152.5	26117.2	26081.9
Beijing	20.6	21.4	22.2	23.1	23.9	24.7	25.5	26.3	27.1
Tianjin	11.4	11.7	12.1	12.4	12.8	13.1	13.5	13.8	14.2
Heibei	138.3	138.9	139.6	140.2	140.8	141.4	142.1	142.7	143.3
Shanxi	102.0	101.3	100.6	99.9	99.2	98.4	97.7	97.0	96.3
Inner Mongolia	451.1	440.9	430.8	420.6	410.5	400.3	390.2	380.0	369.9
Liaoning	248.0	262.2	276.5	290.7	304.9	319.2	333.4	347.7	361.9
Jilin	360.1	368.5	376.8	385.2	393.6	401.9	410.3	418.6	427.0
Heilongjiang	706.0	706.6	707.3	707.9	708.6	709.3	709.9	710.6	711.3
Shanghai	22.4	24.1	25.8	27.5	29.2	30.9	32.6	34.3	36.0
Jiangsu	435.7	433.6	431.4	429.3	427.2	425.0	422.9	420.8	418.6
Zhejiang	712.1	756.0	800.0	843.9	887.8	931.8	975.7	1019.6	1063.6
Anhui	762.2	753.7	745.2	736.7	728.2	719.7	711.2	702.7	694.2
Fujian	1037.5	1052.6	1067.7	1082.8	1098.0	1113.1	1128.2	1143.3	1158.5
Jiangxi	1289.9	1314.3	1338.7	1363.0	1387.4	1411.8	1436.2	1460.6	1485.0
Shangdong	400.6	386.9	373.2	359.5	345.8	332.1	318.4	304.7	291.0
Henan	546.1	521.1	496.0	470.9	445.9	420.8	395.7	370.7	345.6
Hubei	1006.3	994.5	982.8	971.1	959.4	947.7	936.0	924.3	912.6
Hunan	1767.2	1723.8	1680.4	1637.0	1593.6	1550.2	1506.8	1463.4	1419.9
Guangdong	1573.0	1609.8	1646.6	1683.4	1720.2	1757.0	1793.8	1830.6	1867.4
Guangxi	1822.8	1795.8	1768.8	1741.9	1714.9	1687.9	1661.0	1634.0	1607.0
Hainan	200.5	237.8	275.0	312.2	349.4	386.7	423.9	461.1	498.4
Chongqing	557.1	548.8	540.4	532.1	523.8	515.5	507.2	498.9	490.5
Sichuan	2518.9	2493.3	2467.7	2442.1	2416.6	2391.0	2365.4	2339.8	2314.3
Guizhou	968.3	955.2	942.1	929.0	915.9	902.9	889.8	876.7	863.6
Yunnan	1968.6	1946.7	1924.9	1903.1	1881.3	1859.5	1837.7	1815.9	1794.1
Tibet	4575.8	4541.2	4506.6	4472.0	4437.4	4402.9	4368.3	4333.7	4299.1
Shanxi	389.2	399.1	409.0	418.9	428.8	438.7	448.6	458.5	468.4
Gansu	217.9	217.7	217.6	217.5	217.3	217.2	217.0	216.9	216.8
Qinghai	646.8	662.2	677.6	693.0	708.4	723.8	739.2	754.6	770.0
Ningxia	10.9	10.6	10.3	10.0	9.7	9.4	9.1	8.8	8.6
Xinjiang	897.3	898.6	900.0	901.3	902.7	904.0	905.4	906.7	908.1

Appendix 2

Fitted value of total water needs from 2003 to 2011

Model	Gray Model								
Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	5320.4	5584.8	5658.6	5733.2	5808.9	5885.6	5963.3	6042.0	6121.7
Beijing	35.0	34.3	34.5	34.7	34.9	35.1	35.3	35.5	35.7

Tianjin	20.5	22.7	22.7	22.8	22.8	22.9	22.9	23.0	23.0
Heibei	199.8	201.1	200.1	199.2	198.3	197.3	196.4	195.5	194.6
Shanxi	56.2	53.5	55.2	57.1	58.9	60.9	62.9	65.0	67.1
Inner Mongolia	166.9	173.2	174.7	176.2	177.8	179.3	180.9	182.5	184.1
Liaoning	128.3	133.8	135.6	137.4	139.2	141.0	142.9	144.8	146.7
Jilin	104.0	93.5	97.4	101.5	105.8	110.2	114.8	119.6	124.6
Heilongjiang	245.8	259.0	269.7	280.9	292.5	304.7	317.3	330.4	344.1
Shanghai	109.0	118.0	119.1	120.1	121.2	122.2	123.3	124.4	125.5
Jiangsu	433.5	530.0	534.4	538.9	543.4	547.9	552.5	557.1	561.8
Zhejiang	206.0	211.8	210.3	208.8	207.3	205.9	204.4	202.9	201.5
Anhui	178.6	207.8	219.8	232.4	245.7	259.8	274.7	290.4	307.0
Fujian	182.8	183.9	187.2	190.5	193.9	197.3	200.8	204.4	208.0
Jiangxi	172.5	201.4	208.7	216.2	224.0	232.1	240.4	249.1	258.1
Shangdong	219.4	215.4	216.6	217.8	219.1	220.3	221.5	222.8	224.1
Henan	187.6	203.8	207.9	212.1	216.3	220.7	225.1	229.6	234.2
Hubei	245.1	243.2	250.1	257.2	264.5	272.0	279.7	287.7	295.9
Hunan	318.8	325.7	325.6	325.4	325.3	325.1	325.0	324.8	324.7
Guangdong	457.5	460.6	461.3	462.0	462.6	463.3	464.0	464.7	465.4
Guangxi	278.4	306.2	306.1	305.9	305.7	305.6	305.4	305.3	305.1
Hainan	46.3	46.2	46.0	45.8	45.6	45.4	45.2	45.0	44.8
Chongqing	63.2	68.8	71.5	74.2	77.1	80.0	83.1	86.3	89.6
Sichuan	209.9	207.1	210.2	213.4	216.6	219.8	223.1	226.5	229.9
Guizhou	93.7	97.1	97.5	98.0	98.4	98.9	99.3	99.7	100.2
Yunnan	146.1	147.4	147.7	148.0	148.4	148.7	149.1	149.4	149.8
Tibet	25.3	32.7	32.9	33.1	33.3	33.5	33.8	34.0	34.2
Shanxi	75.1	78.0	79.3	80.6	81.9	83.2	84.6	86.0	87.4
Gansu	121.6	122.3	122.3	122.2	122.2	122.1	122.1	122.0	122.0
Qinghai	29.0	31.2	31.2	31.2	31.2	31.2	31.2	31.2	31.2
Ningxia	64.0	76.0	75.5	74.9	74.4	73.9	73.3	72.8	72.3
Xinjiang	500.7	503.7	508.1	512.5	517.0	521.5	526.0	530.6	535.2

Model	Linear Regression Model								
Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	5434.4	5523.6	5612.7	5701.9	5791.0	5880.2	5969.3	6058.5	6147.6
Beijing	34.4	34.6	34.7	34.9	35.0	35.1	35.3	35.4	35.6
Tianjin	21.8	22.0	22.2	22.4	22.6	22.8	23.0	23.2	23.4
Heibei	201.2	200.4	199.6	198.8	198.0	197.3	196.5	195.7	194.9
Shanxi	53.4	54.9	56.5	58.1	59.7	61.2	62.8	64.4	65.9
Inner Mongolia	169.8	171.7	173.5	175.4	177.3	179.2	181.0	182.9	184.8
Liaoning	130.5	132.6	134.7	136.8	138.9	141.0	143.1	145.1	147.2
Jilin	94.8	98.1	101.4	104.7	108.0	111.3	114.6	117.9	121.2
Heilongjiang	245.7	257.7	269.8	281.8	293.9	305.9	318.0	330.0	342.1
Shanghai	114.0	115.5	117.1	118.7	120.3	121.9	123.5	125.1	126.7
Jiangsu	490.5	501.2	511.9	522.6	533.3	544.0	554.6	565.3	576.0
Zhejiang	210.5	209.5	208.5	207.5	206.5	205.5	204.5	203.5	202.5

Anhui	185.8	200.9	216.0	231.1	246.2	261.4	276.5	291.6	306.7
Fujian	181.2	184.5	187.8	191.0	194.3	197.6	200.9	204.2	207.4
Jiangxi	184.9	194.3	203.7	213.1	222.5	231.9	241.4	250.8	260.2
Shangdong	216.1	217.0	217.9	218.8	219.7	220.6	221.4	222.3	223.2
Henan	194.6	199.8	204.9	210.1	215.3	220.4	225.6	230.7	235.9
Hubei	238.8	245.7	252.5	259.3	266.2	273.0	279.8	286.7	293.5
Hunan	323.2	323.5	323.9	324.2	324.5	324.8	325.1	325.4	325.8
Guangdong	459.0	459.9	460.7	461.5	462.4	463.2	464.1	464.9	465.7
Guangxi	295.8	297.5	299.2	300.9	302.6	304.4	306.1	307.8	309.5
Hainan	46.4	46.2	46.0	45.8	45.6	45.4	45.2	45.0	44.8
Chongqing	64.5	67.6	70.8	73.9	77.1	80.2	83.4	86.5	89.7
Sichuan	206.1	209.0	211.8	214.6	217.4	220.2	223.0	225.8	228.6
Guizhou	95.5	96.2	96.8	97.4	98.1	98.7	99.4	100.0	100.7
Yunnan	146.7	147.1	147.5	147.9	148.3	148.7	149.1	149.5	149.9
Tibet	29.7	30.4	31.1	31.8	32.5	33.2	33.9	34.6	35.3
Shanxi	76.0	77.4	78.9	80.3	81.8	83.2	84.7	86.1	87.6
Gansu	122.1	122.1	122.1	122.1	122.1	122.1	122.1	122.1	122.1
Qinghai	30.3	30.5	30.6	30.8	30.9	31.1	31.2	31.3	31.5
Ningxia	71.8	72.1	72.4	72.7	73.0	73.3	73.6	73.9	74.2
Xinjiang	499.6	504.0	508.4	512.8	517.2	521.6	526.1	530.5	534.9

Appendix 3

Industrial water needs from 2003 to 2011 (100 million cubic meters)

Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	1177.2	1228.9	1285.2	1343.8	1403.0	1397.1	1390.9	1447.3	1461.8
Beijing	7.6	7.7	6.8	6.2	5.7	5.2	5.2	5.1	5.0
Tianjin	4.9	5.1	4.5	4.4	4.2	3.8	4.4	4.8	5.0
Heibei	26.2	25.2	25.7	26.2	25.0	25.2	23.7	23.1	25.7
Shanxi	14.1	13.9	13.9	15.4	14.4	13.5	10.5	12.6	14.3
Inner Mongolia	10.1	10.4	13.2	16.7	17.4	20.5	20.9	22.6	23.6
Liaoning	21.9	19.9	21.1	23.5	24.4	24.7	23.9	25.0	24.0
Jilin	22.2	16.6	18.8	19.1	19.5	19.3	23.6	26.1	26.6
Heilongjiang	52.5	53.0	55.5	57.5	57.5	57.6	55.7	56.0	53.2
Shanghai	72.2	77.9	81.3	78.0	81.3	79.6	84.2	84.9	82.6
Jiangsu	155.8	182.6	207.9	220.3	225.3	209.4	194.6	191.9	192.9
Zhejiang	55.3	55.8	58.1	62.7	64.2	61.0	55.3	59.7	61.8
Anhui	63.1	63.3	67.7	79.6	83.8	85.4	93.7	94.0	90.6
Fujian	60.1	59.3	63.5	66.9	72.8	75.4	77.2	81.3	83.5
Jiangxi	46.7	52.2	51.2	50.6	58.6	59.9	53.2	57.4	60.6
Shangdong	31.6	28.4	21.8	22.5	24.1	24.7	24.7	26.8	29.8
Henan	39.9	40.2	45.9	48.3	51.3	51.4	53.5	55.6	56.8

Hubei	80.7	82.4	82.6	86.9	96.6	97.0	100.8	117.1	120.4
Hunan	68.4	76.4	80.5	82.0	82.5	82.0	83.5	89.8	95.6
Guangdong	130.4	136.6	133.9	135.6	141.1	137.2	136.2	138.8	133.6
Guangxi	37.0	41.6	45.0	46.5	47.8	51.7	54.0	55.2	57.3
Hainan	4.1	3.0	3.2	3.8	4.7	4.9	3.9	3.8	3.9
Chongqing	26.9	31.2	33.0	38.4	40.9	46.0	47.6	47.4	43.3
Sichuan	56.1	56.5	56.8	57.5	59.0	57.7	61.6	62.9	64.6
Guizhou	26.4	26.9	28.1	27.3	31.8	33.8	34.1	34.3	30.7
Yunnan	17.4	17.9	18.4	18.8	22.3	22.1	22.4	25.5	25.3
Tibet	0.3	0.3	0.5	0.8	1.1	1.3	1.4	1.5	1.7
Shanxi	13.0	12.5	12.8	13.2	11.7	12.9	11.4	12.1	13.2
Gansu	16.3	15.9	15.8	15.8	14.0	13.1	13.1	13.8	15.5
Qinghai	4.2	5.2	6.3	7.0	7.2	7.9	3.0	3.3	3.5
Ningxia	3.5	3.2	3.5	3.5	3.5	3.3	3.7	4.1	4.6
Xinjiang	8.3	8.0	8.2	8.6	9.2	9.8	10.1	11.2	12.6

Appendix 4

Agricultural water needs from 2003 to 2011(100 million cubic meters)

Province	2003	2004	2005	2006	2007	2008	2009	2010	2011
China	3432.8	3585.7	3580.0	3664.5	3599.5	3663.5	3723.1	3689.1	3743.6
Beijing	12.9	13.0	12.7	12.1	11.7	11.4	11.4	10.8	10.2
Tianjin	11.2	12.0	13.6	13.4	13.8	13.0	12.8	11.0	11.6
Heibei	149.6	147.1	150.2	152.6	151.6	143.2	143.9	143.8	140.5
Shanxi	33.3	32.9	32.7	34.2	34.3	32.9	34.4	38.0	43.4
Inner Mongolia	146.1	149.4	143.9	142.2	141.8	134.1	138.7	134.5	135.9
Liaoning	83.5	85.7	87.2	91.5	91.7	90.9	91.1	89.8	89.7
Jilin	67.5	66.4	66.4	70.4	67.5	69.3	71.2	73.8	81.6
Heilongjiang	171.4	186.3	192.1	208.3	214.8	218.2	237.4	249.6	272.3
Shanghai	16.3	18.8	18.5	18.4	16.2	16.7	16.8	16.8	16.5
Jiangsu	223.1	288.5	263.8	270.7	268.5	287.3	300.1	304.2	307.6
Zhejiang	110.2	107.3	106.7	101.1	100.2	98.7	97.3	94.6	92.1
Anhui	93.8	121.7	113.6	136.4	120.6	151.9	167.2	166.7	168.4
Fujian	101.0	104.2	101.5	98.0	100.9	99.3	100.8	97.2	98.6
Jiangxi	104.1	128.5	134.6	132.9	151.4	148.9	157.2	151.0	171.7
Shangdong	157.0	154.3	156.3	169.4	159.7	157.6	156.4	154.8	148.9
Henan	113.3	124.5	114.5	140.2	120.1	133.5	138.1	125.6	124.6
Hubei	136.2	131.7	142.1	143.0	132.6	142.8	149.4	138.3	142.3
Hunan	209.4	202.3	201.3	198.4	193.9	193.2	189.3	185.8	183.1
Guangdong	242.6	240.3	230.7	226.9	224.8	227.7	228.7	227.5	224.2
Guangxi	205.4	210.1	225.4	222.3	208.4	202.9	195.3	194.6	193.2
Hainan	35.7	37.9	35.1	36.7	35.8	35.6	34.0	33.9	33.8
Chongqing	20.7	20.3	21.4	18.1	18.7	18.9	19.0	19.8	23.6

Sichuan	121.7	121.2	121.8	121.2	118.7	113.6	123.6	127.3	128.4
Guizhou	52.2	51.9	50.5	54.3	48.7	51.6	50.8	50.1	49.7
Yunnan	109.6	109.7	108.4	105.6	105.9	105.1	103.5	95.3	96.1
Tibet	22.6	25.7	30.3	31.8	33.4	33.9	27.5	31.7	27.4
Shanxi	50.7	49.7	52.2	56.8	55.5	57.7	57.2	55.5	56.2
Gansu	96.4	96.7	95.0	94.3	96.1	96.9	93.8	94.3	93.8
Qinghai	21.7	21.8	21.1	21.8	20.5	22.4	21.6	23.2	23.5
Ningxia	58.4	68.6	72.3	71.7	64.8	68.0	65.3	65.1	66.1
Xinjiang	454.9	457.0	464.4	470.0	476.8	486.2	489.4	484.6	488.4

Appendix 5

Total sewage discharge from 2006 to 2011
(100 million cubic meters)

Province	2006	2007	2008	2009	2010	2011
China	536.8	571.7	571.7	589.1	617.3	659.2
Beijing	10.5	11.3	11.3	14.1	13.6	14.5
Tianjin	5.9	6.1	6.1	6.0	6.8	6.7
Heibei	22.2	23.5	23.5	24.5	26.3	27.9
Shanxi	10.3	10.7	10.7	10.6	11.8	11.6
Inner Mongolia	6.2	7.0	7.0	7.3	9.3	10.0
Liaoning	21.3	21.2	21.2	21.7	21.8	23.2
Jilin	9.7	10.8	10.8	11.0	11.4	11.6
Heilongjiang	11.6	11.1	11.1	11.1	11.9	15.1
Shanghai	22.4	22.4	22.4	23.1	24.8	21.4
Jiangsu	51.6	51.0	51.0	52.2	55.6	59.3
Zhejiang	33.1	35.0	35.0	36.5	39.5	42.0
Anhui	16.6	16.9	16.9	18.0	18.5	24.3
Fujian	21.6	23.6	23.6	24.6	23.9	31.6
Jiangxi	13.5	13.9	13.9	14.7	16.1	19.4
Shangdong	30.3	35.9	35.9	38.7	43.6	44.3
Henan	27.8	30.9	30.9	33.4	35.9	37.9
Hubei	24.0	25.9	25.9	26.6	27.1	29.3
Hunan	24.4	25.0	25.0	26.0	26.8	27.9
Guangdong	65.4	67.7	67.7	68.7	72.3	78.6
Guangxi	26.0	34.5	34.5	30.6	31.3	22.2
Hainan	3.5	3.6	3.6	3.8	3.7	3.6
Chongqing	15.1	14.5	14.5	14.7	12.8	13.1
Sichuan	25.2	26.2	26.2	26.3	25.6	28.0
Guizhou	5.5	5.6	5.6	5.9	6.1	7.8
Yunnan	8.0	8.4	8.4	8.8	9.2	14.8
Tibet	0.3	0.3	0.3	0.3	0.4	0.5

Shanxi	8.7	10.5	10.5	11.1	11.6	12.2
Gansu	4.6	4.7	4.7	4.9	5.1	5.9
Qinghai	1.9	2.0	2.0	2.2	2.3	2.1
Ningxia	3.2	3.8	3.8	4.1	4.1	3.9
Xinjiang	6.5	7.5	7.5	7.7	8.4	8.3

Appendix 6

Codes of Gray Model:

```
clear
clc
disp('gray model')
disp('input 1 for predicting the use of water')
disp('input 2 for predicting the storage of water')
disp('input 3 for predicting the use of water for industry')
disp('input 4 for predicting the use of water for agriculture')
disp('input 5 for predicting the amount of sewage')
input=input('please input a number (1~5) : ');
load('water_use.mat')
load('water_storage.mat')
load('sewage.mat')
load('agriculture')
load('industry')
P=ones(32,12);
if input==5
    P=ones(32,9);
end
for jj=1:32
    if input==1
        A=water_consum(jj,:);
    end
    if input==2
        A=water_storage(jj,:);
    end
    if input==3
        A=industry(jj,:);
    end
    if input==4
       A=agriculture(jj,:);
    end
    if input==5
       A=sewage(jj,:);
    end
```

```
R1=ones(32,1);
B=cumsum(A);
n=length(A);
    for i=1:n-1
      C(i)=(B(i)+B(i+1))/2;
    end
D=A;D(1)=[];
D=D';
E=[-C;ones(1,n-1)];
c=(E*E'\E*D)';
a=c(1);b=c(2);
F=[];F(1)=A(1);
    for i=2:(n+14)
      F(i)=(A(1)-b/a)./exp(a*(i-1))+b/a;
    end
G=[];G(1)=A(1);
    for i=2:(n+14)
      G(i)=F(i)-F(i-1);
    end
    P(jj,:)=[G(1:n),G(n+4),G(n+9),G(n+14)];
R1(jj)=sum(abs(G(1:n)./A-1))/n;
end
```

Codes of Linear Regression Model:

```
clear
clc
disp('Linear Regression')
disp('input 1 for predicting the use of water')
disp('input 2 for predicting the storage of water')
input=input('please input 1 or 2 : ');
load('water_use.mat')
load('water_storage.mat')
P=ones(32,12);
R2=ones(32,1);
for jj=1:length(water_consum(:,1))
    x=1:1:9;
    if input==1
    y=water_consum(jj,:);
    end
    if input==2
    y=water_storage(jj,:);
    end
    [a,ST]=polyfit(x,y,1);
```

```
    x1=1:1:23;
    [y1,sp3]=polyval(a,x1,ST);
    R2(jj)=sum(abs(y1(1:9)./y-1))/9;
    P(jj,:)=[y1(1:9),y1(13),y1(18),y1(23)];
end
P(P<=0)=0;
```

Codes of AHP:

```
clear
clc
load('A2.mat')
w=weight(A);
n=length(A(1,:));
lamda=0;
for i=1:n
    for j=1:n
        lamda=lamda+A(i,j)*w(j)/w(i);
    end
end
lamda=lamda/n;
RI=[0 0 0.58 0.90 1.12 1.24 1.32 1.41 1.45 1.49 1.51 1.54 1.56 1.58 1.59];
RI1=RI(n);
CI1=(lamda-n)/(n-1);
CR1=CI1/RI1;
load('B.mat')
N=length(B(:,1));
w1=zeros(N,n);
for i=1:n
    a=B(:,i);
    b=zeros(length(a),length(a));
        for j=1:length(a)
            for k=1:length(a)
                b(j,k)=a(j)/a(k);
            end
        end
    w1(:,i)=weight(b);
end
W=w1*w;
CR=CR1;
[~,order]=sort(W,'descend');
order=order';
tmp1=zeros(1,N);
for i=1:N
```

```
    tmp=order(i);
    tmp1(tmp)=i;
end

function w=weight(A)
n=length(A(1,:));
w=zeros(n,1);
for i=1:n
    for j=1:n
        w(i)=w(i)+prod(A(j,:))^(1/n);
    end
    w(i)=(prod(A(i,:))).^(1/n)/w(i);
end
```