For office use only

T1 ________________

T2 ________________

T3 ________________

T4 ________________

Team Control Number

22599

Problem Chosen

A

For office use only

F1 ________________

F2 ________________

F3 ________________

F4 ________________

2013

Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

(Attach a copy of this page to your solution paper.)

Heat Radiation in The Oven

Heat distribution of pans in the oven is quite different from each other, which depends on their shapes. Thus, our model aims at two goals. One is to analyze the heat distri- bution in different ovens based on the locations of electrical heating cubes. Further- more, a series of heat distribution which varies from circular pans to rectangular pans could be got easily. The other is to optimize the pans placing, in order to choose a best way to maximize the even heat and the number of pans at the same time.

Mathematically speaking, our solution consists of two models, analyzing and optimi- zing. In part one, our whole-local approach shows the heat distribution of every pan. Firstly, we use the Stefan-Boltzmann law and Fourier theorem to describe the heat distribution in the air around the electrical heating tube. And then, based on plane in- tercept method and simplified Monte Carlo method, the heat distribution of different shapes of pans is obtained. Finally, we explain the phenomenon that the corners of a pan always get over heated with water waves stirring by analogy. In part two, our discretize- convert approach optimizes the shape and number of the pans. Above all, we discre- tize the side length of the oven, so that the number and the average heat of the pans vary linearly. In the end, the abstract weight P is converted into a specific length, in order to reach a compromise between the two factors.

Specially, we create a unique method to convert the variables from the whole space to the local section. The special method allows us to draw the heat distribution of every single section in the oven. The algorithm we create does a great job in flexibility, which can be applied to all shapes of pans.

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Heat Radiation in The Oven

Summary

Heat distribution of pans in the oven is quite different from each other, which depends on their shapes. Thus, our model aims at two goals. One is to analyze the heat distribution in different ovens based on the locations of electrical heating cubes. Furthermore, a series of heat distribution which varies from circular pans to rectangular pans could be got easily. The other is to optimize the pans placing, in order to choose a best way to maximize the even heat and the number of pans at the same time.

Mathematically speaking, our solution consists of two models, analyzing and optimizing. In part one, our whole-local approach shows the heat distribution of every pan. Firstly, we use the Stefan-Boltzmann law and Fourier theorem to describe the heat distribution in the air around the electrical heating tube. And then, based on plane intercept method and simplified Monte Carlo method, the heat distribution of different shapes of pans is obtained. Finally, we explain the phenomenon that the corners of a pan always get over heated with water waves stirring by analogy. In part two, our discretize- convert approach optimizes the shape and number of the pans. Above all, we discre- tize the side length of the oven, so that the number and the average heat of the pans vary linearly. In the end, the abstract weight P is converted into a specific length, in order to reach a compromise between the two factors.

Specially, we create a unique method to convert the variables from the whole space to the local section. The special method allows us to draw the heat distribution of every single section in the oven. The algorithm we create does a great job in flexibility, which can be applied to all shapes of pans.

Keywords: Monte Carlo thermal radiation section heat distribution discretization

Introduction

Many studies on heat conduction wasted plenty of time in solving the partial differential equations, since it's difficult to solve even for computers. We turn to another way to work it out. Firstly, we study the heat radiation instead of heat conduction to keep away from the sophisticated partial differential equations. Then, we create a unique method to convert every variable from the whole space to section. In other words, we work everything out in heat radiation and convert them into heat contradiction.

Assumptions

We make the following assumptions about the distribution of heat in this paper.

- Initially two racks in the oven, evenly spaced.

- When heating the electrical heating tubes, the temperature of which changes from room temperature to the desired temperature. It takes such a short time that we can ignore it.

- Different pans are made in same material, so they have the same rate of heat conduction.

- The inner walls of the oven are blackbodies. The pan is a gray body. The inner walls of the oven absorb heat only and reflect no heat.

- The heat can only be reflected once when rebounded from the pan.

Heat Distribution Model

Our approach involves four steps:

- Use the Fourier theorem to calculate the loss energy when energy beams are spread in the medium. So we can get the heat distribution around each electrical heating tube. The heat distribution of the entire space could be go where the heat of two electrical heating tubes cross together.

- When different shapes of the pans are inserted into the oven, the heat map of the entire space is crossed by the section of the pan. Thus, the heat map of every single pan is obtained.

- Establish a suitable model to get the reflectivity of every single point on the pan with the simplified Monte Carlo method. And then, a final heat distribution map of the pan without reflection loss is obtained.

- A realistic conclusion is drawn due to the results of our model compared with water wave propagation phenomena.

First of all, the paper will give a description of the initial energy of the electrical heating tube. We see it as a blackbody who reflects no heat at all. Electromagnetic knowledge shows that wavelength of the heat rays ranges from $10^{-1}um$ to $10^{2}um$ as shown below[1]:

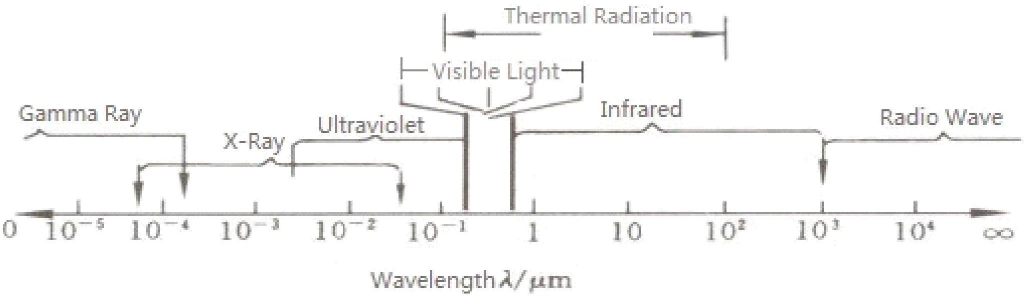


Figure 1.

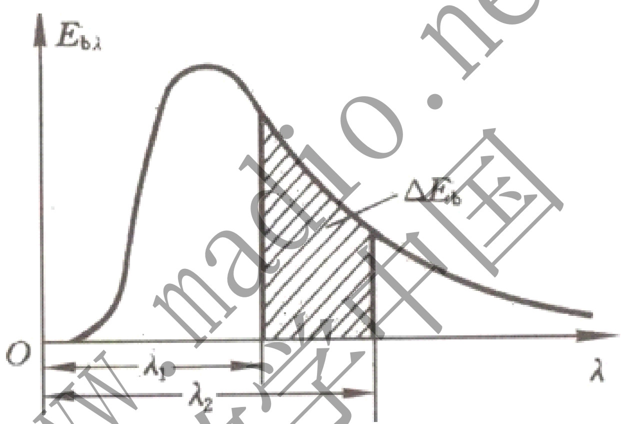


Figure 2.

We apply the Stefan-Boltzmann's law[2] whose solution is

$$E_{b\lambda} = \frac{c_1\lambda^{-5}}{e^{c_2/(\lambda T)} - 1} \tag{1}$$

$$E_b = \int_0^\infty E_{b\lambda} d\lambda = \int_0^\infty \frac{c_1\lambda^{-5}}{e^{c_2/(\lambda T)} - 1} d\lambda \tag{2}$$

Where E_b means the ability of blackbody to radiate. c_1 and c_2 are constants. Obviously,, the initial energy of a black body is $E_{b0} = 3.2398e + 012(w \times m^2)$.

Combine Figure 1 with Figure 2, we integrate (1) from λ_1 to λ_2 to get the equation as follow:

$$E_{b(\lambda_1 - \lambda_2)} = \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda \tag{3}$$

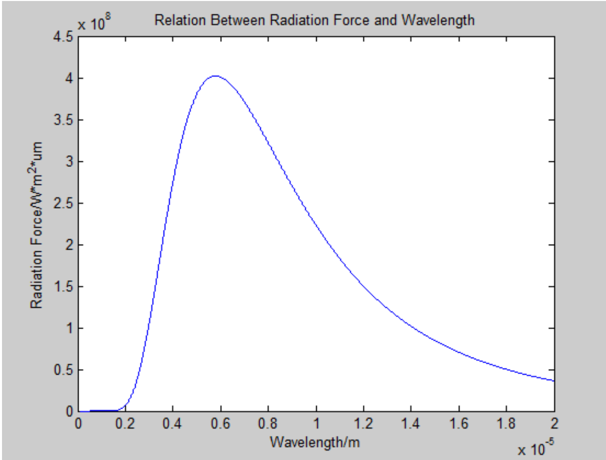


Figure 3.

From Figure 3, it can be seen how the power of radiation varies with wavelength.

Secondly, based on the Fourier theorem, the relation between heat and the distance from the electrical heating tubes is:

$$Q = -\lambda S \frac{dt}{dx} \tag{4}$$

Where Q is the power of heat ($J/s = W$), S is the area where the energy beam radiates (m^2), $\frac{dt}{dx}$ represents the temperature gradient along the direction of energy beam. [3]

It is known that the energy becomes weaker as the distance becomes larger. According to the fact we know:

$$\rho = \frac{dQ}{dx} \tag{5}$$

Where ρ is the rate of energy changing.

We assume that the desired temperature of electrical heating tube is 500k. With the two equations, the distribution of heat is shown as follow:

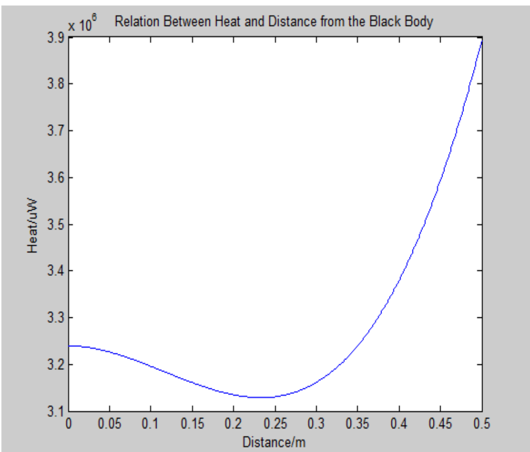


Figure 4. (a)

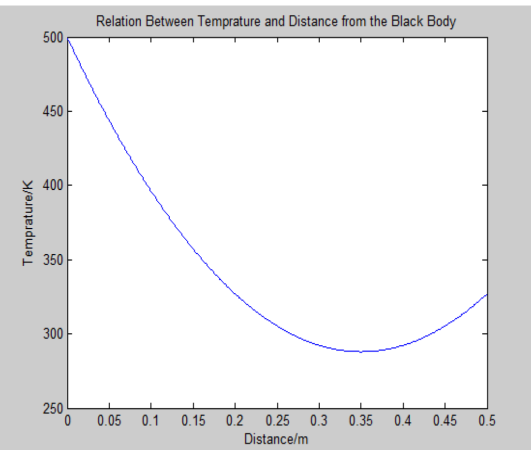


Figure 4. (b)

In order to draw the map of heat distribution in the oven, we use MATLAB to work on the complicated algorithm. The relation between the power of heat and the distance is shown in Figure 4(a). The relation between temperature and distance is presented in Figure 4(b). The spreading direction of energy beam is presented in Figure 5.

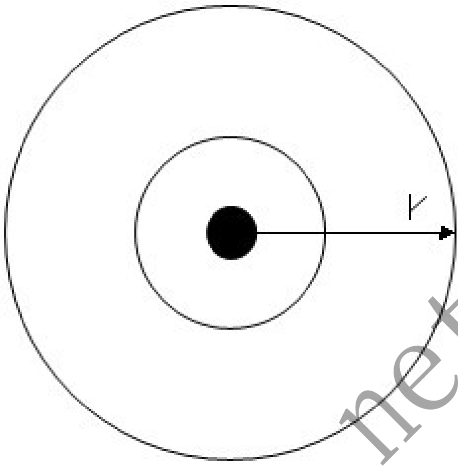


Figure 5.

The shape of electrical heating tube is irregular. The heat distribution of a single electrical heating tube can be draw in 3D space with MATLAB. The picture is shown in Figure 6. After superimposing, the total heat distribution of two tubes is shown below in Figure 7 and Figure 8.

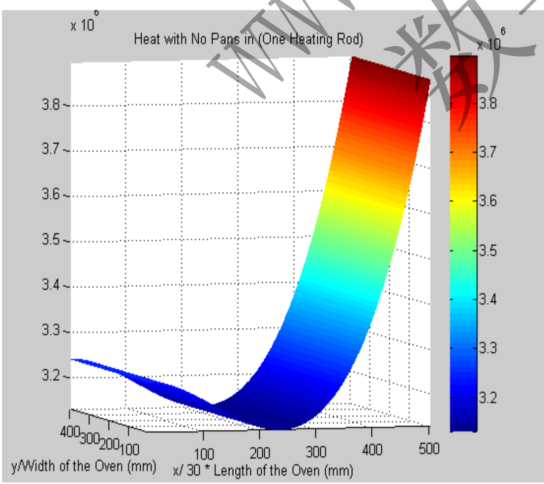


Figure 6.

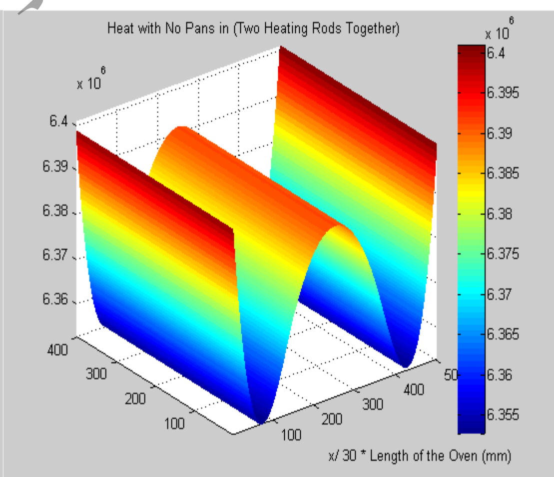


Figure 7.

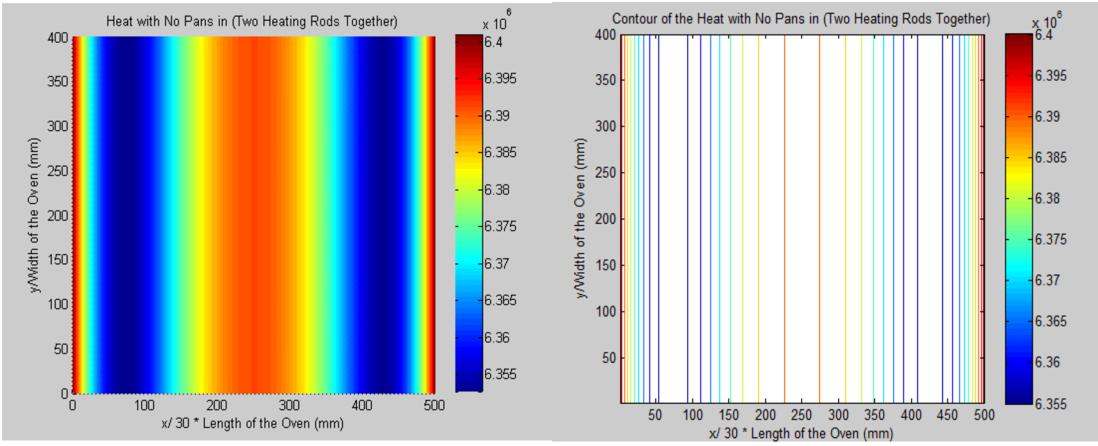


Figure 8.

The pictures above show the energy in an oven with no pan. We put in a rectangular pan whose area is A, and intercept the maps with MATLAB. The result is show in Figure 9.

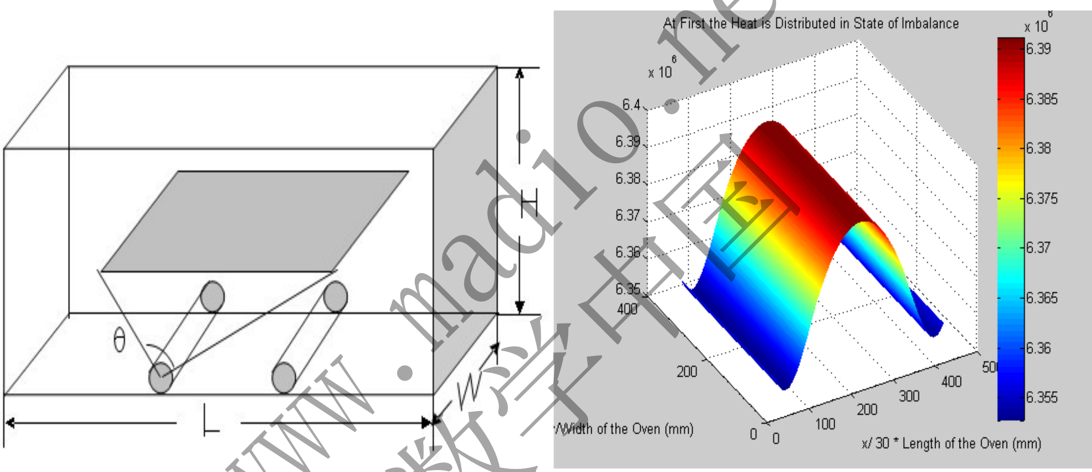


Figure 9.

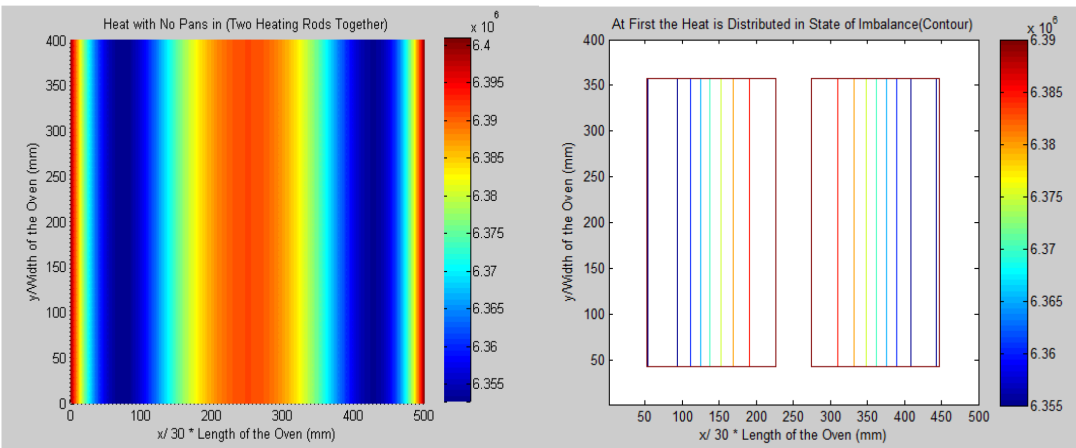


Figure 10.

Put in a circular pan to intercept the maps, whose area is A, also. The distribution of heat is shown in Figure 11.

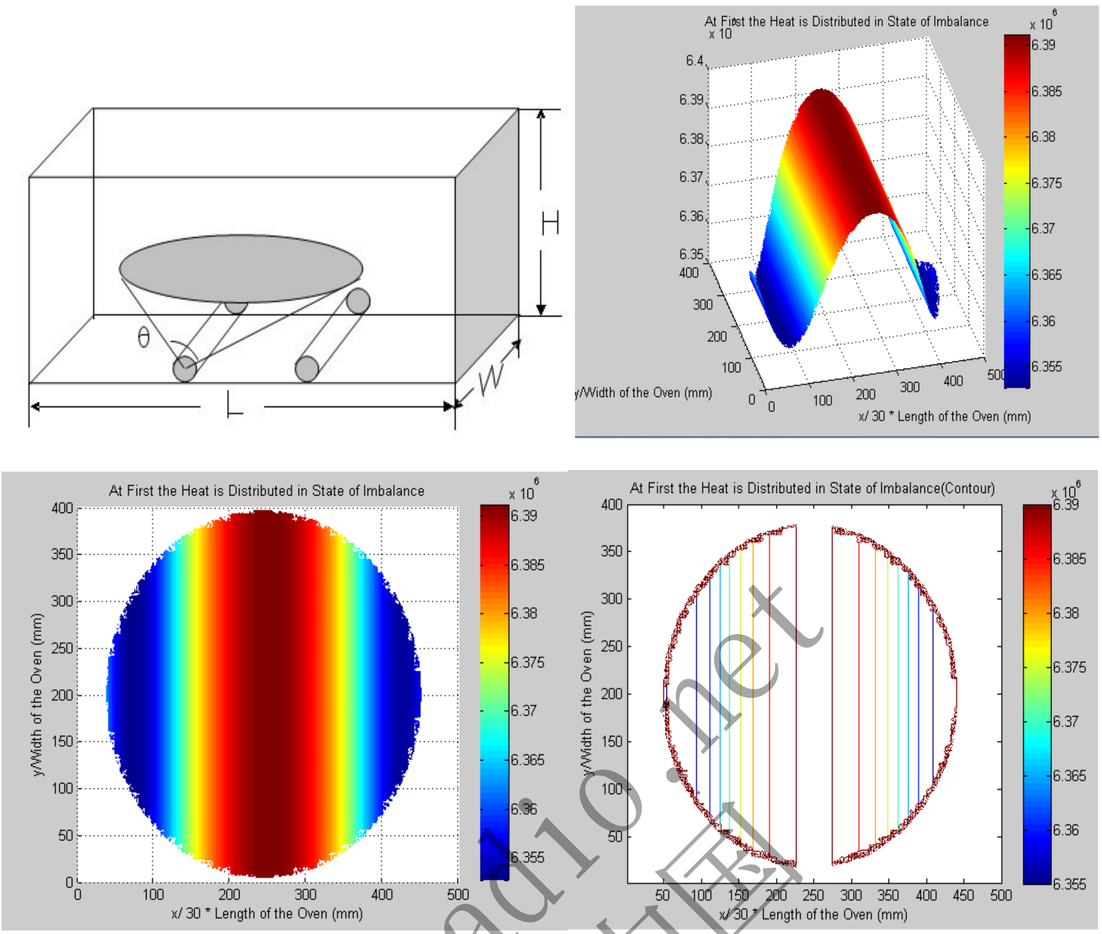


Figure 11.

When put in a pan in transition shape, which is neither rectangular nor circular. The area of it is A, also. The heat distribution on such a pan is shown as follow:

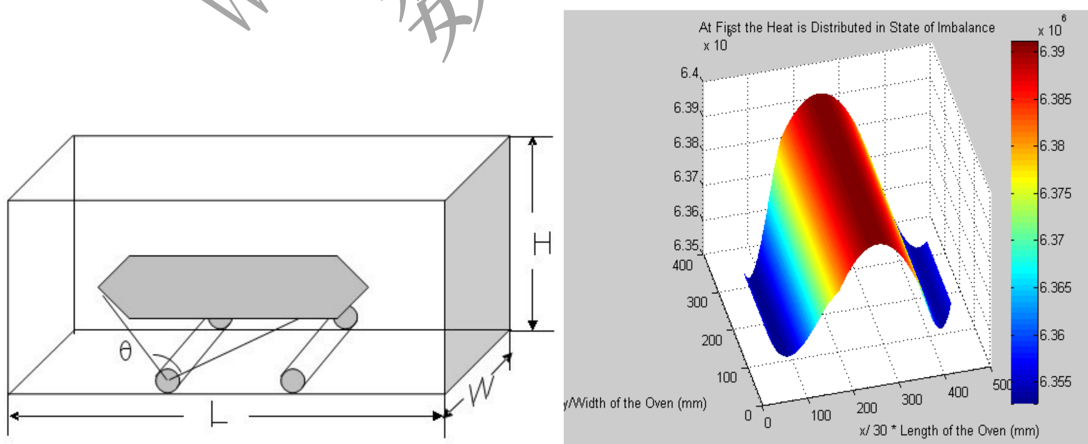


Figure 12(a)

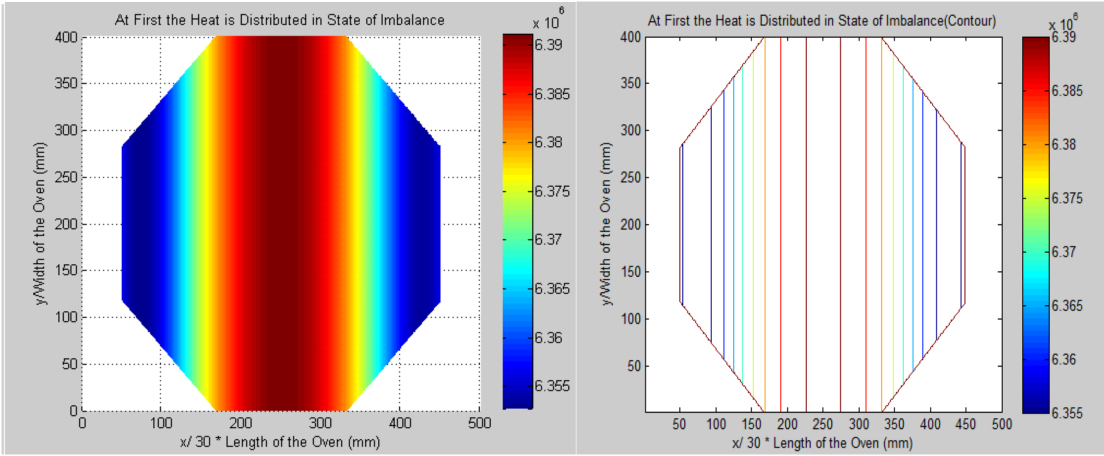


Figure 12(b).

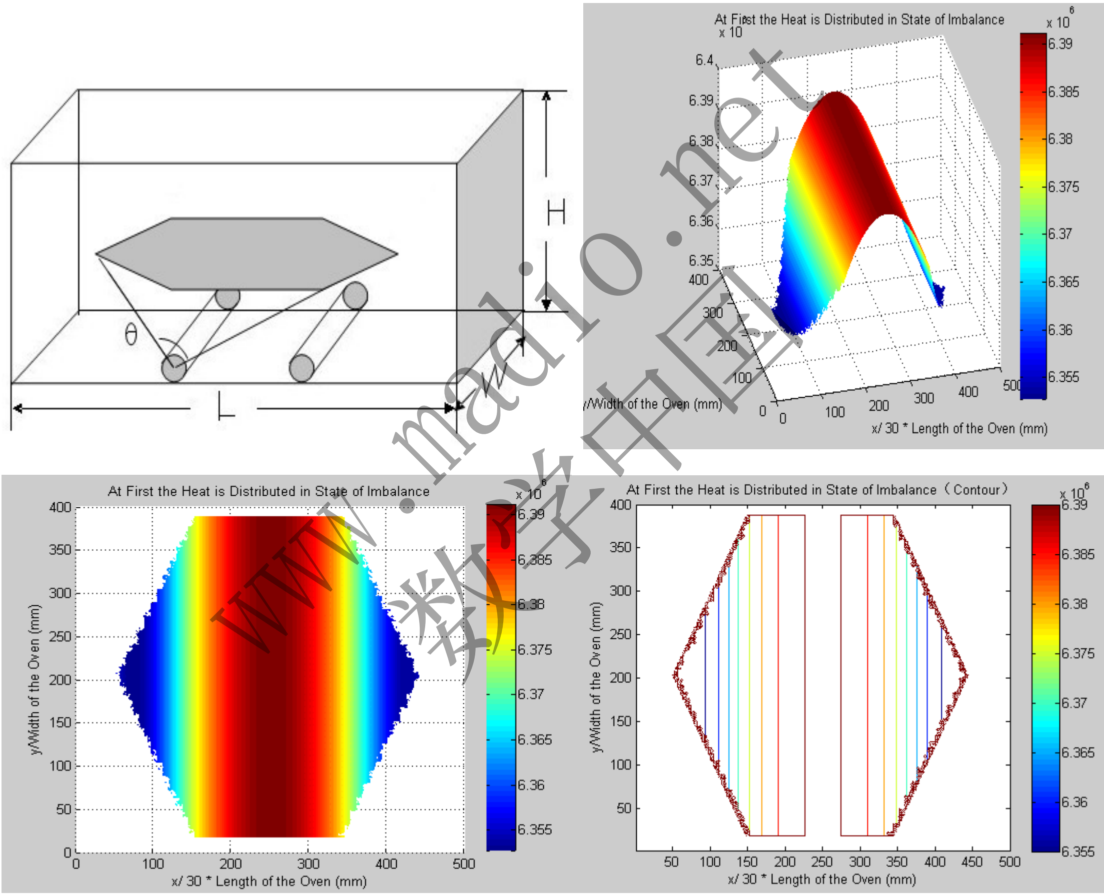


Figure 13.

Next, learning from the Monte Carlo simulation[4], a model is established to get obtain the reflectivity. We generate a random number between 0 and 1 to determine if the energy beam on certain point is reflected.

- Firstly, to demonstrate the question better , we construct a simple model:

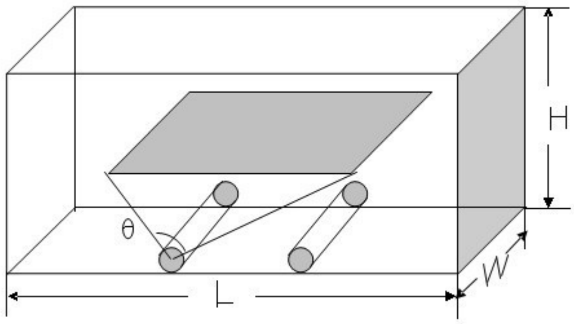


Figure 14.

Where θ is the viewing angle from electrical heating tube to the pan. $R = \frac{\theta}{360}$ is the proportion of the beams radiated to the pan.

• What is more, we assume the total beam is M_1. Ideally, the number of absorption is $M_1 \times \frac{\theta}{360}$. Then, each element of the pan is seen as a grid point. Each grid point can generate a- $M_1 \times \frac{\theta}{360}$ -random-number vector between 0 and 1 in MATLAB.

• After MATLAB simulating, the number of beams decreased by M_2, due to the reflection. So we define a probability $\rho = \frac{360 \times M_2}{M_1 \theta}$ to describe the number of beams reflected.

The conclusion is :

• If $\rho \le R$, the energy beam is absorbed.

• If $\rho > R$, the energy beam is reflected.[5]

Based on the analysis above, our model get a final result of heat-distribution on the pan as shown below:

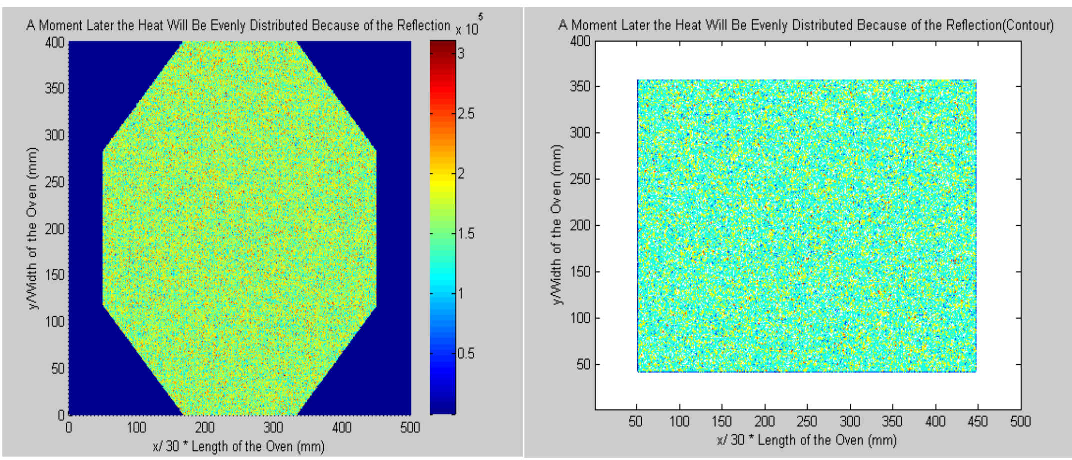


Figure 15(a)

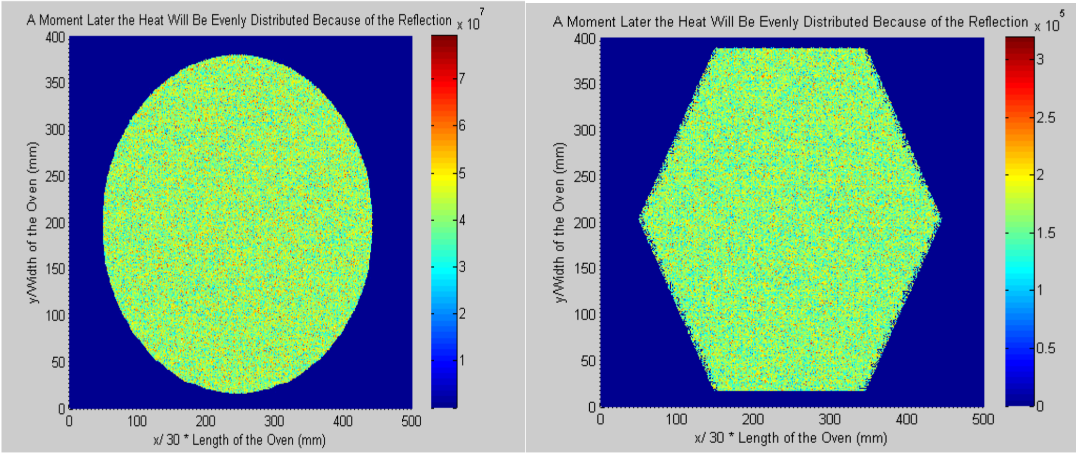


Figure 15(b).

The conclusion is known that the closer the shape of pans is to circle, the more evenly the heat is distributed. Moreover, the phenomenon that the corners always get over heated can be explained by water wave propagation in different containers.

When there is a fluctuation in the center of the water, the ripples will fluctuate and spread in concentric circles, as shown in Figure 16. The fluctuation stirs waves up when contacting the borderlines. Compared with the waves with one boundary, the waves in corner make a higher amplitude.

The thermal conduction on the pan is exactly the inverse process of the waves propagation. The range of thermal motion is much smaller than it on the side. That's why the corners is easy to get over heated.

In order to make the heat evenly distributed on the pan, the sides of the pan should be as few as possible. Therefore, if nothing is considered about the utilization of space, a circle pan is the best choice.

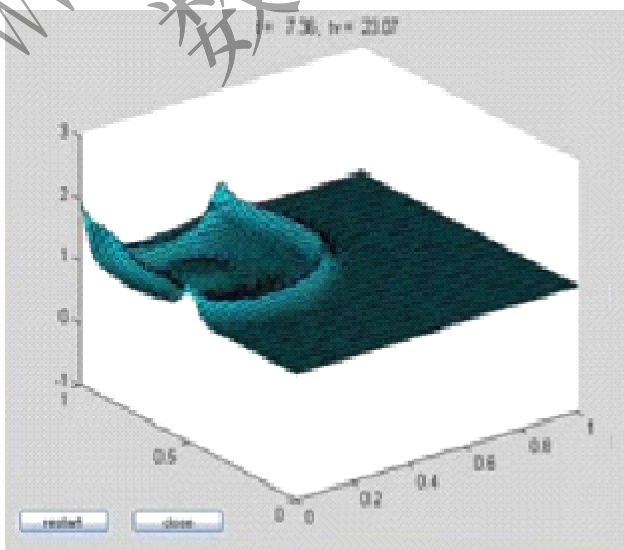


Figure 16, the water waves propagation[6]

According to the analysis above and Figure 7, the phenomenon shows that the heat conduction is similar to water waves propagation. So it is proved that heat concentrates in the four corners of the rectangular pan.

The Super Pan Model

Assumptions

• The width of the oven (W) is $100mm$, the length is L.

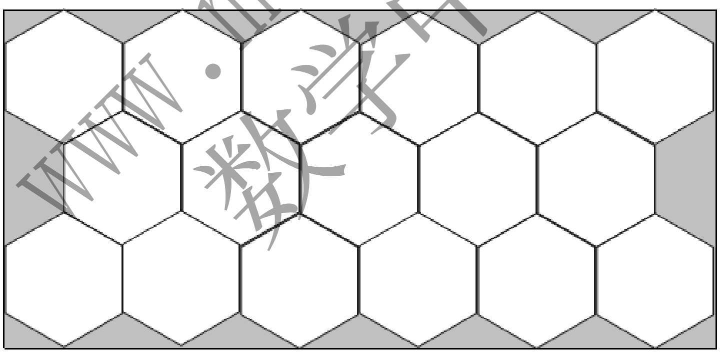
• There are three pans at most in vertical direction.

• Each pan's area is A.

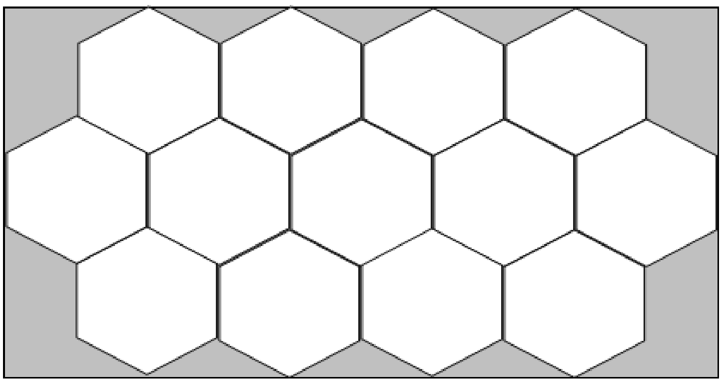
The first part.
Calculate the maximum number of pans in the oven. Different shapes of pan have different heat distribution which affects the number of pans, judging from the previous solution. According to the conclusion in the first model, the heat is distributed the most evenly on a circular pan rather than a rectangular one. However, the rectangular pans make fuller use of the space the space than circular ones. Both factors considered, a polygonal pan is chosen.

A circle can be regard as a polygon whose number of boundaries tends to infinity. Except for rectangle, only regular hexagon and equilateral triangle can be closely placed. Because of the edges of equilateral triangle, heat dissipation is worse than rectangle. So, hexagonal pans are adopted after all the discussion.

Considering the gaps near boundaries, we place the hexagonal pans closely attached each other on the long side L. There are two kinds of programs as shown below.



Program 1.



Program 2.

Obviously, Program2 is better than Program1 when considering space utilization. So scheme 1 is adopted.

Then, design a size of each hexagonal pan to make the highest space utilization. With the aim of utilization, hexagonal pans has to be placed contact closely with each other on both sides. It is necessary to assume a aspect ratio of the oven to work out the number of pans(N).

Assume that the side length of a regular hexagon is a , the length-width ratio of the oven is λ and ΔL is the increment in discretization. Because the number of pans can not change continuously when $n \in (m, m+1), m = 1,2,3\cdots$, the equations would be as follows.

$$\begin{cases} \dfrac{W}{5} = a \\ 0 < \dfrac{W}{L} < 1 \\ L = L_0 + k \cdot \Delta L \\ \Delta L = \dfrac{\sqrt{3}}{2} \cdot a; \\ 0 \le \dfrac{L + k \cdot \Delta L}{W} - \left[\dfrac{L + k \cdot \Delta L}{W} \right] < 1 \\ \dfrac{W}{L + k \cdot \Delta L} = \lambda \\ N_0 = 8, L_0 = W \\ n = \left[\dfrac{L - W}{\dfrac{\sqrt{3}}{2} \cdot a} \right] \end{cases} \tag{6}$$

Result:

$$\begin{cases} N_1 = N_0 + 3 \cdot \dfrac{n-1}{2} + 1 & n = 2k-1, k = 1,2,3\cdots \\ N_2 = N_0 + 3 \cdot \dfrac{n}{2} & n = 2k, k = 1,2,3\cdots \end{cases}$$

Where N_1 represents the number of pans when n is odd, N_2 represents the number of pans when n is even. The specific number of pans is depended on the width-length ratio of oven.

The second part.
Maximum the heat distribution of the pans. We define the average heat(H) as the ratio of total heat and total area of the pans. Aiming to get the most average heat, we set the width-length ratio of the oven λ . Space utilization is not considered here.

A conclusion is easy to draw from Figure 8 that a square area in the oven from 150 mm to 350 mm in length shares the most heat evenly. So the pans should be placed mainly in this area. From model 1 we know that the corners of the oven are apt to gather heat. Besides, four more pans are added in the corners to absorb more heat. Because heat absorbing is the only aim, there is no need to consider space utilization. Circular pans can distribute heat more evenly than any other shape due to model 1. So circular pans are used in Figure 17.

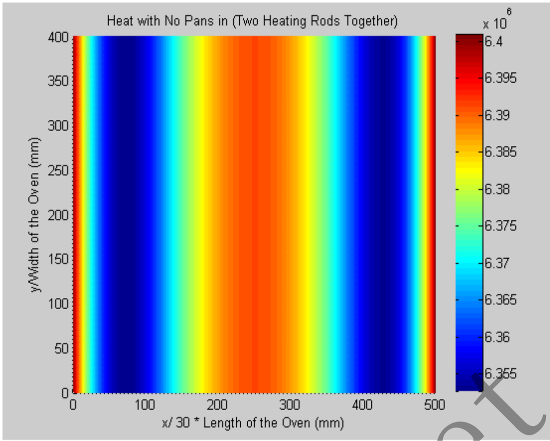


Figure 8.

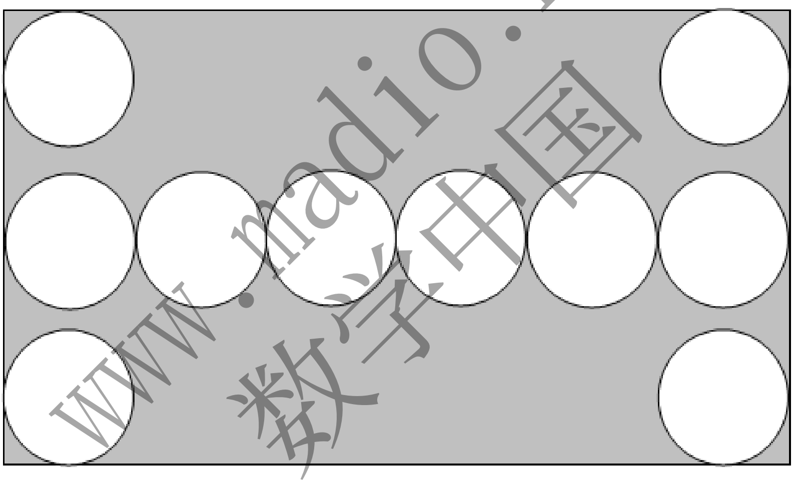


Figure 17.

We set the heat of the pans in the most heated area (the middle row) as Q. Pans in the corners receive more heat but uneven theoretically. And the square of the four pans in the corners is so small compared with the total square that we set the heat of the four as Q too. When the length of oven (L) increases, the number of pans increases too. It makes the square of the gaps between pans bigger, meanwhile. If each pan has a same radius (r) and square (A), the equation about average heat, length-width ratio and number of pans would be (7).

$$\begin{cases} r=\frac{1}{4}\cdot\frac{3}{5}\cdot W \\ A=\pi\cdot r^2 \\ 0<\frac{W}{L}<1 \\ L=L+k\cdot\Delta L \\ \Delta L=2\cdot r \\ 0\le\frac{L+k\cdot\Delta L}{W}-\left[\frac{L+k\cdot\Delta L}{W}\right]<1 \\ \frac{W}{L+k\cdot\Delta L}=\lambda \\ N_0=7; L_0=W \\ n=\left[\frac{L-W}{2\cdot r}\right] \\ N=N_0+n \\ k=1,2,3\ldots \end{cases} \tag{7}$$

Here we get the most average heat (H):

$$H=\frac{400}{9\pi}\cdot\frac{Q}{W^2}$$

The third part.

We discussed two different plans in the previous parts of the paper. One is aimed to get the most average heat, while the other aimed to place the most pans. The two plans are contradictory with each other, and can not be achieved together.

Firstly, the weight of plan 1 is P and the weight of plan 2 is $1-P$. Obviously, this kind optimization has difficulty in solving and understanding. So we turn to another way to make it a easier and linear question. It has been set that the width of the oven is a constant W and there should be three pans at most in vertical direction. We make the weight P a proportion of the two plans. Thus the two plans could be achieved together due to proportion P and $1-P$, as shown in Figure 18.

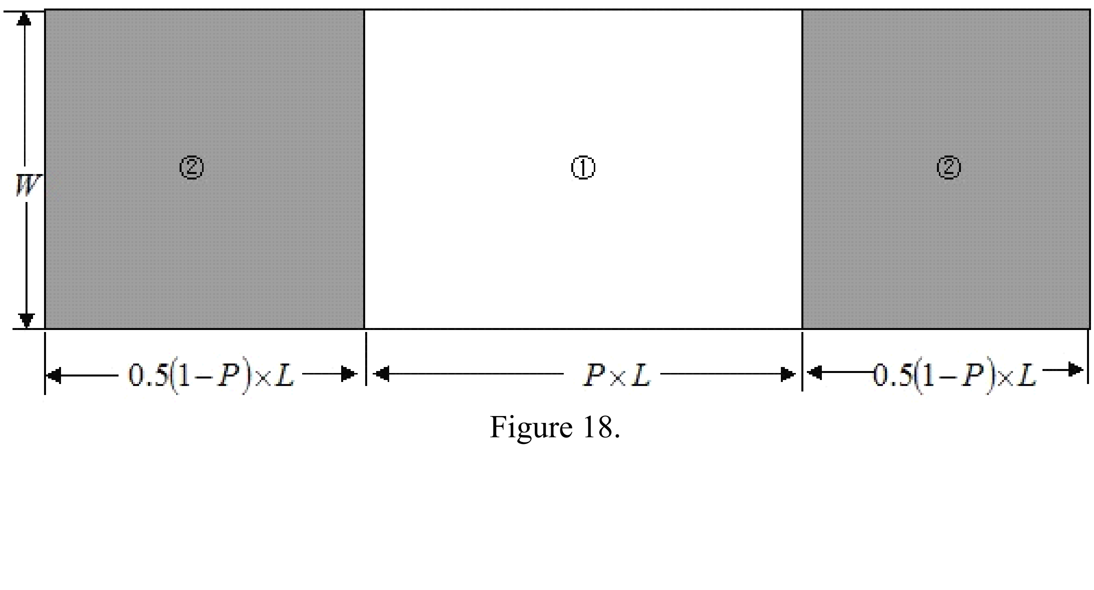


Figure 18.

As been told in model 1, the corners have a higher temperature than other parts of the oven. So plan 1 is used in district 1 (in Figure 10) and plan 2 is used in district 2 (in Figure 10). A better compromise could be reached in this way, as shown in Figure 19.

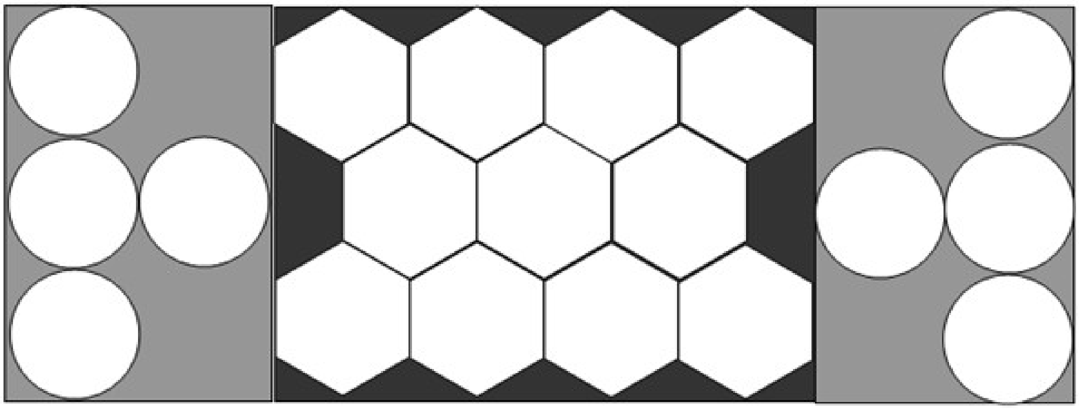


Figure 19.

Every pan has a square of A. Radius of circular ones is r. Side length of regular hexagon is a.

$$\pi \cdot r^2 = \frac{3\sqrt{3}}{2} \cdot a^2 \quad \Rightarrow \quad a : r = 1.1 \tag{8}$$

Based on the equation (8), if the pans are placed as shown in Figure 19, regular hexagons are placed full of district 1, the circular ones will be placed beyond the border line. If the circular ones are placed full of district 2, there will be more gaps in district 1, which will be wasted. So we change our plan of placing pans as Figure 20.

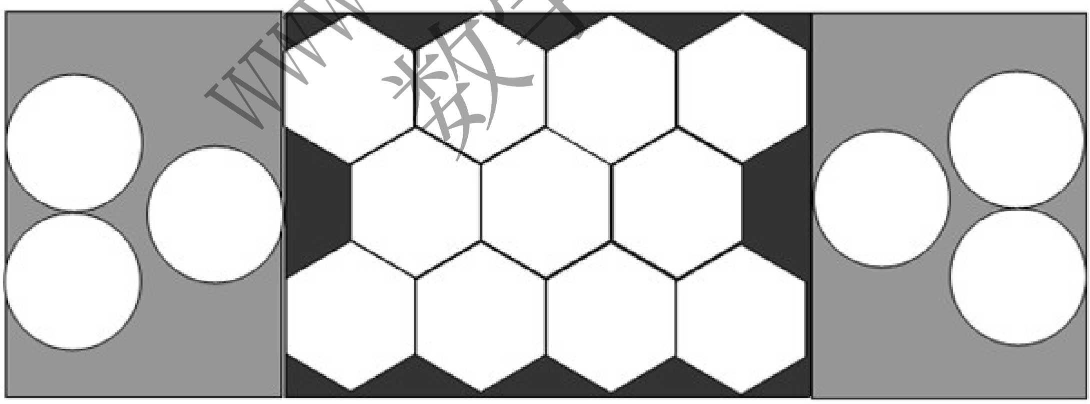


Figure 20.

The number of circular ones decreases by two, but the space in district 1 is fully used, and no pan will be placed beyond the borderline.

We assume that P is bigger than $1-P$, so that, the heat in district 1 will be fully used. By simple calculating, we know that the ratio of the heat absorbed in circular pan (H_1) and in regular hexagon (H_2) is 1.2:1.

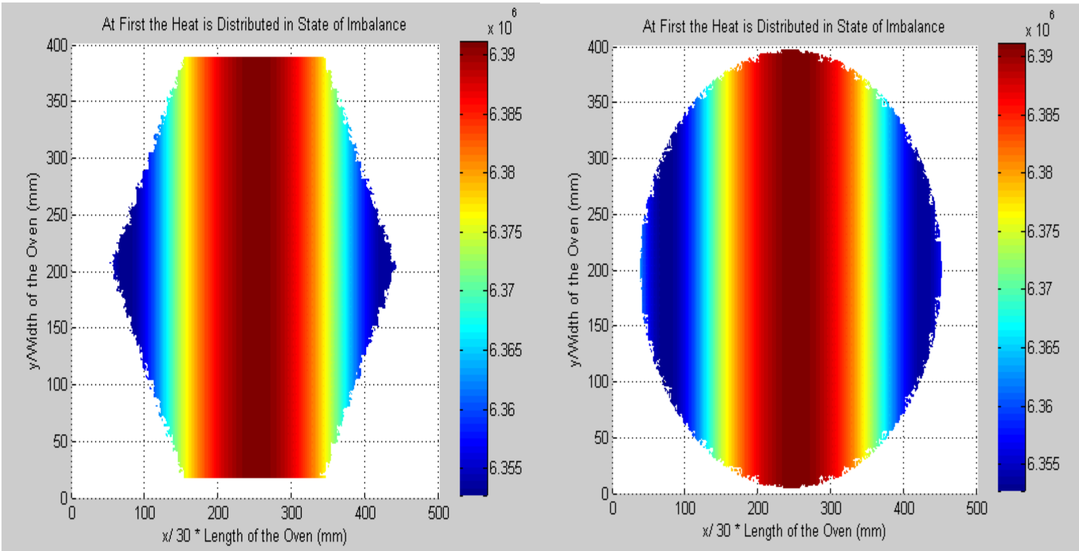


Figure 21.

So, based on the pans placing plan, a equation on heat can be got as follow:

$$
\begin{cases}
\dfrac{\sqrt{3}}{2}a \approx r = x; \\
A = \dfrac{3\sqrt{3}}{2}\cdot a = \pi \cdot r^2 \\
0 < \dfrac{W}{L} < 1 \\
L = L_0 + k \cdot \Delta L \\
\Delta L = x \\
0 \le \dfrac{L + k\cdot \Delta L}{W} - \left[\dfrac{L + k\cdot\Delta L}{W}\right] < 1 \\
\dfrac{W}{L + k \cdot \Delta L} = \lambda \\
N_0 = 9, L_0 = W \\
n_1 = \left[P \cdot \dfrac{L - W}{x}\right] \\
n_2 = \left[(1-P)\cdot \dfrac{L-W}{x}\right] \\
k = 1,2,3\ldots
\end{cases}
\tag{9}
$$

Resolution:

$$\begin{cases} N_1 = N_0 + 2\cdot n_2 + 3\cdot \dfrac{n_1-1}{2} + 1 & (n_1 = 2k-1) \\ N_2 = N_1 + 2\cdot n_2 + 3\cdot \dfrac{n_1}{2} & (n_1 = 2k) \\ H_1 = \dfrac{(5+\frac{n_1-1}{2}\cdot 3)\cdot Q + 1.2Q\cdot(4+2\cdot n_2)}{N_1\cdot A} \\ H_2 = \dfrac{(5+\frac{n_1}{2}\cdot 3)\cdot Q + 1.2Q\cdot(4+2\cdot n_2)}{N_2\cdot A} \\ k = 1,2,3... \end{cases} \tag{10}$$

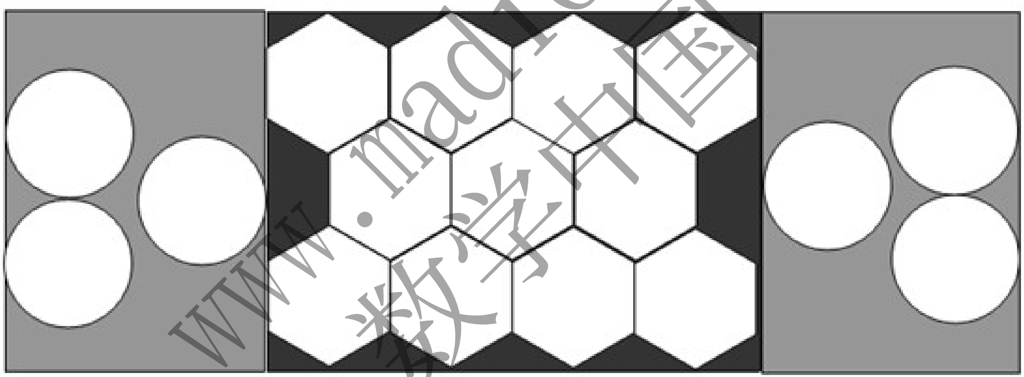
N_1 and H_1 means the number of pans and average heat absorbed when n is odd.

N_2 and H_2 means the number of pans and average heat absorbed when n is even.

For example:

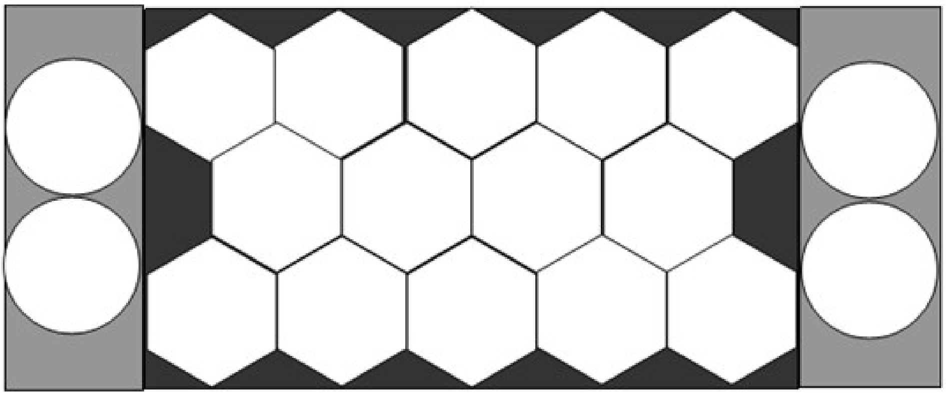
(1) When $\lambda = 0.37$, $P = 0.6$:

$N = 16$, $H = 1.075\dfrac{Q}{A}$. The best placing plan is:



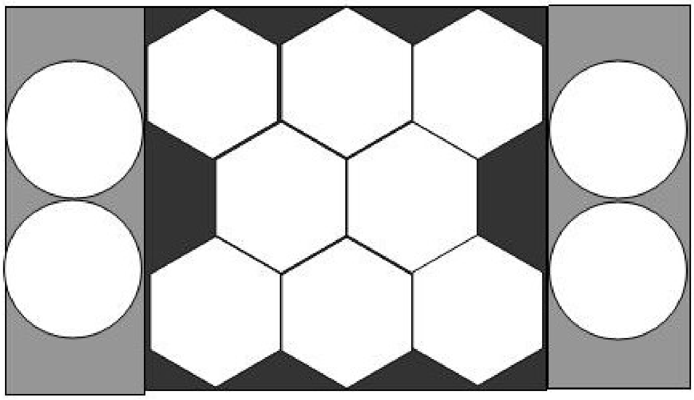
(2) When $\lambda = 0.37$, $P = 0.7$:

$N = 18$, $H = 1.044\cdot\dfrac{Q}{A}$. The best placing plan is:



(3) When $\lambda = 0.58$, $P = 0.6$:

$N = 12$, $H = 1.067 \cdot \dfrac{Q}{A}$. The best placing plan is:



A conclusion is easy to draw that when the ratio of width and length of the oven (λ) is a constant, the number of pans increases with an increasing P, but the average heat decreases (example (1) and (2)). When the weight P is a constant, the number of pans decreases with an increasing λ, and the average heat decreases also. So, the actual plan should be base on your specific needs.

Conclusion

In conclusion, our team is very certain that the method we came up with is effective in heat distribution analysis. Based on our model, the more edges the pan has, the more evenly the heat distribute on. With the discretize-convert approach, we know that when the ratio of width and length of the oven (γ) is a constant, the number of pans increases with an increasing P, but the average heat decreases. When the weight P is a constant,the number of pans decreases with an increasing γ, and the average heat decreases also. So, the actual plan should be base on your specific needs.

Strengths & Weaknesses

Strengths

• Difficulties Avoided.

In model 1, we turn to another way to work simulate the heat distribution instead of work on heat conduction directly. Firstly, we simulate heat radiation not heat conduction to keep away from the sophisticated partial differential equations. Then, we create a unique method to convert every variable from the whole space to section. In other words, we work everything out in heat radiation and convert them into heat contradiction.

•Close to Reality.
Our model considers both the thermal radiation and surface reflection, which is relatively close to the actual situation.

•Flexibility Provided.
Our algorithm does a great job in flexibility. The heat distribution map on sections are intercepted from the heat distribution maps of the entire space. All shapes of sections can be used in the algorithm. The heat distribution in the whole space is generated based on the location of the electrical heating tubes and the decay curve of the heat, which can be modified at any time.

•Innovation.
Based on our model, the space of an oven can be divided into six parts with different hear distribution. In order to make full use of the inner space, we invent a new pan which allows users to cook six different kinds of food at same time. An advertisement is published in the end of the paper.

Weaknesses

• Pan's Thermal Conductivity Ignored.
The heat comes from not only the electrical heating tubes, but also heat conduction of the pans themselves. But the pan's thermal conduction is ignored in the model, which may cause little inaccuracy.

• Thermal Conductivity of Electrical Heating Tubes Ignored.
it is assumed that there are two electrical heating tubes in the oven and placed in a specific location. The initial temperature of the tubes is a desired constant temperature. In other words, the time electrical heating tubes spend to heating themselves is ignored. The simplification can cause some inaccuracy.

• Linear simplification.
In model 2, the length of the oven is discretized, so that the number of pans will changes linearly. calculating through simple integer linear method. This will lead to the result of our model is not accurate enough.

Application

We have discussed the heat distribution in the oven in model 1. The heat distribution is shown in figure 1 and figure 2.

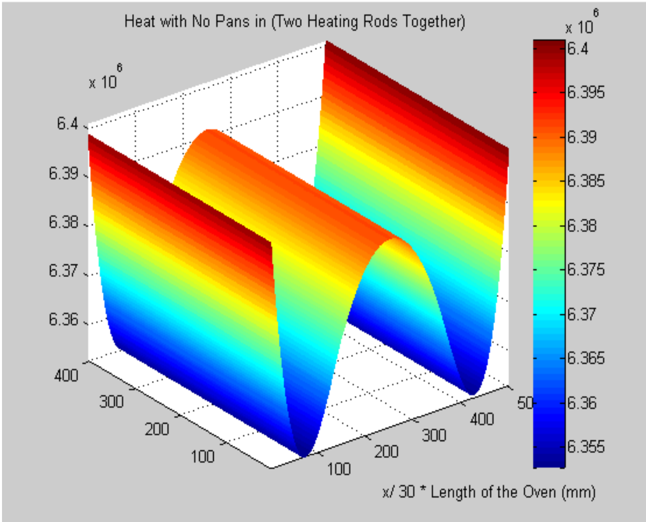


Figure 1

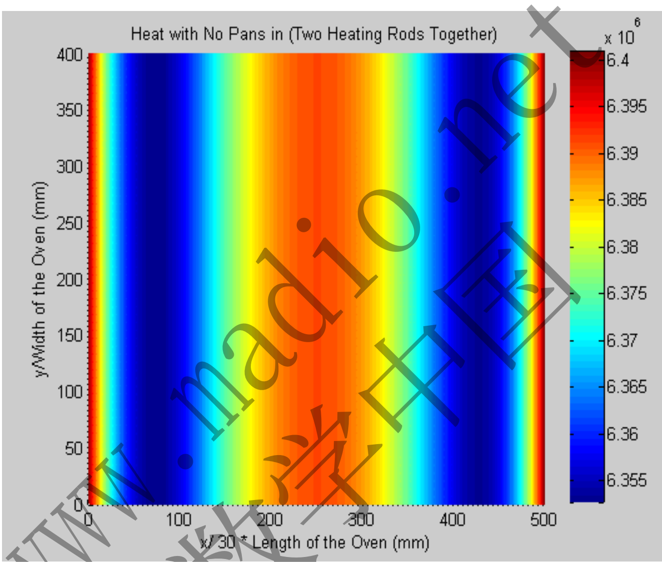
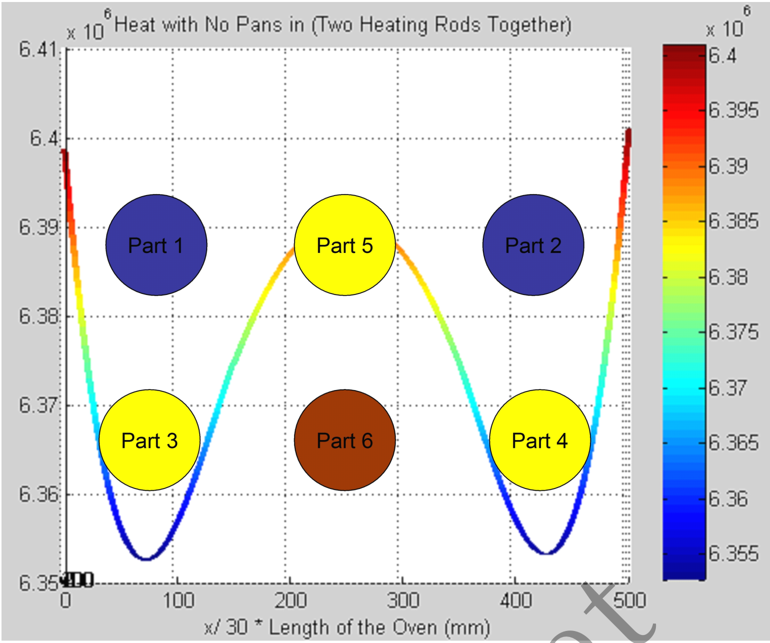


Figure 2

As shown, the edges of the oven are distributed the most heat. Areas on both sides of the, is distributed the least heat. While the middle area absorbs little less than the edges. So, we can separate the oven area into six parts, as shown bellow.



Part 1 and part 2 are distributed the least heat and located the furthest from the heat source (the electrical heating tubes locate on the bottom of the oven). So these two parts absorb the least heat. Part 3 and part 4 are distributed the least heat but locating the nearest to the heat source. Part 5 located far from the bottom but distributed the most heat. So simply, we regard the heat of part 3, part 4 and part 5 as the same. Part 6 is distributed the most heat, and locating nearest to the bottom. So, the heat part 6 absorbs is the most in the oven.

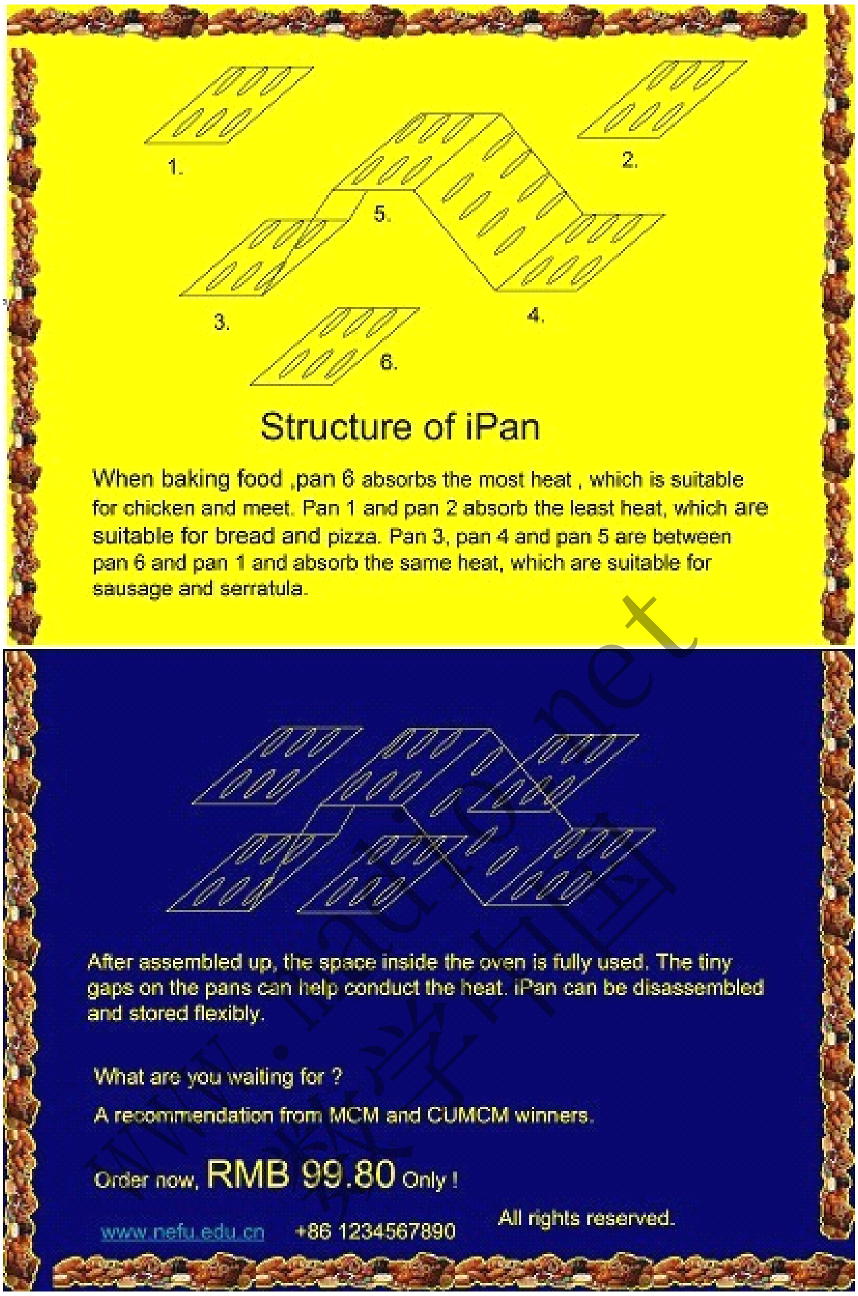
Based on our conclusion above, we invent the iPan, a new combined pan, which can bake three kinds of food at the same time. For example, one wants to have a little bread, pieces of sausage, a chicken wing and a pizza for lunch. He will have to wait 30 minutes at least for his lunch, if he just has one oven. As the Chinese saying goes, '*Bear paws and fish never come together*' .

By using iPan can solve the issue for him, he could put the bread in pan 1, pizza in pan 2, sausage in pan 5 and chicken wing in pan 6, and power on. Thus, he can have his delicious lunch in at least 10 minutes. So, bear paws and fish come together.

We make an advertisement for *Brownie Gourmet Magazine* in the end of the paper.

Advertising Sheets


BHM®
Get
FISH and BEAR
鱼 & 熊掌
in One Pan
with
iPAN
Little Bread ?
One Pizza ?
A Few Sausage ?
Pieces of Hum ?
Tow or Three Chicken Wings ?
Wanna a Taste of Serratula ?
What if you just have one oven ?
Waiting 30 minutes for dinner ?
NO !
iPan is designed based on our model of the heat distribution inside the oven .
With the design ,we can use three different intensity of heat in the same time .
In other word, we can make up a delicious dinner with at least six kinds of cuisine in ten minutes !



References

[1] Heat Radiation, http://wenku.baidu.com/view/f5ed1619cc7931b765ce1599.html, Page. 4

[2] G. S. Ranganath, *Black-body Radiation*, http://link.springer.com/article/10.1007%2Fs12045-008-0028-7?LI=true#, February, 2013

[3] Kaiqing Lu, *The Chemical Basis of Heat Transfer*, Journal of Higher Correspondence Education(Natural Sciences Edition), Vol.3: p.33, 1996

[4] Mark M. Meerschaert, *Mathematical Modeling(Third Edition)*, China: China Machine Press, May. 2009

[5] Jianzhong Zhang, *Monte Carlo Method*, Mathematics in Practice and Theory,

Vol.1 p.28, 1974
[6] Shallow water equations, http://en.wikipedia.org/wiki/Shallow_water_equations