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Problem Chosen

B

2011 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

KEY PARAMETERS

N-Number of users

R-Communication range of a repeater

r-Communication range of a user

C-Capacity of a repeater

KEY CONCEPT

Relay Networks

Two-Tiered Model

Covering Problem

Voronoi Diagram

KEY TECHNIQUES

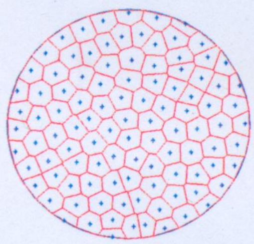
Iterative Refinement

Extremal Optimization

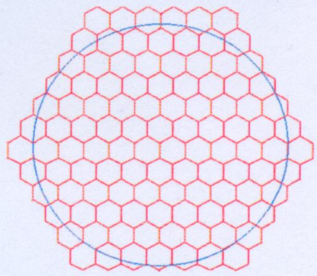
Ploting Voronoi Diagram

Determining Circumcircle

Minimum Spanning Tree



Voronoi Diagram Beats Cellular Network



104 Repeaters from out algorithm

108 Repeaters from the naïve solution

Problem Clarification. In a wireless communication system, amplify-and-forward repeaters are of great significance in supporting long-distance communication between low-power users. Repeaters are of limited capacities and may interfere with one another if they locate nearby and use close transmitter frequencies. Given a circular flat area, we are asked to determine the least number of repeaters, as well as their working frequencies and private line tones, to accommodate a number of simultaneous users

Assumptions. The crucial assumptions in modeling the system are threefold: (i) users are uniformly distributed in the considered area and prefer to communicate with the nearest repeaters; (ii) two repeaters will interfere with each other if the difference between their transmitter frequencies is less than 0.6MHz, they share the same PL tone and their distance is less than twice of the repeater's communication range; (iii) in the considered area, the wireless signals can fade freely.

Model. We propose a two-tiered network model consisted of two kinds of nodes: normal users and repeaters. User's and repeater's communication ranges are respectively *r* and *R*. Users can send signals to repeaters within a distance *r* and repeaters can transmit signals to other repeaters and users up to a distance *R*. A repeater's capacity is C. Besides the capacity constrain and the interference avoidance, every user should be covered by at least one repeater and in a stronger requirement, every user can communicate with another user in any position of the considered area.

Methods. (i) Inspired by the cellular network structure, we give a naïve solution where the number of repeaters and their positions can be obtained analytically. (ii) We propose an iterative refinement algorithm consisting of three fundamental modules: to draw the Voronoi diagram, to determine the circumscribed circles of Voronoi areas, and to escape the local optimum by using the idea of extremal optimization. (iii) We propose a greedy algorithm based on the minimal spanning tree to rearrange the frequencies and PL tones of repeaters.

Results. The naïve solution: 12 repeaters for 1000 users and 108 repeaters for 10000 users. The proposed algorithm: 11 repeaters for 1000 users and 104 repeaters for 10000 users. We have proved that the 11 repeaters is the optimal solution. Without adding any more repeaters, after the rearrangement of the 104 repeaters' frequencies and PL tones by the greedy algorithm, with probability 97.12%, a randomly selected user can successfully send signals to a random position in the considered area.

Strengths. The model is very general and can be naturally mapped to a well-defined optimization problem. The proposed algorithm is very effective and efficient: it runs much faster than the simulated annealing approach and is of higher ability to escape the local optimum than other iterative refinement and machine learning algorithms.

Weaknesses. We have not developed an effective method to make sure every user can communicate with users in any positions of the considered area.

Least Number of Repeaters Covering a Circular Area: Iterative Extremal Optimization Based on Voronoi Diagram

Abstract

In this paper, we propose a two-tiered network model, where lower-power users can communicate with one another through repeaters, which amplify weak signals and retransmit them. A repeater is of limited capacity and may interfere with a nearby repeater if their transmitter frequencies are close and they share the same private line tone. Our objective is to place the least number of repeaters that can satisfy the users' communication requirement: the weak requirement is that every user must be covered by at least one repeater while the strong requirement is that every user can communicate with another user in any position of the considered area. Motivated by the structure of cellular networks, we give a naïve solution where the number of repeaters and their positions can be obtained analytically. In a circular area with radius 40 miles, 12 repeaters are enough to accommodate 1000 simultaneous users. We further propose an iterative refinement algorithm consisting of three fundamental modules: to draw the *Voronoi diagram*, to determine the circumscribed circles of Voronoi areas and move the repeaters to the centers of these circumscribed circles, and to escape the local optimum by using the idea of *extremal optimization*. The proposed algorithm outperforms the naïve solution. For the case of 1000 users, it obtains a solution with 11 repeaters, which can be proved to be the real optimum. For the case of 10000 users, it obtains a solution with 104 repeaters, better than the naïve solution with 108 repeaters. We further discuss how to assign frequencies and private line tones based on maximum and minimum spanning tree technique, the effects of simultaneous users need to be accommodated, the fluctuation of user density in reality, how the landscape can affect the design of repeaters' locations, and the strengths and weaknesses of the model and algorithms.

1 Introduction

The design of amplify-and-forward relay networks has attracted increasing attention recently [1-3]. The relay networks are very helpful for long-distance communication since the relay nodes can amplify the weak signals and forward them to other relay nodes or send them directly to the destinations. Through several relay nodes, low-power users can communicate with one another in situation where direct user-to-user communication would not be possible. The relay networks have already found wide applications in cooperative communication [4]. For example, in amateur radio activities hams can communicate with distant hams through one or more relay nodes (also called radio repeaters) [5], and in wireless sensor networks relay nodes are used to help information dissemination between distant sensors [6].

In many amplify-and-forward cooperative communication networks, the nodes are homogeneous, namely a node can simultaneously play the roles as source node, sink node and relay node. A typical example is the mobile communication, where a cell phone can also be an amplify-and-forward relay node [4]. In another scenario, the relay nodes usually do not initiate communications but only help communications between other nodes. For example, in amateur radio activities, radio repeaters can be considered as relay nodes, and hams usually carry their own radios (transceivers). Repeaters can pick up weak signals from radios, amplify and retransmit them. Different from the homogeneous scenario, the functionalities of repeaters and radios are different, and the specifications of repeaters and radios are also different. For example, in Utah, the power of a radio is usually a few watts, while most repeaters are of power no less than a hundred watts.

Pan *et al.* [7] proposed a two-tiered relay network model, where the base stations are considered as low power users and some inter-application nodes are playing the role of relay nodes. However, they did not address the issue of covering users by relay nodes. Gupta and Younis considered the fault-tolerant [8] and traffic load balance [9] problems in a two-tiered relay network model, yet they did not address the placement problem of relay nodes. Tang *et al.* [10] proposed two algorithms for the relay nodes placement problem, aiming at placing the fewest number of relay nodes in the considered area such that each user can communicate with at least one relay node and the network of relay nodes is connected.

The above-mentioned works are different from ours because in a more general scenario that includes the amateur radio communication, the capacity of a repeater and the interference among nearby repeaters should be taken into account. In this paper, we propose a more general two-tiered network model, where each repeater can simultaneously manage at most C users and two nearby repeaters will interfere with each other if their transmitter frequencies are close and they share the same private line tone. Our objective is to place the least number of repeaters that can satisfy the users' communication requirement. We respectively consider two different requirements: the weak requirement is that every user must be covered by at least one repeater and the strong requirement is that every user can communicate with another user in any position of the considered area. Our analysis indicates that the former requirement is equivalent to a circle covering problem while the solution on the latter requirement is based on the answer to the former requirement.

The geometric covering problem is known to be NP hard [11], and the algorithm is usually very time-consuming even for a small number of circles. For example, Nurmela and Ostergard [12] proposed a simulated annealing algorithm to obtain near-optimal solutions for the unit square covering problem with up to 30 equal circles. Their result are known to be one of the most accurate results yet the algorithm has to run more than 2 weeks to obtain the solution for 27 circles. In contrast, considering the limited capacity of a repeater, when the number of users being accommodated increases, we have to place remarkably more repeaters than being considered in the previous works.

Motivated by the structure of cellular networks (i.e., networks with beehive-like structure), we give a naïve solution where the number of repeaters as well as their positions can be obtained analytically. In numerical simulation, this naïve solution performs unexpectedly well. In a circular area with radius 40 miles, 12 repeaters are enough to accommodate 1000 simultaneous users and for 10000 users, 108 repeaters are sufficient. In comparison, in the former case, 11 repeaters are the optimum and in the later case. We further propose an iterative refinement algorithm consisting of three fundamental modules: to draw the *Voronoi diagram* [13], to determine the circumscribed circles [14] of Voronoi areas and move the repeaters to the centers of these circumscribed circles, and to escape the local optimum by using the idea of *extremal optimization* [15,16]. This method outperforms the naïve solution. For the case of 1000 users, it obtains a solution with 11 repeaters, which can be proved to be the real optimum. For the case of 10000 users, it obtains a solution with 104 repeaters, better than the naïve solution.

Taking into account the capacity limitation and interferences, we proposed a more general model than most of previous models. Two methods are designed to place the repeaters in a typical area. They both perform very well. This work provides a general model and a novel algorithm to an extensively studied problem: how to place relay nodes in a cooperative communication system. This work can find applications in not only the amateur radio activities, but also the design of sensor wireless networks and mobile wireless networks.

This paper is organized as follows. In Section 2, we clarify the considered problem, and in Section 3, we descript the proposed two-tiered amplify-and-forward network model, as well as the definitions of solution and strong solution. Section 4 presents the calculation of the communication range of repeaters and users, the capacity of a repeater. In this section, we apply the continuous approximation and show how to map this problem to a “circles covering circle” problem. The naïve solution is also given in Section 4. We present the algorithms respectively to determine the requirement number of repeaters as well as their positions and to assign frequencies and private line tones to repeaters in Section 5. In Section 6, we give the solutions obtained by our algorithm, compare them with the naïve solution, and prove that the solution for the small population case is the optimal one. In Section 7, we discuss the effects of simultaneous users need to be accommodated, the fluctuation of user density in reality, and how the landscape can affect the design of repeaters’ locations, and finally, in Section 8, we summarize the paper and discuss the strengths and weaknesses of the model and algorithms.

Parameters, mathematical symbols and assumptions appear everywhere in this paper. Therefore, for convenience, in Appendix A, we list all the parameters, in Appendix B, we list the mathematical symbols, and in Appendix C, we summarize the important assumptions made in this paper.

2 Problem Description

Given a circular flat area of radius Φ (this circular area is denoted by Γ), we are asked to determine the least number of radio repeaters necessary to accommodate a given number of users *N*. A repeater is a combination of a radio receiver and a radio transmitter that picks up weak signals, amplify them, and retransmit them on a different frequency. In this problem, the three most fundamental parameters characterizing a repeater are the receiver frequency f_r, the transmitter frequency f_t and the private line (PL) tone n_{PL}. A repeater responds only to signals on its receiver frequency and containing the same PL tone. If so, the signals will be amplified and transmitted on the repeater's transmitter frequency and the PL tone will not be changed. In this problem, both f_r, and f_t belong to the range [145MHz, 148MHz], $|f_r - f_t|$=0.6MHz, and there are in total N_{PL}=54 different PL tones available.

For example, a repeater located in Mong Kok, Hong Kong has the receiver frequency 145.025MHz, the transmitter frequency 145.625MHz, and the PL tone 110.9Hz. That is to say, only the signals on 145.025MHz and with PL tone 110.9Hz can be amplified by this repeater, and after amplification, the signals will be transmitted with frequency 145.625MHz and PL tone 110.9Hz.

Notice that, two repeaters will interfere with each other if they satisfy all the following three conditions: (i) they carry exactly the same PL tone; (ii) their distance is smaller than the twofold of the communication radius *R* of the repeater (for simplicity, we assume every repeater has the same communication radius *R*); (iii) the difference between their transmitter frequencies is smaller than a threshold f_c. It is necessary to guarantee that no interference will happen between any pair of repeaters.

Each user can directly contact other nearby users and reach distant users through one or several repeaters. The maximal communication distance from a user to a repeater, *r*, is supposed to be identical for every user, which is remarkably smaller than the communication radius of a repeater (i.e., $r<R$). Every user should be covered by at least one repeater, namely at least one repeater locates inside the circle with radius *r* and centered at the user's location. Under a stronger requirement, each user is able to send information to users at any positions in the considered area through one or several repeaters.

Since the communication ranges, *R* and *r*, are finite, and the capacity of a repeater, *C*, is very limited (i.e., a repeater can manage at most *C* different users simultaneously), usually a number of repeaters are required to accommodate users in a given area. The primary problem is to determine the minimum number of repeaters that can satisfy the communication requirement of a given number of users. Of course, a complete solution should also contain the positions, receiver and transmitter frequencies, as well as the PL tones of repeaters. The effects of the number of users and the landscape are emphasized in this problem. More detailed description of the meanings and effects of the parameters, and the in-depth analysis on the problem constrains are presented in the following sections.

3 Model

To present the system, we use a two-tiered directed network $D(V_u, V_r, E_{ur}, E_{rr})$, where $V_u = \{u_1, u_2, ..., u_N\}$ and $V_r = \{r_1, r_2, ..., r_M\}$ denote the sets of users and repeaters, and E_{ur} and E_{rr} are the sets of directed links from users to repeaters and between repeaters. Each user u_i is identified by her/his location $(x(u_i), y(u_i))$ in a 2-dimensional plane, and each repeater r_j is identified by its location, receiver frequency, transmitter frequency and PL tone as $(x(r_j), y(r_j), f_r(r_j), f_t(r_j), n_{\text{PL}}(r_j))$. As mentioned above, the frequencies of an arbitrary repeater r_j should obey the constrains $f_r(r_j), f_t(r_j) \in [145\text{MHz}, 148\text{MHz}]$ and $\left| f_r(r_j) - f_t(r_j) \right| = 0.6\text{MHz}$. Since the considered area is a circular area of radius Φ, the location for arbitrary user or repeater should satisfy the inequality

$$x^2 + y^2 \le \Phi^2 \tag{1}$$

where $\Phi = 40$ is given a priori. A directed link from u_i to r_j exists (i.e., $(u_i, r_j) \in E_{ur}$) if the distance between u_i and r_j is no more than a user's communication range r. A directed link from r_j to r_k exists (i.e., $(r_j, r_k) \in E_{rr}$) if the distance between r_j and r_k is no more than a repeater's communication range R, the transmitter frequency of r_j equals the receiver frequency of r_k (i.e., $f_t(r_j) = f_r(r_k)$) and they share the same PL tone (i.e., $n_{\text{PL}}(r_j) = n_{\text{PL}}(r_k)$). Clearly, the sets of directed links are determined when the information about users and repeaters is known.

A network D is called a *solution* (in section 4, we will discuss a more practical definition of solution that does not depend on a specific distribution of users' locations) if the following three conditions ($\Omega_1, \Omega_2, \Omega_3$) are all satisfied.

Ω_1 **-Capacity constrain**. For simplicity, we assume users are uniformly distributed in the considered area Γ and each of them prefers to communicate with the nearest repeater. Each repeater can manage at most C users at the same time (the capacities of different repeaters are supposed to be the same, and the estimation of C will be given in the next section). For a repeater r_j, there exists a connected area $S_V(r_j)$ inside Γ such that for every point inside this area, r_j is the nearest repeater and for every point outside this area, r_j is definitely not the nearest repeater. This area is called the *Voronoi area* [13] of r_j. For every repeater, the number of users inside its Voronoi area must be no more than its capacity C.

Ω_2 **-Interference avoidance**. If two repeaters share the same PL tone and are with distance less than $2R$, the difference between their transmitter frequencies must be no less than a given threshold f_c. In the spectrum 145-148MHz, the standard threshold is f_c=0.6MHz.

Ω_3**-Connectivity**. Every user is covered by at least one repeater. That is to say, $\forall u_i, \exists r_j$, such that $(u_i, r_j) \in E_{ur}$.

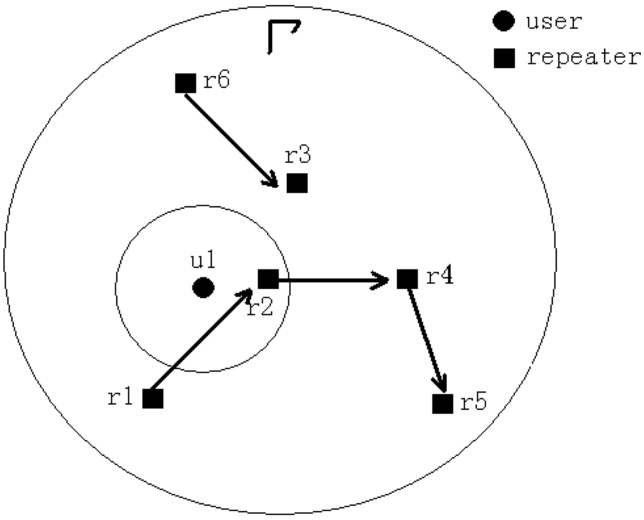


Figure 1. An illustration about the reachable repeaters of u_1. The circle centered at u_1 is the communication of u_1 with radius *r*.

Our goal is to find out the solution with the minimum number of repeaters *M*. In a solution, although every user is covered by at least one repeater, her/his signals may not reach the desired position in Γ since the coverage of a repeater is also limited and the solution does not guarantee the existence of a multi-hop path through several repeaters to reach the desired position. Ignoring the small area that can be directly reached by a user without the help of any repeater, the reachable area of a user u_i, denoted by $S_r(u_i)$, is defined as the area in Γ that can be covered by at least one reachable repeater of u_i (each repeater covers a circle with radius *R*). The set of reachable repeaters for u_i, denoted by $R_r(u_i)$, consists of two parts: One is the repeaters that can be directly reached by u_i (i.e., the repeaters located within the circle with radius *r* and centered at u_i) and another is the repeaters that can be further reached through directed paths consisted of directed links in E_{rr} and started from the directly reachable repeaters. Figure 1 illustrates a simple example where r_2 can be directly reached by u_1, and r_4 and r_5 can be further reached starting from r_2. Two nearby repeaters, like r_2 and r_3, may not have a link in between since they may not match with each other in frequency or PL tone. The reachable repeaters of u_1 are r_2, r_4 and r_5, and thus the reachable area of u_1 is the union of the coverage areas of r_2, r_4 and r_5. The following condition must be satisfied to guarantee every user can in principle reach any position of the considered area through multi-hop repeaters.

Ω_4**-Global Reachability**. The reachable area of every user is equal to the considered area Γ.

A network *D* is called a *strong solution* if the following three conditions ($\Omega_1, \Omega_2, \Omega_4$) are all satisfied. Under these definitions, two propositions obviously hold, that is

Proposition 1: A strong solution is a solution.

Proposition 2: When $R \geq 2\Phi$, any solution is a strong solution.

To find a strong solution is much more difficult than to find a solution, and these two tasks are equivalent to each other only if $R \geq 2\Phi$. We will respectively study the cases of $R \geq 2\Phi$ and $R < 2\Phi$, emphasizing on the strong solutions.

4 Analysis

In this section, we will give some elementary analysis about the model. Firstly, in subsection 4.1 and 4.2, we will calculate the communication ranges for repeaters and users, as well as the repeater's capacity according to the Shannon's theory. Taking into consideration of the mobility of users, in subsection 4.3, we will show that the continuous approximation of the distribution of users' locations is necessary to address the problem. In subsection 4.4, we will present a naïve solution where repeaters are arranged in a cellular network (i.e., a beehive-like structure). In spite of the simplicity, the naïve solution performs unexpectedly well, which will play a role of benchmark solution in evaluating the algorithmic output.

4.1 Communication Radius

We apply a simple model to determine the effective radiated distance (i.e., communication range) of repeaters and users. According to the problem description, we assume that this circular area is a place where wireless signals can fade freely. That is to say, there are not any other sorts of interference such as fogs, rivers, hills, buildings, activities of sun and so forth. Therefore the fading of signals under such situation is only due to the distance across which the signal is transmitted.

Let $P_{r,out}$ be the power of the signal transmitted by a repeater. Since the signal can fade freely, its average power P within one unit area at a distance d from the repeater is

$$P = \frac{P_{r,out}}{4\pi d^2} \tag{2}$$

According to the antenna theory [17], the effective receiving area of an antenna is $\lambda^2/4\pi$, where λ is the wave length of the signal. So the receiving power of the signal is

$$P' = \frac{P_{r,out}}{4\pi d^2} \bullet \frac{\lambda^2}{4\pi} \tag{3}$$

Substituting λ with c/f, where c is the velocity of light and f is the frequency of the signal, one obtains

$$P' = P_{r,out}\left(\frac{c}{4\pi df}\right)^2 \tag{4}$$

In terms of the communication theory (Shannon's thoery), the loss L_s is

$$L_s = 10\lg\left(\frac{P_{r,out}}{P'}\right) = 92.4 + 20\lg d + 20\lg f \tag{5}$$

where L_s is measured by dB, d is measured by km, f is in GHz. The actual power of received signal $P_{r,in}$ is

$$P_{r,in} = P_{r,out} + (G_{out} + G_{in}) - (L_{f,out} + L_{f,in}) - (L_{b,out} + L_{b,in}) - L_s \tag{6}$$

From one repeater (transmitter) to another repeater (receiver), the equations hold as $L_{f,out} = L_{f,in}$ (the loss of feed system), $L_{b,out} = L_{b,in}$ (other loss of the system), $G_{out} = G_{in}$ (the gain of antenna). Normally, $L_{f,out} = L_{f,in} = 2dB$, $L_{b,out} = L_{b,in} = 1dB$, $G_{out} = G_{in} = 39dB$. For this problem, the frequency of signals is about 0.1465GHz (i.e., the middle point of the available spectrum 145MHz~148MHz), and thus

$$d = 10^{\frac{10\lg\frac{P_{r,out}}{P_{r,in}} - 37.2328}{20}} \tag{7}$$

Table 1: Effective radiated powers of repeaters in Utah (http://www.ussc.com/~uvhfs/rptr.html)

Call No.	**ERP**	**Call No.**	**ERP**	**Call No.**	**ERP**	**Call No.**	**ERP**	**Call No.**	**ERP**
K7UCS	120 W	WA7UAH	30 W	KD7NX	58 W	K7JL	100 W	K7JL	500 W
KF6RAL	100 W	KD7HUS	25 W	K7JH	100 W	K7MLA	100 W	K7SDC	100 W
W7IHC	100 W	WV7H	40 W	K7QEQ	100 W	W7WAC	50 W	K7MLA	100 W
AA7JR	100 W	N7BSA	80 W	W7SP	100 W	N7KM	40 W	N7NKK	50 W
NZ6Z	100 W	W7BAR	100 W	WV7H	100 W	W7EO	40 W	K7MLA	100 W
KE7FO	20 W	K7HEN	100 W	AE7TA	120 W	KC7SNN	100 W	K7JL	100 W
KR7D	100 W	W7DRC	48 W	N6EZO	100 W	K7OGM	100 W	AC7O	10 W
K7JL	50 W	K7SDC	100 W	W7SU	100 W	K7MLA	75 W	N7ZOI	100 W
N7WFM	100 W	W7SP	100 W	NR7K	100 W	K7DAV	50 W	W7EO	100 W
WB7REL	160 W	WA7MXZ	100 W	N7PCE	100 W	W7BAR	183 W	AB7TS	100 W
K7SDC	100 W	W7JVN	100 W	K7JL	100 W	K7SDC	100 W	KJ7VO	100 W
WA7KMF	100 W	W7DRC	60 W	KD7YE	97 W	N7DZP	40 W	WA7YZR	100 W
N7BYU	100 W	W7DHH	100 W	W7NRC	100 W	KB7ZCL	200 W	K7HEN	100 W
K7ACA	100 W	N7JSQ	100 W	KD0J	100 W	K7CEM	40 W	N7PQD	147 W
W7EO	160 W	KD0J	100 W	K7QEQ	100 W	K7SDC	100 W	N7WPF	125 W
KA7LEG	200 W	KA7STK	25 W	W7SU	100 W	WX7Y	100 W	AC7O	200 W
AC7O	100 W	W7NRC	100 W	NR7K	20 W	K7SDC	60 W	K7SG	25 W
WB7TSQ	100 W	AC7O	100 W	N7TOP	175 W	N7GGN	20 W	K7UCS	120 W
W7DES	25 W	WB7CBS	100 W	W7BYU	50 W	W7BAR	100 W	W7CWK	100 W
W7EO	100 W	K7UCS	120 W	WA7GIE	100 W	KB7WQD	48 W	WB7TSQ	100 W
K7SDC	100 W	KK7EX	50 W	WI7M	67 W	WA7GIE	100 W	WA7GIE	100 W
KK7DO	800 W	NR7K	100 W	WR7AAA	100 W	AG7BL	100 W	W7NRC	100 W

In the light of the statistics about the 2-meter repeaters in Utah, USA (see Table 1), the effective radiated power of most of the repeaters is $P_{r,out} = 100W$, and normally a repeater can receive the signal with power no less than $1\mu W$ (i.e., $P_{r,in} \geq 1\mu W$). According to Eq. (7), the communication radius of a repeater is $R = 85.45$ miles (the original result is measured by kilometers, and we have already changed it to miles). Analogously, the average working power for a user (according to several wireless devices) is $P_{u,out} = 3.2W$ and $P_{u,in} \geq 1\mu W$, resulting in the user's communication radius $r = 15.28$ miles. Although our calculation indicates that $R \geq 2\Phi$ and the global reachability (Ω_4) is equivalent to the connectivity (Ω_3), we will discuss in this paper the more complicated case when $R < 2\Phi$.

4.2 Repeater's Capacity

In this subsection, we will calculate the maximal number of users that can be simultaneously accommodated in a repeater, namely the capacity *C*.

Ignoring the background noise and the interference, namely we assume that signals from one repeater will not affect others. Considering the transmitter frequency in a repeater is an exact value rather than in a broad band, we come up with a method to calculate the capacity through the gain. A mainstream method to estimate the capacity of information over a noisy channel is the famous Shannon theory, as:

$$\varphi = B\log_2(1+SNR) \tag{8}$$

where φ is the information bit rate, measured by watt (dB), *SNR* is the signal-to-noise ratio, which is dimensionless, and *B* is the total bandwidth, measured by Hz.

Inspired by Shannon Theory, Gilhousen *et al.* [18] put forward a method to estimate the capacity of a cellular CDMA System. Of greater importance for reliable system operation is the bit energy-to-noise density ratio, whose numerator is obtained by dividing the desired signal power by the information bit rate φ, and dividing the noise by the total bandwidth W, which results in the equation

$$\frac{E_b}{N_0} = \frac{P_{ur}/\varphi}{(C-1)P_{ur}/B} = \frac{B/P_{ur}}{C-1} \tag{9}$$

where P_{ur} is the power of signal come from a single user and received by a repeater, measured by bps; C is the maximum number of users that can be simultaneously managed by a repeater, E_b/N_0 level is assumed which ensures operation at the level of bit error performance required for digital voice transmission, which is always set ranging from 5 to 30 dB. E_b/N_0 is a standard for digital voice transmission, and can take different values according to different circumstances.

Different from the CDMA System, the transmitter frequency in a repeater is an exact value rather than in a broad band. We prefer to take the following equation

$$\frac{E_b}{N_0} = \frac{G_{out}}{V(C-1)(1+I_{other}/I_{self})} \tag{10}$$

where G is the gain of antenna, V is the gain of voice, I_{other} is the interference come from other repeaters, and I_{self} is the interference of itself.

In the situation that a repeater can accommodate C users simultaneously, SNR can be regarded as the ratio of the effective information in the total received signal, say

$$SNR = \frac{P_{ur}}{(C-1)P_{ur}} = \frac{1}{C-1} \tag{11}$$

Normally, $G_{out} = 39dB = 7966.40W$ and $V = 0.4dB = 1.07W$ (in the calculation, the unit must be watt). We ignore the interference come from other repeaters, namely $I_{other}/I_{self} = 0$. Setting $E_b/N_0 = 18dB = 63.01W$ (the bit energy-to-noise density ratio is always ranging from 5 to 30dB, and for transceiver we set it as the middle point 18dB), we can get the capacity of a single repeater through:

$$C = 1 + \frac{G_{out}}{V(1 + I_{other}/I_{self})\,E_b/N_0} \approx 119 \tag{12}$$

4.3 Continuous Approximation

Taking into account the mobility of users, the repeaters have to cover all the considered area Γ. Given distributed users V_u, a solution $D(V_u, V_r)$ may be not a solution for another distribution of users. That is to say, the network $D(V_u^{'}, V_r)$ may not be a solution although the user locations of both V_u and $V_u^{'}$ are sampled from an identical uniform distribution. Therefore, a practical solution should not depend on a specific distribution.

To make it possible, we use a continuous approximation instead of the uniform distribution of users. Under this approximation, the number of users is considered as a real variable (do not be surprised if we talk about 94.40 users) and the N users are distributed completely uniformly in the considered area. At any point inside the considered area, the user density, denoted by ρ, is a constant as

$$\rho = \frac{N}{\pi\Phi^2} \tag{13}$$

where $\pi\Phi^2$ is the area of Γ. Except the irrelevant constrain Ω_2, the other three constrains are changed correspondingly as

Ω_1^***-Capacity constrain**. For every repeater r_j, if the area of its Voronoi area is $S_V(r_j)$, it must satisfy the inequality

$$\rho S_V(r_j) \le C \tag{14}$$

Ω_3^***-Connectivity**. Every point in the considered area is covered by at least one repeater.

Ω_4^* **-Global Reachability**. The reachable area of every point (considering that at every point, there can be user) is equal to the considered area Γ.

Hereinafter, the terms *solution* and *strong solution* are all corresponding to the continuous case. Although we assume the users are uniformly distributed, in the real discrete case, the distribution can never be as uniform as the continuous case. The effects of fluctuation in the real distribution of users' locations (where the number of users is always an integer) will be discussed in the Section 7.

As mentioned in **Proposition 1** and **2**, if we only consider the solution or in the case with $R \geq 2\Phi$, under continuous approximation, our problem is equivalent to the "circles covering circle" problem if we can easily make sure any two repeaters will not interfere with each other. In a word, we should determine the least number of circles with radius *r'* to cover a circle with radius Φ. Notice that, *r'* may be smaller than *r* due to the capacity limitation.

In the frequency range [145MHz, 148MHz] with f_c=0.6MHz, in a PL tone, if $R \geq 2\Phi$ (when $R \geq \Phi$, any pair of repeaters in the considered area may interfere with each other), there are at most six different repeaters without interference, whose transmitter frequencies are 145.0MHz, 145.6MHz, 146.2MHz, 146.8MHz, 147.4MHz and 148.0MHz. However, according to the *pigeonhole principle*, there must be a pair of repeaters with inverse frequency pair as f_1-f_2 and f_2-f_1 (Here, a repeater represented by f_1-f_2 has a receiver frequency f_1 and a transmitter frequency f_2). It is very possible that the former repeater amplifies signals and sends to the latter repeater, and the latter repeater amplifies the received signals and sends back to the former repeater, again the former repeater will amplify the signals and send to the latter repeater, and so on. To avoid such a problem, we consider a set of non-interacting repeaters in a PL tone. Obviously, the maximum number of such a group of repeaters is five. An example set is 145.6-145.0, 146.3-145.7, 147.0-146.4, 147.7-147.1 and 147.4-148.0. Therefore, we can conclude that when $R \geq \Phi$, with the help of $N_{\text{PL}} = 54$ different PL tones, the maximum number of repeaters without any interference is $54 \times 6 = 324$, and the maximum number of repeaters without any interactions is $54 \times 5 = 270$. Therefore, if the required number of small circles is no more than 324, we do not need to consider the interference avoidance constrain but just set these repeaters to make sure they do not interfere with each other, while if the required number is no more than 270, we can make sure there will not be any needless interactions between repeaters. Of course, if we consider the strong solution with $R < 2\Phi$, the situation is much more complicated, and interactions between repeaters are generally necessary.

According to the connectivity constrain, in any solution, the number of repeaters *M* should satisfy the inequality

$$M \geq \frac{\pi\Phi^2}{\pi r^2} = \frac{\Phi^2}{r^2} = \frac{40^2}{15.28^2} = 6.85 \qquad (15)$$

Since *M* is an integer, we have $M \geq 7$. According to the capacity constrain, in any solution, the number of repeaters *M* should satisfy the inequality

$$M \geq \frac{N}{C} = \frac{N}{118} \tag{16}$$

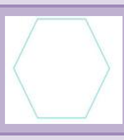
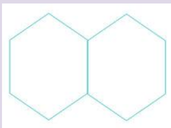
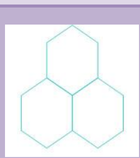
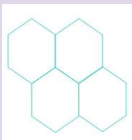
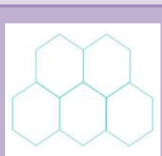
In this problem, for N=1000, the lower boundary of M is 9 (i.e., $M \geq 9$), while for N=10000, it is 85 (i.e., $M \geq 85$).

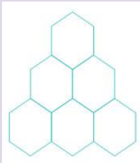
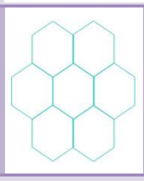
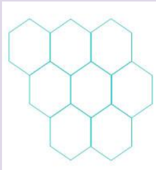
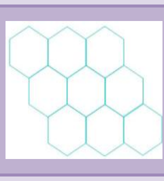
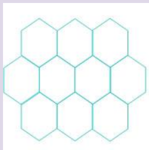
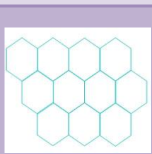
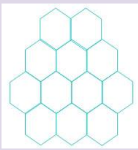
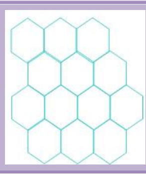
4.4 Naïve Solution

Cellular networks are widely used in placing the relay nodes, especially in the design of the locations of base stations in mobile wireless networks. In this paper, as a naïve solution, we directly apply the cellular network structure to cover the considered area. Table 2 illustrates some small-size cellular networks, each of them can be considered as the *Voronoi diagram* [13] where the center of each regular hexagon lays a repeater. Clearly, the number of regular hexagons is equal to the number of repeaters M.

We firstly consider an inverse problem, that is, given a number of equal regular hexagons (we assume the edge length is a unit), to determine the largest circle that can be fully covered by these regular hexagons (these hexagons must be arranged in a beehive-like pattern). Table 2 gives the results up to 13 regular hexagons, which are calculated by hand.

Table 2: The cellular networks that can cover the largest circle. The third column lists the radiuses of the corresponding circles. Given the number of cells M, sometimes, the cellular networks are not unique, and then we only draw one of them as a representative.

The number of cells, *M*	**The schematic diagram**	**The radius of the largest circle can be covered**
1		$\frac{\sqrt{3}}{2}$
2		$\frac{\sqrt{3}}{2}$
3		1
4		$\frac{\sqrt{7}}{2}$
5		$\frac{7}{5}$

6		$\sqrt{3}$
7		2
8		2
9		2
10		$\frac{\sqrt{19}}{2}$
11		$\frac{\sqrt{19}}{2}$
12		$\sqrt{7}$
13		$\sqrt{7}$

Here we show how to directly obtain the solution or the strong solution with $R \geq 2\Phi$ for the case N=1000 by using Table 2. In this case, the user density is $\rho = \frac{N}{\pi\Phi^2} \approx 0.1989$. The maximal area of the Voronoi area of any repeater in a cellular network is equal to the area of the regular hexagon (in the edge of the considered area, a repeater's Voronoi area may be smaller than the area of the regular hexagon). The edge length of the regular hexagon r_h should not exceed the communication range r to make sure each point is covered by at least one repeater. In addition, according to the capacity limitation, the edge length r_h should satisfy the inequality

$$\frac{3\sqrt{3}}{2} r_h^2 \rho = C \tag{17}$$

These two conditions determine the possibly longest edge length, and of course, to cover as large circle as possible, we always use the longest r_h. In this case, r=15.28 miles, and according to Eq. (17), r_h should be no longer than 15.18 miles. Therefore, the edge length of the regular hexagon is r_h=15.18miles. The circle to be covered is of radius $\Phi = 40$ miles. Since $\frac{\sqrt{19}}{2} < \frac{\Phi}{r_h} < \sqrt{7}$, according to Table 2, 12 repeaters are sufficient to cover the considered area under the constrains of capacity and connectivity, yet 11 repeaters are not possible using the cellular network structure. This solution is shown in Figure 2, and the coordinates of repeaters are presented in Table 3. Notice that, here we only consider the solution or the strong solution with $R \geq 2\Phi$, the frequencies and PL tones of repeaters are easily to be arranged. As an example, we use three PL tones to make sure there is no interaction or interference.

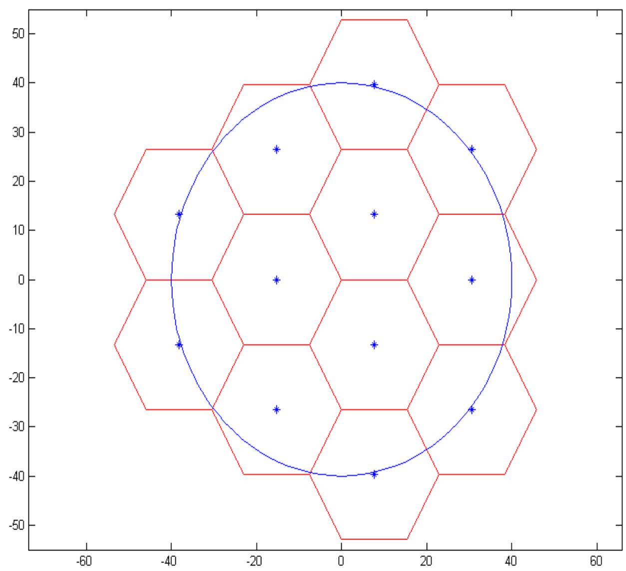


Figure 2: The solution consisted of 12 repeaters arranged in a cellular network. The blue circle denotes the considered area with radius 40miles, the read regular hexagons represent the cellular network, and the blue points in the centers of the regular hexagons denote the positions of repeaters.

As shown above, the basic procedure to get the na ïve solution contains two steps: (i) find the longest r_h, (ii) search the best solution according to Table 2. However, the calculation of the radius of the largest coverable circle is not an easy problem (at least not easy by hand) for many regular hexagons. We only get the analytical results for two particular cases up to 121 circles, where the covered circle's center is either at the center of the central hexagon or at the intersection of the three more central hexagons.

Table 3: The coordinates, frequencies and PL tones of the solution shown in Fig. 2.

Repeater No.	*x*-Coordinate	*y*-Coordinate	Receiver Frequency	Transmitter Frequency	PL Tone
1	-15.1800	0.0000	145.6	145.0	1
2	7.5900	13.1463	146.3	145.7	1
3	7.5900	-13.1463	147.0	146.4	1
4	-15.1800	26.2925	147.7	147.1	1
5	-37.9500	-12.6411	147.4	148.0	1
6	-37.9500	12.6411	145.6	145.0	2
7	-15.1800	-26.2925	146.3	145.7	2
8	7.5900	39.2733	147.0	146.4	2
9	7.5900	-39.2733	147.7	147.1	2
10	30.3600	0.0000	147.4	148.0	2
11	30.3600	26.0436	145.0	145.6	3
12	30.3600	-26.0436	147.4	148.0	3

Table 4: Analytical results for two particular cases. In the left side the largest covered circle is centered at the center of the central hexagon, while in the right side the largest covered circle is centered at the intersection of the three more central hexagons.

Center of a circle		*Intersection of three circle*	
The number of cells, *M*	**The radius of the largest coverable circle**	**The number of cells, *M***	**The radius of the largest coverable circle**
1	$\frac{\sqrt{3}}{2}$	3	1
7	2	6	$\sqrt{3}$
13	$\frac{3\sqrt{3}}{2}$	12	$\sqrt{7}$
19	$\sqrt{13}$	18	3
31	$\frac{5\sqrt{3}}{2}$	27	4
37	5	36	$\sqrt{21}$
55	$\frac{7\sqrt{3}}{2}$	48	$\sqrt{31}$
61	$\sqrt{43}$	60	6

85	$\frac{9\sqrt{3}}{2}$	75	7
91	8	90	$\sqrt{57}$
121	$\sqrt{91}$	108	9

In the case N=10000, $\rho = \frac{N}{\pi\Phi^2} \approx 1.989$, according to the capacity constrain Eq. (17), we have $r_h = 4.80$ miles. Since $8 < \frac{\Phi}{r_h} < 9$, according to Table 4, 108 repeaters are sufficient to cover the considered area under the constrains of capacity and connectivity. Since the number of repeaters is smaller than 324, it is easy to arrange them when considering only a solution or a strong solution with $R \geq 2\Phi$. Figure 3 shows this solution, and the table containing all the coordinates is ignored.

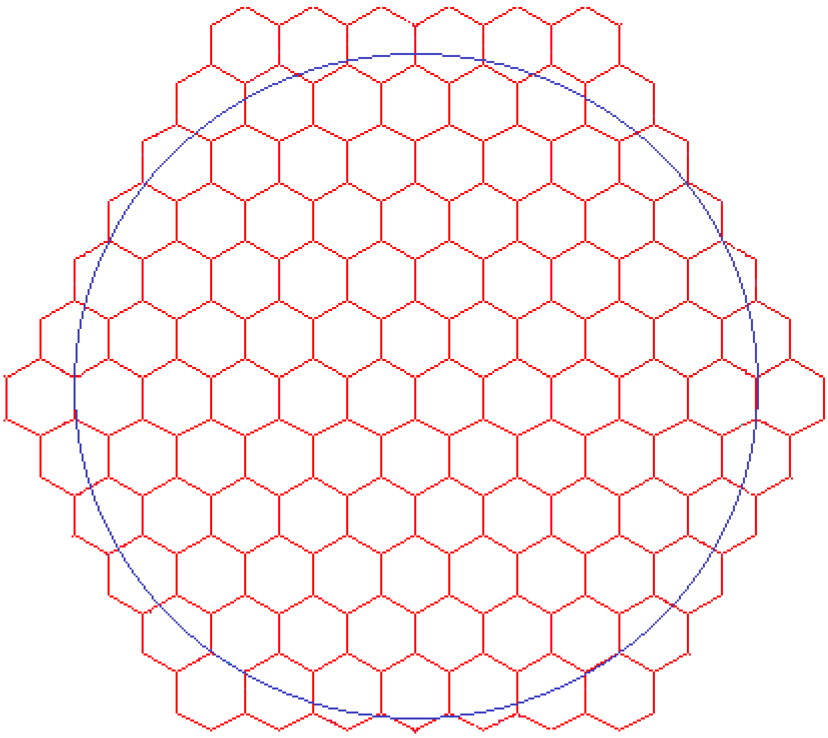


Figure 3: The solution consisted of 108 repeaters arranged in a cellular network. The blue circle denotes the considered area with radius 40miles, the red regular hexagons represent the cellular network. The repeaters are also lay in the centers of the regular hexagons (not shown).

5 Algorithm

This section will present two algorithms. The main algorithm tries to solve the circles covering circle problem with the least number of circles, and the secondary algorithm assigns each repeater the receiver frequency, the transmitter frequency and the PL tone.

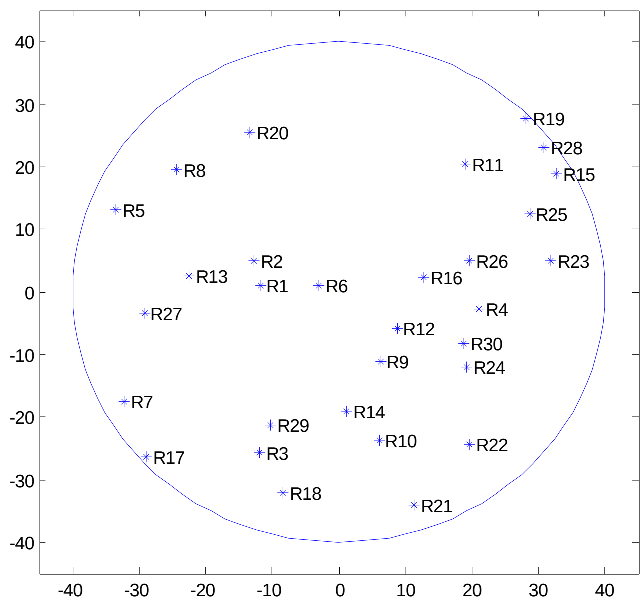


Figure 4: Illustration of an initial configuration with 30 repeaters. The coordinates of each repeater are generated randomly inside the considered area.

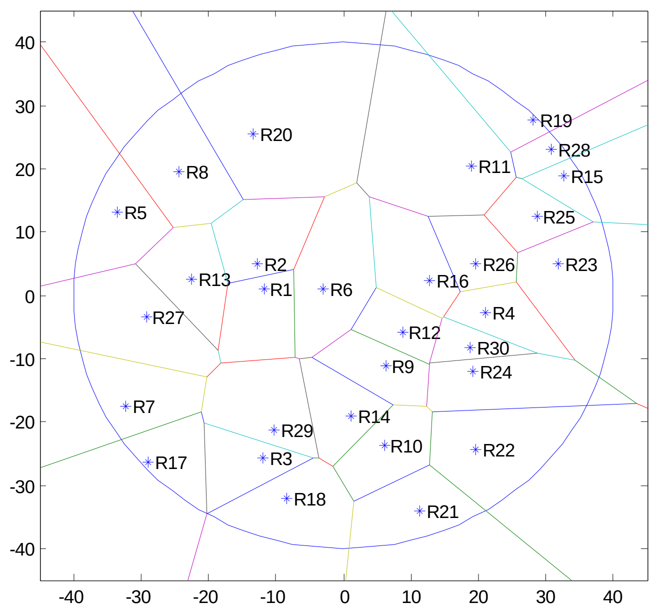


Figure 5: The resulted Voronoi diagram corresponding to the configuration shown in Fig. 4.

Before running the algorithm, we first determine the lower boundary of M according to Eqs. (15) and (16). The boundary is denoted by M_0. The procedure of the algorithm is as follows.

(1) Randomly place M_0 repeaters in the considered area (See Fig. 4).

(2) Divide the area into different parts in terms of Voronoi diagram, that is to say, determine the Voronoi area of each repeater. The algorithm can be found in Ref. [13], and one can see Fig. 5 for the resulted Voronoi diagram of the above configuration. Notice that, for some repeaters close to the edge of the considered area, their Voronoi areas may contain a part of the edge of the considered area.

(3) Determine the circumscribed circle of each Voronoi area (i.e., a circle which can cover the area and meanwhile has the minimal radius). A standard algorithm can be found in Ref. [14], an advanced method can be found in Ref. [19]. We apply the standard method. The corresponding result after Fig. 5 is shown in Fig. 6.

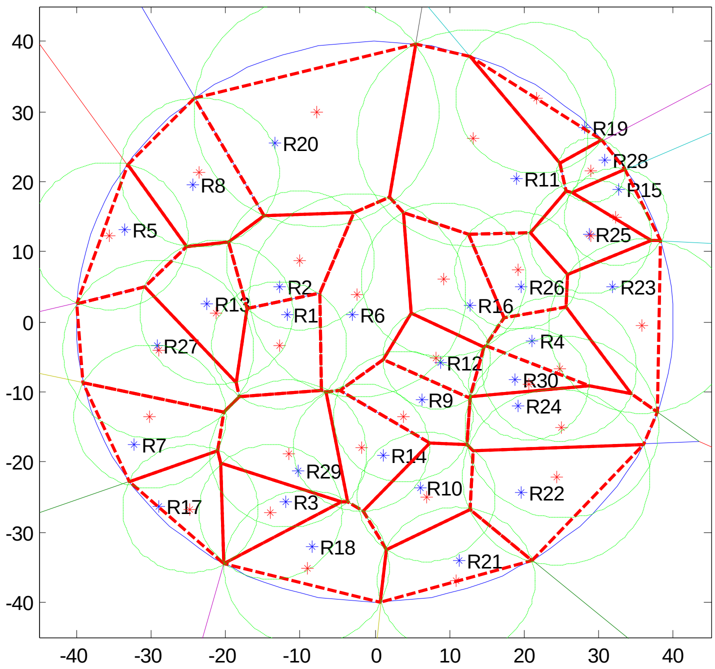


Figure 6: The circumscribed circles of the Voronoi areas, corresponding to the ones shown in Fig. 5. The circumscribed circles are colored in green while the Voronoi areas are colored in red. Notice that, for the some repeaters close to the edge of the considered area, their Voronoi areas may contain a part of the edge of the considered area.

(4) Calculate the coordinates of the center of each circumscribed circle.

(5) Calculate the distance from each repeater's current location to the centers of the corresponding circumscribed circle. Sum up all the distances and compare the sum to a threshold ξ. If the sum is less than ξ, the current set of repeaters' locations is considered to be converged. If it is converged, go to step (6), otherwise, move each repeater to the center of the corresponding circumscribed circle (see figure 7) and go

back to step (2).

(6) Calculate the number of users in each Voronoi region and see if the number is less than the repeater's capacity, and the radius of the circumscribed circle should be less than the communication radius of users. If so, stop the algorithm and output the current solution. Otherwise, go to step (7)

(7) With the current number of repeaters, if the number of extremal optimization operations is less than a threshold T_c, pick up the repeater with the smallest Voronoi area, move it to a random position in the considered area and go back to step (2) and count an extremal optimization operation. Otherwise, add one more repeater, randomized the positions of all repeaters, and go back to step (2).

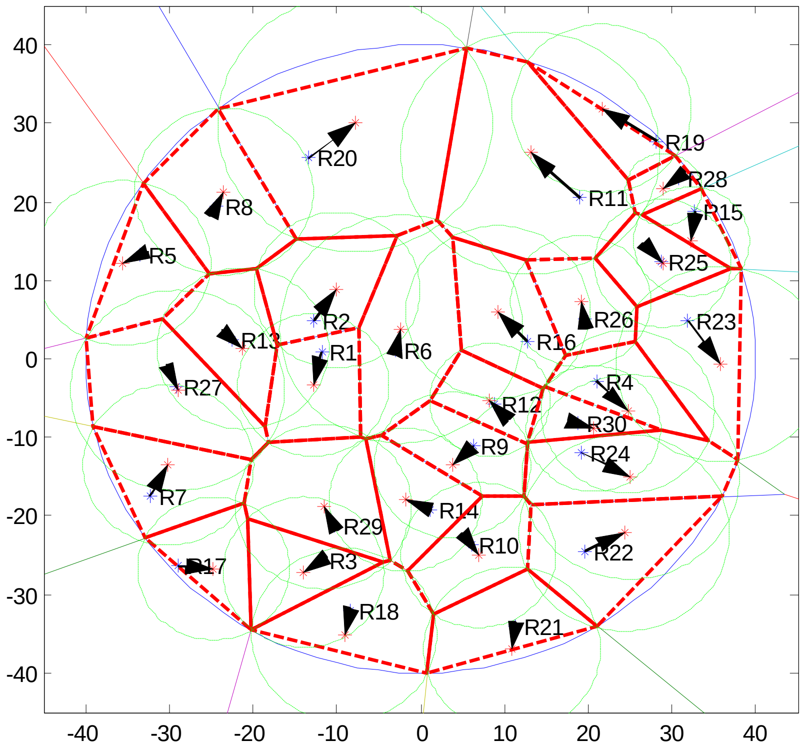


Figure 7: The black asterisks stand for the locations of repeaters, the red ones represent the centers of the circumscribed circles, and the black arrows show the directions towards which the repeaters will move. This illustration corresponds to the result shown in Fig. 6.

Here, the Voronoi area can be considered as the contribution of the corresponding repeater, and thus a repeater with smaller Voronoi area is less effective. The so-called extremal optimization is a method to escape the local optimum by every time change the individual with the least fitness. This idea originally comes from the Bak-Sneppen model [20] that describes the punctuated equilibrium in evolution caused by the annihilation of the least fit species. In the simulation, we set $\xi = 0.01$ and $T_c = 100$.

Next we present the secondary algorithm, whose task is to assign each repeater a receiving frequency, a transmitting frequency and a PL tone. This algorithm will not change the locations of the repeaters obtained from the first algorithm, and will not add new repeaters. That is to say, this algorithm tries to maximize the reachable area of users just by rearranging the frequencies and PL tones of repeaters. The procedure of the algorithm is presented as follows.

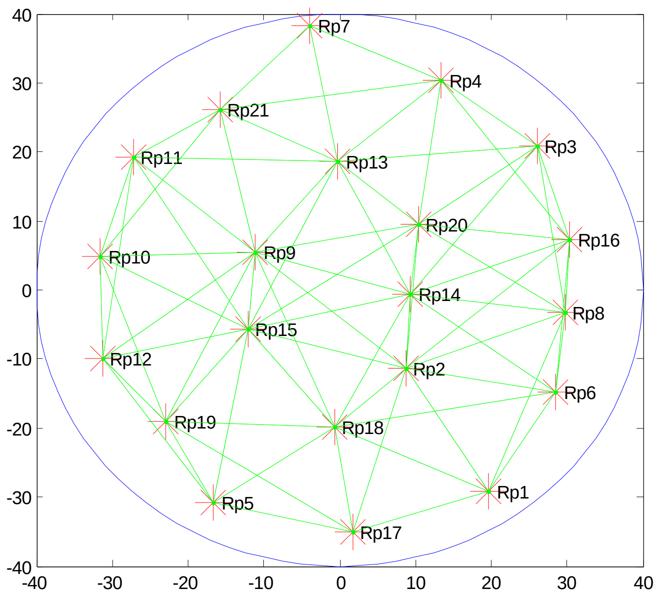
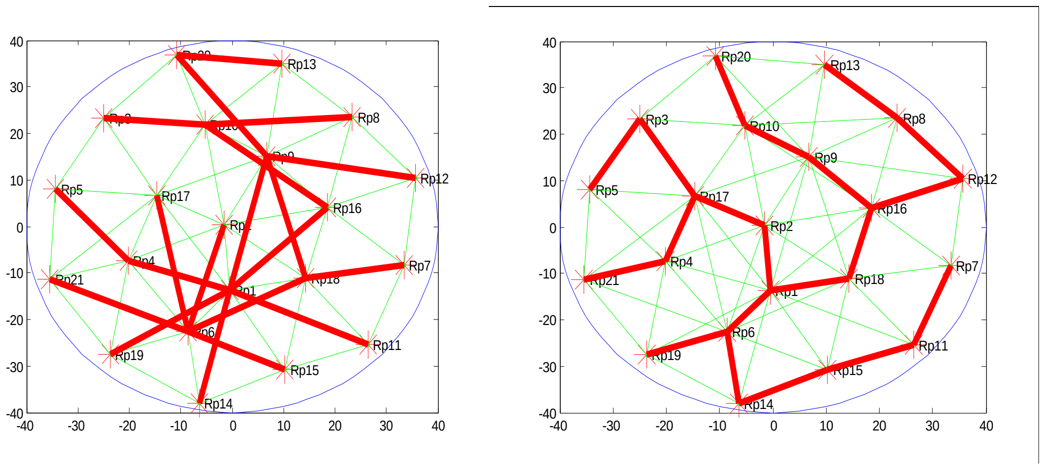


Figure 8: The red asterisks represent the repeaters while a green line connecting two repeaters if they have the chance to transmit signals (i.e., within a distance *R*).



(a)A maximum spanning tree (b) A minimum spanning tree

Figure 9: Illustration of the maximum spanning tree (a) and the minimum spanning tree (b) of G The red lines are the edges in the spanning trees and green lines are the other edges in the graph G.

(1) Construct a graph G to represent the relation that two repeaters have the ability to transmit signal to each other. That is to say, if the distance between two repeaters is less than *R*, we add one edge between them (see Fig. 8)

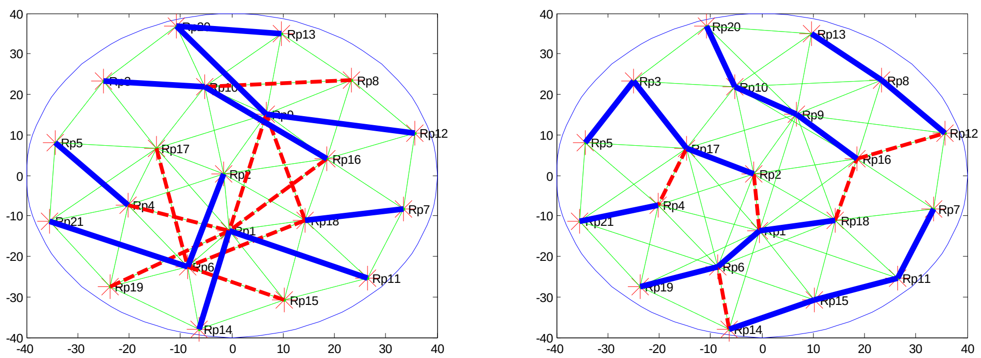
(2) Find the minimum spanning tree T or the maximum spanning tree T' of G. From

the illustration shown in Fig. 9, one can see that edges in the minimum spanning tree will not intersect. If we build the signal transmission paths along the minimum spanning tree, the signals received by repeaters are fewer than the case of maximum spanning tree. However, since the distance between two adjacent repeaters along the tree is the shortest, the received signals will be much stronger than other type of assignment. So the assignment based on the minimum spanning tree is suitable for communication in local area. In contrast, most edges in maximum spanning tree intersect with each other, the number of signals in many areas is very large and signals can cover larger areas. However, this increases the possibility of frequency interference. For different purposes, one can choose different spanning tree to continue our algorithm.

(3) Remove edges from the spanning tree. For any node *i* whose degree *k* is larger than 3, we delete *k*-2 edges. Then the node *i* will be divided apart from *k*-2 connected components. Let the size of component *j* be SC_j, then the method of removing edges can be presented as follows: find *k*-2 edges to remove in order to minimize $\sum_{a,b} | SC_a - SC_b |$.

The results can be found in Fig. 10.

(4) After the step 3, we can obtain several distinct signal routes that do not connect. Assign a different PL tone to each distinct route. Then for the repeaters in each different route, assign its transmitting frequency and receiving frequency. Make sure the transmitting frequency of a repeater is the receiving frequency of the repeater's neighbors in the same signal route.



(a)Signal routes in maximum spanning tree (b) Signal routes in minimum spanning tree

Figure 10: The resulted routes after the removal of edges, corresponding to the spanning tree in Fig 9. The red dashed lines represent the removed edges, while the blue solid lines are the reserved ones. The repeaters on the same blue line share the same tone.

6 Simulation Results

In this section, we present the strong solutions obtained by the proposed algorithm. The analysis in subsection 4.1 indicates that *R* is larger than the diameter of the considered area, and thus a solution is also a strong solution. In subsection 6.1 and 6.2, we will show the algorithmic results for *N*=1000 and *N*=10000, with *R*=85.45miles. The number if required repeaters are 11 and 104 respectively, better than the ones obtained by the naïve solution, 12 and 108. In subsection 6.1, we will prove that 11 is the optimal solution. Notice that, the calculation made in subsection 4.1 is based on an ideal circumstance, while the fog or rain can largely affect the transmitting of signal and reduce the effective communication range of a repeater. Therefore, in subsection 6.3 and 6.4, we will discuss the cases with shorter *R*. Since the communication range *R* will be affected by the weather, when discussing the interference, we should consider the case with large *R* to avoid possible interference in good weather while when discussing the global reachability, we should consider short *R* to make sure any point-to-point communication is possible even under bad weather. Therefore, as in the ideal case the communication range is larger than the diameter (2Φ) of the considered area (when $R \geq \Phi$, any two repeaters may interfere with each other) we always guarantee that the transmitter frequency difference between any two repeaters sharing the same PL tone is no less than 0.6MHz. In subsections 6.3 and 6.4, we assume that *R*=40miles.

6.1 *N*=1000, *R*=85.45

Figure 11 reports a solution with 11 repeaters obtained by our algorithm. The maximal Voronoi area is 560.56, the user density is 0.1989, and thus the largest capacity demand is 112, smaller than the repeater's capacity *C*=119. The advantage of this result compared with the naïve solution is twofold: (i) the number of required repeaters is smaller; (ii) the sizes of Voronoi areas of repeaters are more homogeneous. Detailed information of the repeaters is presented in Table 5.

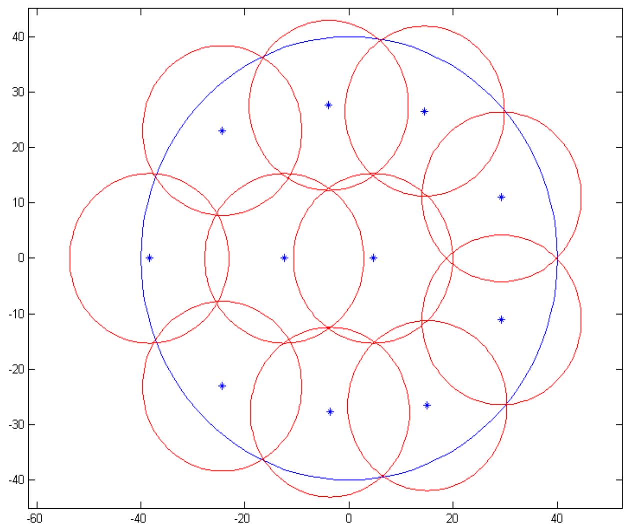


Figure 11: The solution consisted of 11 repeaters obtained by our algorithm. The blue circle denotes the considered area with radius 40miles, the red small circles represent the ability of receiving signals from users, and the blue points in the centers of the small red circles denote the positions of repeaters.

Table 5: The coordinates, frequencies and PL tones of the solution shown in Fig. 11.

Repeater No.	*x*-Coordinate	*y*-Coordinate	Receiver Frequency	Transmitter Frequency	PL Tone
1	-38.3765	0.0000	145.6	145.0	1
2	-24.3880	-23.0549	146.3	145.7	1
3	-24.3880	23.0549	147.0	146.4	1
4	-12.4503	0.0000	147.7	147.1	1
5	-3.6744	-27.7292	147.4	148.0	1
6	-3.9222	27.5753	145.6	145.0	2
7	4.6059	0.0000	146.3	145.7	2
8	14.5012	26.5499	147.0	146.4	2
9	15.0139	-26.5499	147.7	147.1	2
10	29.2673	-11.0660	147.4	148.0	2
11	29.2673	11.0660	145.0	145.6	3

We next prove that the solution shown in Table 5 with 11 repeaters is the optimal solution, namely 10 repeaters with radius no more than 15.28miles cannot cover a circle with radius 40miles.

Lemma 1 [21]: Let $r(n)$ be the maximum radius of a circular disc that can be covered by n closed unit circles, then $r(n)=1+2\cos(2\pi/(n-1))$ for $n=8$, $n=9$, and $n=10$.

Proposition 3: To cover a circular area with radius 40, the least number of circles with radius 15.28 is 11.

Proof. Table 5 has already shown a possible solution, and thus we only need to prove that to cover with 10 circles is not possible. According to **Lemma 1**, $r(10)=1+2\cos(2/9\pi)=2.532$, which is smaller than the ratio 40/15.28=2.618, and thus the coverage with 10 circles is not possible.

■

6.2 *N*=10000, *R*=85.45

Figure 12 reports a solution with 104 repeaters obtained by our algorithm, with the detailed information of repeaters shown in Table 6. 21 PL tones are used to guarantee that every pair of repeaters will not interact each other.

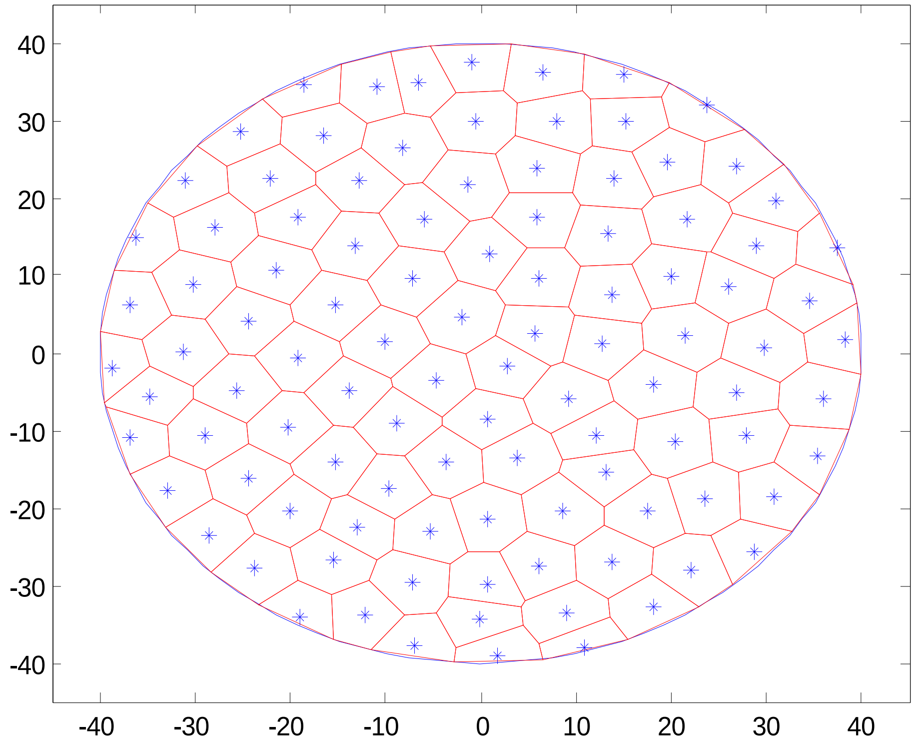


Figure 12: The solution consisted of 104 repeaters obtained by our algorithm. The blue circle denotes the considered area with radius 40miles, the red areas are the corresponding Voronoi areas and the blue points denote the positions of repeaters.

Table 6: The coordinates, frequencies and PL tones of the solution shown in Fig. 12.

Repeater No.	*x*-Coordinate	*y*-Coordinate	Receiver Frequency	Transmitter Frequency	PL Tone
1	−21. 4846	10. 7841	145.6	145.0	1
2	20. 0881	9. 8614	146.3	145.7	1
3	−24. 3006	−16. 2601	147.0	146.4	1
4	13. 8192	−27. 0256	147.7	147.1	1
5	−38. 7037	−1. 9022	147.4	148.0	1
6	1. 0018	12. 7802	145.6	145.0	2
7	−13. 2011	13. 9291	146.3	145.7	2
8	26. 0811	8. 6698	147.0	146.4	2
9	26. 8280	−5. 1802	147.7	147.1	2
10	−18. 9492	−33. 9863	147.4	148.0	2
11	−19. 2527	17. 5280	145.6	145.0	3
12	−36. 7795	−10. 8329	146.3	145.7	3
13	27. 8500	−10. 6307	147.0	146.4	3
14	19. 6493	24. 5973	147.7	147.1	3
15	−6. 8453	−37. 6773	147.4	148.0	3

16	−12.0455	−33.8082	145.6	145.0	4
17	18.2067	−4.1299	146.3	145.7	4
18	−31.0283	22.1830	147.0	146.4	4
19	14.0491	22.4823	147.7	147.1	4
20	−25.2712	28.5517	147.4	148.0	4
21	6.0148	9.6461	145.6	145.0	5
22	−5.2833	−22.9543	146.3	145.7	5
23	−24.2711	4.0665	147.0	146.4	5
24	−7.2155	−29.6266	147.7	147.1	5
25	23.5724	−18.8907	147.4	148.0	5
26	0.7092	−8.6258	145.6	145.0	6
27	−22.0810	22.4277	146.3	145.7	6
28	−4.6952	−3.5107	147.0	146.4	6
29	−28.4376	−23.5828	147.7	147.1	6
30	15.0935	36.0433	147.4	148.0	6
31	−10.0930	1.3872	145.6	145.0	7
32	8.9930	−33.6010	146.3	145.7	7
33	−10.8101	34.2136	147.0	146.4	7
34	−16.5338	28.0647	147.7	147.1	7
35	2.7253	−1.7304	147.4	148.0	7
36	−28.9439	−10.5425	145.6	145.0	8
37	28.9259	13.9310	146.3	145.7	8
38	17.4628	−20.3533	147.0	146.4	8
39	−18.4778	34.4848	147.7	147.1	8
40	−15.3172	−13.9991	147.4	148.0	8
41	−13.8680	−4.9981	145.6	145.0	9
42	−7.1204	9.4823	146.3	145.7	9
43	35.9930	−5.8130	147.0	146.4	9
44	−12.9073	−22.4915	147.7	147.1	9
45	6.5647	36.2554	147.4	148.0	9
46	23.7103	31.8433	145.6	145.0	10
47	−12.7061	22.2901	146.3	145.7	10
48	3.9099	−13.5329	147.0	146.4	10
49	−20.2166	−9.6509	147.7	147.1	10
50	−19.1212	−0.5781	147.4	148.0	10
51	1.7840	−38.9554	145.6	145.0	11
52	6.1130	−27.4748	146.3	145.7	11
53	13.3601	15.2740	147.0	146.4	11
54	−15.5091	−26.6073	147.7	147.1	11
55	35.3109	−13.2947	147.4	148.0	11
56	−25.5448	−4.7593	145.6	145.0	12

57	30.9858	19.5100	146.3	145.7	12
58	8.6603	−20.2950	147.0	146.4	12
59	−15.3072	6.1877	147.7	147.1	12
60	−8.2420	26.4597	147.4	148.0	12
61	−1.3574	21.7607	145.6	145.0	13
62	−19.9827	−20.4432	146.3	145.7	13
63	−30.1025	8.9472	147.0	146.4	13
64	5.9444	17.4680	147.7	147.1	13
65	−0.5221	29.9967	147.4	148.0	13
66	0.6271	−29.8848	145.6	145.0	14
67	−3.6197	−13.9964	146.3	145.7	14
68	5.9811	23.8501	147.0	146.4	14
69	−32.7771	−17.8837	147.7	147.1	14
70	9.2566	−6.0159	147.4	148.0	14
71	20.3993	−11.3469	145.6	145.0	15
72	28.6670	−25.6027	146.3	145.7	15
73	38.1683	1.7115	147.0	146.4	15
74	−36.8485	6.2865	147.7	147.1	15
75	12.1489	−10.6071	147.4	148.0	15
76	−36.2190	14.8745	145.6	145.0	16
77	15.3432	29.9396	146.3	145.7	16
78	37.3272	13.5208	147.0	146.4	16
79	12.7538	1.2800	147.7	147.1	16
80	−8.7399	−8.9681	147.4	148.0	16
81	−23.6555	−27.8664	145.6	145.0	17
82	22.1340	−28.0080	146.3	145.7	17
83	30.7725	−18.5245	147.0	146.4	17
84	−9.5450	−17.3707	147.7	147.1	17
85	13.0948	−15.5244	147.4	148.0	17
86	34.6099	6.7702	145.6	145.0	18
87	18.0988	−32.7824	146.3	145.7	18
88	13.6878	7.6081	147.0	146.4	18
89	5.6810	2.5887	147.7	147.1	18
90	−31.1704	0.1887	147.4	148.0	18
91	10.9276	−37.9560	145.6	145.0	19
92	21.3901	2.2075	146.3	145.7	19
93	−0.1086	−34.3627	147.0	146.4	19
94	7.9409	29.8451	147.7	147.1	19
95	0.6967	−21.3952	147.4	148.0	19
96	−27.7998	16.0646	145.6	145.0	20
97	−1.9265	4.5995	146.3	145.7	20

98	−5.8836	17.3017	147.0	146.4	20
99	21.6476	17.3278	147.7	147.1	20
100	29.7908	0.7205	147.4	148.0	20
101	−6.6019	34.9874	145.6	145.0	21
102	26.8732	24.1059	146.3	145.7	21
103	−0.9480	37.5368	147.0	146.4	21
104	−34.8281	−5.6874	147.7	147.1	21

6.3 *N*=1000, *R*=40

As mentioned in Section 5, we will not add more repeaters to make sure every user can communicate with other users at any positions in the considered area. Instead, our secondary algorithm tries to maximize the reachable areas of users. Therefore, the repeaters' locations of the solutions in subsections 6.3 and 6.4 are the same to those in subsections 6.1 and 6.2. However, the frequencies and PL tones are different.

Table 7: The coordinates, frequencies and PL tones of the solution shown in Fig. 11.

Repeater No.	*x*-Coordinate	*y*-Coordinate	Receiver Frequency	Transmitter Frequency	PL Tone
1	-38.3765	0.0000	145.0	145.6	1
2	-24.3880	-23.0549	145.0	145.6	2
3	-24.3880	23.0549	145.0	145.6	3
4	-12.4503	0.0000	145.6	146.2	1
5	-3.6744	-27.7292	145.6	146.2	2
6	-3.9222	27.5753	145.6	146.2	3
7	4.6059	0.0000	146.2	146.8	1
8	14.5012	26.5499	146.2	146.8	3
9	15.0139	-26.5499	146.2	146.8	2
10	29.2673	-11.0660	146.8	147.4	2
11	29.2673	11.0660	146.8	147.4	3

Table 7 presents the result for *N*=1000. To test the effectiveness of the secondary algorithm, we randomly pick up 100000 ordered pairs of points (*u*,*v*) inside the considered area to see in how many pairs, *u* can send to *v* signals. The answer is 90708, namely given a user, the probability that the system can satisfy this user's requirement to communicate with a random user is about 90.71%.

6.4 *N*=10000, *R*=40

Table 8 reports the result for N=10000. Similar to the solution in subsection 6.4, this solution could not guarantee that a user can communicate with other users in any positions in the considered area. However, it seems that when the required number of repeaters increases, the reachable area of a user increases. For example, in this case, for the solution shown in Table 8, the corresponding probability is 97.12%。

Table 8: The coordinates, frequencies and PL tones of the solution shown in Fig. 12.

Repeater No.	*x*-Coordinate	*y*-Coordinate	Receiver Frequency	Transmitter Frequency	PL Tone
1	−21.4846	10.7841	145.0	145.6	1
2	20.0881	9.8614	145.6	146.2	2
3	−24.3006	−16.2601	145.6	146.2	3
4	13.8192	−27.0256	145.0	145.6	4
5	−38.7037	−1.9022	145.0	145.6	5
6	1.0018	12.7802	145.6	146.2	6
7	−13.2011	13.9291	145.0	145.6	7
8	26.0811	8.6698	146.2	146.8	2
9	26.8280	−5.1802	145.6	146.2	8
10	−18.9492	−33.9863	145.0	145.6	9
11	−19.2527	17.5280	145.6	146.2	1
12	−36.7795	−10.8329	146.2	146.8	5
13	27.8500	−10.6307	145.0	145.6	8
14	19.6493	24.5973	146.2	146.8	10
15	−6.8453	−37.6773	146.2	146.8	9
16	−12.0455	−33.8082	145.6	146.2	9
17	18.2067	−4.1299	145.6	146.2	11
18	−31.0283	22.1830	145.0	145.6	12
19	14.0491	22.4823	145.0	145.6	13
20	−25.2712	28.5517	146.8	147.4	1
21	6.0148	9.6461	146.2	146.8	6
22	−5.2833	−22.9543	145.6	146.2	14
23	−24.2711	4.0665	146.2	146.8	15
24	−7.2155	−29.6266	146.2	146.8	14
25	23.5724	−18.8907	145.6	146.2	16
26	0.7092	−8.6258	145.6	146.2	17
27	−22.0810	22.4277	146.2	146.8	1
28	−4.6952	−3.5107	146.2	146.8	18
29	−28.4376	−23.5828	145.6	146.2	19
30	15.0935	36.0433	145.0	145.6	10
31	−10.0930	1.3872	145.6	146.2	18
32	8.9930	−33.6010	145.6	146.2	20
33	−10.8101	34.2136	145.6	146.2	21
34	−16.5338	28.0647	146.2	146.8	22
35	2.7253	−1.7304	146.2	146.8	17
36	−28.9439	−10.5425	146.2	146.8	3
37	28.9259	13.9310	146.8	147.4	2
38	17.4628	−20.3533	145.0	145.6	16
39	−18.4778	34.4848	145.0	145.6	21

40	−15.3172	−13.9991	146.2	146.8	24
41	−13.8680	−4.9981	145.6	146.2	22
42	−7.1204	9.4823	145.6	146.2	7
43	35.9930	−5.8130	145.0	145.6	1
44	−12.9073	−22.4915	145.6	146.2	25
45	6.5647	36.2554	145.0	145.6	26
46	23.7103	31.8433	146.8	147.4	10
47	−12.7061	22.2901	145.6	146.2	22
48	3.9099	−13.5329	145.0	145.6	17
49	−20.2166	−9.6509	145.6	146.2	24
50	−19.1212	−0.5781	145.0	145.6	22
51	1.7840	−38.9554	145.0	145.6	27
52	6.1130	−27.4748	145.0	145.6	20
53	13.3601	15.2740	145.6	146.2	13
54	−15.5091	−26.6073	145.0	145.6	25
55	35.3109	−13.2947	145.6	146.2	1
56	−25.5448	−4.7593	145.0	145.6	24
57	30.9858	19.5100	145.0	145.6	28
58	8.6603	−20.2950	145.0	145.6	29
59	−15.3072	6.1877	145.0	145.6	18
60	−8.2420	26.4597	145.0	145.6	22
61	−1.3574	21.7607	145.6	146.2	30
62	−19.9827	−20.4432	145.0	145.6	3
63	−30.1025	8.9472	145.6	146.2	15
64	5.9444	17.4680	145.0	145.6	6
65	−0.5221	29.9967	145.0	145.6	31
66	0.6271	−29.8848	146.2	146.8	27
67	−3.6197	−13.9964	145.0	145.6	5
68	5.9811	23.8501	146.2	146.8	26
69	−32.7771	−17.8837	145.0	145.6	19
70	9.2566	−6.0159	146.8	147.4	29
71	20.3993	−11.3469	146.2	146.8	11
72	28.6670	−25.6027	146.8	147.4	4
73	38.1683	1.7115	145.0	145.6	32
74	−36.8485	6.2865	145.0	145.6	15
75	12.1489	−10.6071	146.2	146.8	29
76	−36.2190	14.8745	146.2	146.8	12
77	15.3432	29.9396	145.6	146.2	10
78	37.3272	13.5208	145.6	146.2	32
79	12.7538	1.2800	145.6	146.2	10
80	−8.7399	−8.9681	146.2	146.8	23

81	−23. 6555	−27. 8664	146.2	146.8	19
82	22. 1340	−28. 0080	146.2	146.8	4
83	30. 7725	−18. 5245	146.2	146.8	16
84	−9. 5450	−17. 3707	146.2	146.8	25
85	13. 0948	−15. 5244	145.6	146.2	29
86	34. 6099	6. 7702	145.6	146.2	32
87	18. 0988	−32. 7824	145.6	146.2	4
88	13. 6878	7. 6081	145.0	145.6	2
89	5. 6810	2. 5887	145.0	145.6	14
90	−31. 1704	0. 1887	145.6	146.2	10
91	10. 9276	−37. 9560	146.2	146.8	20
92	21. 3901	2. 2075	145.0	145.6	11
93	−0. 1086	−34. 3627	145.6	146.2	27
94	7. 9409	29. 8451	145.6	146.2	26
95	0. 6967	−21. 3952	145.0	145.6	14
96	−27. 7998	16. 0646	145.6	146.2	12
97	−1. 9265	4. 5995	146.2	146.8	7
98	−5. 8836	17. 3017	145.0	145.6	30
99	21. 6476	17. 3278	145.0	145.6	14
100	29. 7908	0. 7205	146.2	146.8	8
101	−6. 6019	34. 9874	146.2	146.8	21
102	26. 8732	24. 1059	145.6	146.2	28
103	−0. 9480	37. 5368	145.6	146.2	31
104	−34. 8281	−5. 6874	145.6	146.2	5

7 Sensitivity Analysis

In subsection 7.1, we will investigate the sensitive of the algorithmic output versus the change of model parameters. In subsection 7.2, we will analysis the error caused by the continuous approximation of user density. In subsection 7.3, we will discuss the possible defects in line-of-sight propagation caused by mountainous areas, as well as some promising ways in solving the repeater placement problem in such circumstance.

7.1 Sensitivity of Parameters

Most parameters hereinbefore are regarded as constants. However, parameters can be regulated in a certain range, which may have a huge impact on the algorithmic results. Here we discuss to what extent the results depend on the parameters.

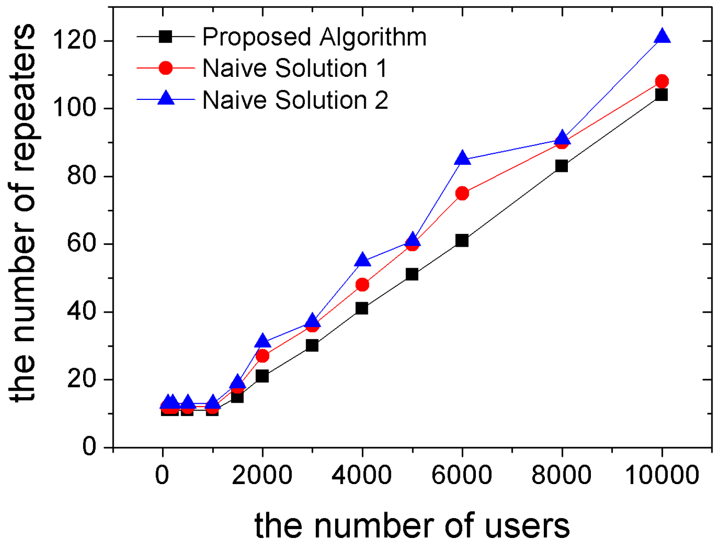


Figure 13: The Comparison between the least numbers of required repeaters obtained by the proposed algorithm and the Naïve solution. The largest coverable circle in naïve solution 1 is centered at the center of the central hexagon, while the largest coverable circle in naïve solution 2 is centered at the intersection of the three more central hexagons.

Figure 13 shows the least number of repeaters versus the number of users. The number of users varies from 100 to 10000. It is clear that the proposed algorithm is better than the Naïve solution. Notice that there is a transition point at about N=1000, which approximately satisfies the equation

$$\frac{N}{\pi\Phi^2} = \frac{C}{\pi r^2} \tag{18}$$

Before this transition point, the number of repeaters mainly depends on the communication range r, while after this transition point, capacity becomes the bottleneck determining the least number of require repeaters. Actually, given the considered area, the least number of repeaters keeps as a constant before the transition point. Since the capacity limitation will play the major role when many users are presented, it is not a surprise that the growing tendency of the number of repeaters is linear versus the number of users.

Figure 14 reports the relation between the least number of required repeaters and the user's communication range r. The curves have been normalized through dividing by their respective largest value. In the case N=10000, the number of repeaters never changes with r, again indicating that the capacity limitation determines the result, while when N=1000, the number of repeaters decreases with the increase of user's communication range.

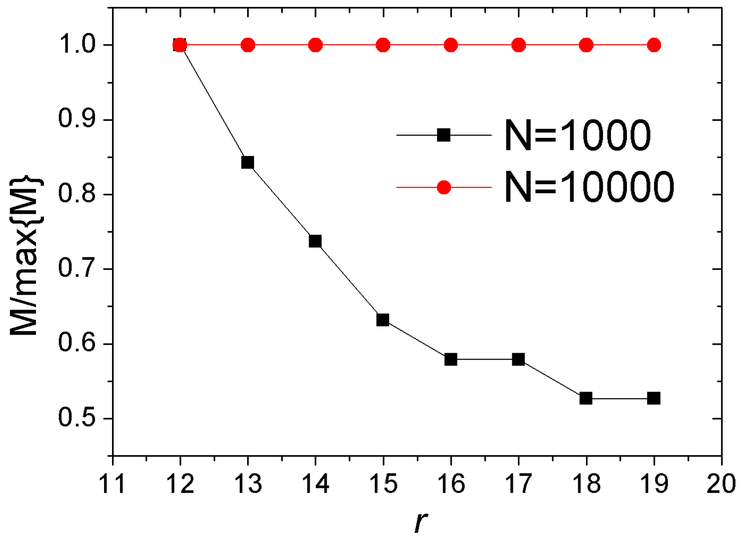


Figure 14: The Comparison between the least numbers of required repeaters obtained by the proposed algorithm and the Naïve solution. The red and black points correspond to the case of N=10000 and N=1000, respectively. Two curves have been normalized.

7.2 User Density Fluctuation

In the analysis of our model, the user density is a constant in the considered area, behaving like a real variable. However, in reality, the number of users can only be an integer. If we consider the discrete case where users are distributed uniformly, then in a Voronoi area S_v centered by a repeater. Each user belongs to this area with the probability $S_v / \pi\Phi^2$, and thus the number of users in this area, X, obeys a Bernoulli distribution

$$P(X) = \binom{N}{X} \left(\frac{S_v}{\pi\Phi^2} \right)^X \left(1 - \frac{S_v}{\pi\Phi^2} \right)^{N-X} \tag{19}$$

whose mathematical expectation and standard deviation are

$$E(X) = NS_v / \pi\Phi^2 = \rho S_v \tag{20}$$

and

$$\sigma(X) = \sqrt{\frac{NS_v}{\pi\Phi^2}\left(1 - \frac{S_v}{\pi\Phi^2}\right)} = \sqrt{\rho S_v}\sqrt{1 - \frac{S_v}{\pi\Phi^2}} \tag{21}$$

When the total number of users N is small, the capacity is not a big problem, and when N is big (corresponding to high user density), the area S_v should be small due to the capacity limitation.

Therefore, the standard deviation is about the square root of the expected number of users in the Voronoi area. Of course, we are interested in the case when the number of users in S_v is approaching to the capacity limitation. For example, in this problem, *C*=119, and thus the standard deviation is about 11. If we would like to set up a tolerance for two times of the standard deviation, the tolerant capacity *C'* should satisfy the approximate equation

$$C' + 2\sqrt{C'} = C \tag{22}$$

leading to an answer *C'*=99. The qualitative relation is trivial, namely higher tolerance leads to lower capacity and generally corresponds to more repeaters.

7.3 Effects of Landscape: Discussing the Mountainous Areas

Completely flat area is just a kind of ideal assumption, but the complex landscape is common which should be taken into consideration. Actually, there might be defects in line-of-sight propagation caused by mountainous areas. Here we analyze the impact of landscape on signal transmission, and based on the analysis, we put forward a proposal on how to arrange the position and tone of repeaters.

Because the VHF radio spectrum involves line-of-sight transmission and reception, the mountain will obstruct the signal's diffusion along the straight line. On the other hand, it makes a contribution to avoiding the interference between the two repeaters, even if they are geographically nearby and use close transmitter frequencies. The available spectrum is in the range from 145 to 148MHz and the wavelength is about 2 meters, so the signal wave have a strong ability to bypass the a dwarf, small or single mountain according to diffraction properties of wave. In a word, the situation that the area is full of rolling mountains is the point we should focus on.

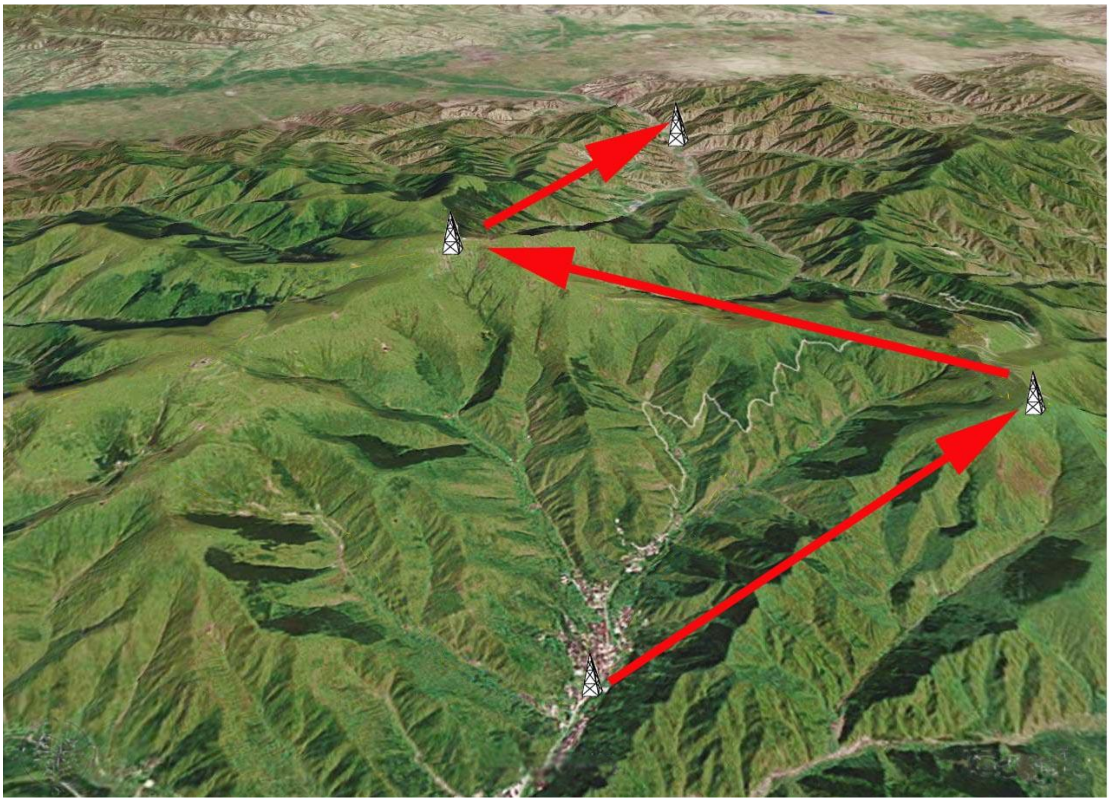


Figure 15: An illustration about how a signal can be transmitted from a hollow area to a distant place with the help of a hierarchically organized relay network.

The proposal put forward here is based on hierarchical organization. We divide all the repeaters into two levels. The repeaters of the first (lower) level, are always located at hollows to make sure all the users can be covered by at least one repeater. The repeaters of the second (higher) level are always located at the top of maintain, who play a central role in the information transmission. The higher altitude the repeater locates at, the wider its communication range is. Comparatively, the signal from hollow is difficult to be received by repeaters. According to the connectivity constrain Ω_3, we must design a suitable arrangement of repeaters covering all the users, so we first put some repeaters in the hollow, and erect them highly enough to make sure the signal they received can be transmitted to a second-lever repeater. The second-level repeaters are always put on the top of maintains to expand the range of radiation, who can communicate with other second-level repeaters. Through such a hierarchically organized relay network, a signal can be transmitted from a hollow area to a distant place, just like what is shown in Fig. 15.

The mountains narrow the range of radiation, so we must use more repeaters in mountainous area than flat area and this make arrangement of PL tones difficult. Considering the mountain between first-level repeaters can avoid the interference, they can share the same tone and the same transmitter frequency. If a first-level repeater can be covered by two second-level repeaters, it chooses the nearest repeater to send signals. So that a second-level repeater may receive signals from several first-level repeaters simultaneously, and the signals transmitted to several first-level repeaters can pass through a second-level repeater. We set all the first-level repeaters covered by one second-level repeater share the same transmitter frequency, and share the PL same tone with the second-level repeater. In this way, we reduce the number of PL tones. We only need to distribute PL tones to secondary repeaters using the same way in the flat area. Then we set the receiver frequency of a first-level repeater being equal to the transmitter frequency of the corresponding second-level repeater, and set the transmitter frequency of a first-level repeater being equal to the receiver frequency of the corresponding second-level repeater.

8 Conclusion and Discussion

In this paper, we propose a two-tiered network model, where lower-power users can communicate with one another through repeaters. The capacity limitation and interference have been taken into consideration in our model. Motivated by the structure of cellular networks, we give a naïve solution where the number of repeaters and their positions can be obtained analytically. We further propose an iterative extremal optimization algorithm based on the Voronoi diagram. The proposed algorithm outperforms the naïve solution. For the case of 1000 users, it obtains a solution with 11 repeaters, which has been proved to be the real optimum. For the case of 10000 users, it obtains a solution with 104 repeaters, better than the naïve solution with 108 repeaters. Moreover, we proposed an algorithm based on maximum and minimum spanning tree to assign frequencies and private line tones. This algorithm does not introduce any new repeaters yet can largely broaden the reachable areas of users. For example, in the case of 10000 users, given a user, the probability this user can successfully send signal to a random position in the considered area is about 97.12%.

Compared with the related model for sensor wireless networks and mobile communication networks, our model is more general. The proposed algorithm is very effective and efficient: it runs much faster than the simulated annealing approach [12] and is of higher ability to escape the local optimum than another iterative refinement algorithm [22]. Concepts and methods from disparate scientific domains are well integrated in this algorithm, such as the extremal optimization from statistical physics community, the Voronoi diagram from geographic information science, and iterative refinement framework from computer science. Another advantage of the algorithm is that it can be widely applied since it does not require any specific geographical features of the considered area, while many efficient circle-covering algorithms only work well for squares.

There are also two considerable weaknesses in this work. Firstly, we have not developed an effective algorithm to satisfy the constrain of global reachability without adding too many repeaters. Secondly, our model is largely simplified, which have not taken into account the heterogeneity of users and repeaters, or the wave reflection and refraction by atmosphere.

Appendix A Table of Parameters

For convenience, here we list parameters used in this paper. The detailed definitions can be found in the main content. The values of the parameters, expect for those free parameters, are also presented in this Table. Some values are given in the contest problem itself or directly obtained from literatures, while others are calculated according to our model. Details can be found in the main content.

Table 9: Parameters appeared in this paper.

Parameter	**Brief Description**	**Value/Range**
Φ	Radius of the considered circular area	40 miles
N	Number of users need to be accommodated	Tunable parameter. Two cases, N=1000 and N=10000, are emphasized in this paper
f_r	Receiver frequency of a repeater	$\in[145\text{MHz},148\text{MHz}]$
f_t	Transmitter frequency of a repeater	$\in[145\text{MHz},148\text{MHz}]$
n_{PL}	Specific PL tone of a repeater	$\in\{1,2,...,54\}$
N_{PL}	Total number of available PL tones	54
R	Communication radius of a repeater	85.45 miles
f_c	Threshold of the frequency difference, below which interference may happen	0.6MHz
r	Maximal distance for a user to send signals to a repeater	15.28 miles
C	Capacity of a repeater	119 users
C'	Tolerant capacity of a repeater	99 users
$L_{f,out}$	Loss of feed system for transmitter	2dB
$L_{f,in}$	Loss of feed system for receiver	2dB
$L_{b,out}$	Other loss of the transmitter	1dB
$L_{b,in}$	Other loss of the receiver	1dB
G_{out}	Antenna gain of the transmitter	39dB
G_{in}	Antenna gain of the receiver	39dB
$P_{r,out}$	Effective radiated power of a repeater	100W
$P_{r,in}$	Actual power of received signal for a repeater	$\geq 1\mu W$

$P_{u,out}$	Effective radiated power of a user	3.2W
$P_{u,in}$	Actual power of received signal for a user	$\geq 1\mu W$
E_b/N_0	Bit energy-to-noise density ratio	18dB for transceiver, and always in the range 5-30dB
V	Voice gain	0.4dB
φ	The information bit rate	bps
ξ	Threshold to judge whether the iterative refinement is converged	0.01miles
T_c	Threshold to judge whether we should stop the extremal optimization operation	100

Appendix B Table of Symbols

For convenience, here we list mathematical used in this paper. The detailed definitions can be found in the main content.

Table 10: Mathematical symbols appeared in this paper.

Symbol	**Description**
Γ	The circular area described in the problem
D	A two-tiered directed network
I_{other}	Interference from other repeaters
I_{self}	Interference from itself
V_u	The set of users
V_u'	The set of users
V_r	The set of repeaters
SNR	Signal-to-noise ratio
E_{ur}	The set of directed links from users to repeaters
E_{rr}	The set of directed links from repeaters to repeaters
u_i	A user
r_i	A repeater
$S_V(r_j)$	The Voronoi area of a repeater r_j

$S_r(u_i)$	The reachable area of a user u_i
$R_r(u_i)$	The reachable repeaters of a user u_i
ρ	Density of users
M	Number of repeaters
B	Bandwidth
P_{ur}	Power of signal from a user to a repeater
T	The minimal spanning tree
T'	The maximum spanning tree

Appendix C Summary of Assumptions

For convenience, here we summary all the important assumptions made for our model. Notice that, these assumptions also appeared in the model description of the main content.

[A1]. Users are uniformly distributed in the considered area, and in a more strong assumption, we consider the number of users as a real variable and the user density in the considered area is a constant.

[A2]. Users prefer to communicate with the nearest repeaters.

[A3]. Considering two repeaters sharing the same PL tone and the difference between their transmitter frequencies is less than a threshold f_c=0.6MHz. If the distance between them is less than 2R, they will interfere with each other.

[A4]. The considered circular area is a place where wireless signals can fade freely, which means there are not any other sorts of interference such as fogs, rivers, hills, buildings, activities of sun and so forth, so the fading of signals under such situation is only due to the distance across.

[A5]. There is no background noise in this system.

[A6]. Repeaters don't have noisy impact on others.

[A7]. Functionalities and specifications of users' radios are the same (i.e., homogeneous users' radios). Functionalities and specifications of radio repeaters are the same (i.e., homogeneous repeaters).

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