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2011 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include
the name of your school, advisor, or team members on this page.

In this paper, we study the shape of a snowboard course and its influential factors under energetic view. The main idea is to measure the "vertical air" by final energy. Before calculation, several assumptions are made for simplicity.

- The snowboarder is treated as a mass point;
- The halfpipe is regarded as a perfectly rigid body;
- Sliding friction, collision and air resistance are introduced step by step, while other sources of energy loss are ignored.

With these assumptions, we build our basic model, which is simple but sufficient to offer a fundamental idea and reveal essential interactions among different factors. We make force analysis, set up kinematical equations and obtain the general form of final mechanical energy analytically.

After that, two specific types of transition curve, quarter circle and quarter ellipse, are introduced to calculate the optimal design. Various factors are systematically analyzed in both models. In quarter-circle model, we obtain analytical solutions and find the best radius R^* of the circle. While in quarter-ellipse model, we use numerical simulation to get the optimal semi-major axis a^* and semi-minor axis b^*. Then we further develop the 2D quarter-ellipse model into a more actual and complex case, involving the third dimension of the halfpipe. Through numerical computation, we find the most suitable slope angle of the halfpipe.

Next, by introducing the sensitivity coefficient into these models, the sensitivities of different variables are analyzed both qualitatively and quantitatively. It helps us to compare the significance of these variables in building a best halfpipe.

At last, the tailors and tradeoffs to develop a "practical" course are discussed. To further optimize the shape of the halfpipe, we take various requirements into consideration, including construction difficulty, players' safety and their performance. After adjustment, our result becomes more practical.

In conclusion, through systematic and comprehensive analysis of various factors, we find a reasonable shape of the snowboard course, which fits the real data well.

Higher in the Air: Design of Snowboard Course

Team # 9159

February 15, 2011

Contents

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1 Introduction

Snowboarding is an adventurous and exciting sport that has been contested at the Winter Olympic Games since 1998. Till now, the events are held in three specialities: parallel giant slalom, snowboard cross and halfpipe.[2] What we are interested here is the halfpipe, in which competitors perform tricks while going from one side of a ditch to the other. For both ornamental and safety purpose, the shape of the halfpipe is of vital importance.

Fig 1: Snowboard competetion[1]

In its most basic form, a halfpipe consists of two concave ramps (or quarterpipes), topped by copings and decks, facing each other across a transition and an extended flat ground (the flat bottom) added between the quarterpipes. For snowboarding, the plane of the transition is oriented downhill at a slight grade to allow riders to use gravity to develop speed and facilitate drainage of melt.

There are three main tasks in this paper as follows:

- Determine the shape of a snowboard course which can generate the highest jump above the edge of the halfpipe.
- Optimize the shape to satisfy other requirements of the tricks.
- In practical, what properties of the halfpipe should be abandoned.

We analyze the problem from the viewpoint of energy translation. The competition between the influx from gravitational potential energy and the loss caused by friction and collision determines the process of sliding. To study the energy change in this process, we firstly consider about the 2D case. We build a simple quarter circle-type model to study what factors involve in and what role they play. Next the sliding process is further simulated by introducing an elliptic-type trail. The energy loss is calculated numerically to obtain an ideal shape for 2D trail.

Then we take the third dimension into consideration. We work out the slope angle by balancing the energy gaining and losing in new 3D model, which is developed from former 2D case.

At last, to optimize the shape of the halfpipe, we take various requirements into consideration, including construction difficulty, players' safety and their performance.

2 Symbols, Hypothesis and Explanations

2.1 Symbols Used in this Paper

We list the quantities we use as follows. Some are geometric parameters of the halfpipe, some are physical quantities in the sliding process. Symbols and corresponding parameters are shown in Tab. 1 and Fig. 2.

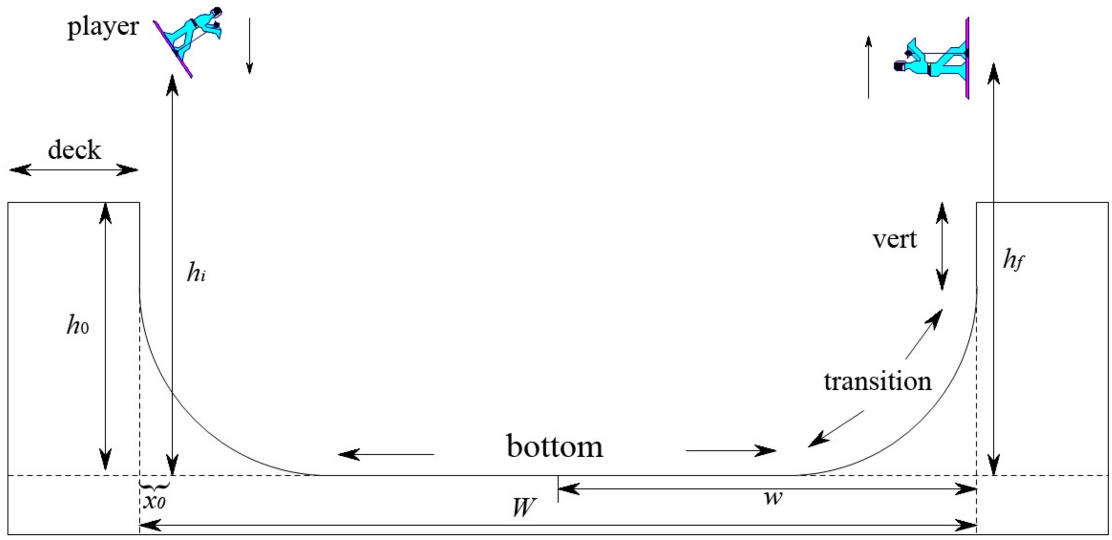


Fig 2: Cross section of the halfpipe, perpendicular to the ground

2.2 Hypothesis

- We treat the snowboarder as a mass point with mass m, ignoring body twists of snowboarder and other geometrical properties.
- The mass point moves right on the snowboard course rather than traveling along the trajectory of the original gravity center of a snowboarder above the snow surface.

Symbol	Quantity
m	Mass of snowboarder and the snowboard
μ	Friction coefficient between snow and the board
N	Pressure on the snow surface caused by snowboarder
E	Total mechanical energy of the snowboarder
h_0	The height of the halfpipe
x_0	Horizontal distance between the landing point and the coping
h_i & E_i	Initial height & energy of the snowboarder
W & w	Width and half width of the pipe
L	Length of the half pipe

Tab 1: Symbols used in this paper

- Only collision and sliding friction account for the mechanical energy loss, ignoring other possible causes such as air friction.
- The halfpipe is treated as perfectly rigid body, which can be modeled just by curve lines.
- The player does free falling from h_i with initial velocity 0.
- We ignore the collision time and treat the collision between snowboarder and halfpipe as perfect inelastic collision.

2.3 Some Explanations

All the Hypothesis above are designed for simplifying the calculation. Qualitative analysis can be made based on these assumptions. Because some of the Hypothesis may seem a little confusing at the first glance, we give some explanations here.

- The mass point is mainly used to simplify calculation. Since our aim is at first to determine the shape of a snowboard course to maximize the production of *vertical air*, all the geometrical properties such as body size, shape and body twists of snowboarders can be regarded as irrelevant, at least not decisive. In fact, since the body of a snowboarder is not rigid, deformation leads to an increase in computation complexity and the uncertainty in centre-of-gravity path, which covers up the essential aspects of this problem. Mass point assumption enables us to focus on the essential interaction between course shape and maximum of *vertical air*.

- In this problem, sliding friction, inelastic collision and air friction are three main factors accountable for the energy loss. As for the most basic model, we just take sliding friction and collision into consideration for simplicity. In further sections, we will introduce air friction to give some more practical results.
- According to the halfpipe standard released by International Ski Federation, in order to build a halfpipe, one should frequently drive on the decks to compact the snow before the pipe is stepped and wet new snow is preferable for construction since it is easy to bond together well. Therefore, the hard snow makes it feasible to assume that the halfpipe is a perfectly rigid body.

3 Basic Model

We use curve $\phi(x), 0 \leq x \leq W$ to denote the halfpipe. According to our assumption, the energy decay after landing is due to the friction between the snowboard and the snow surfaces. Therefore, if we assume μ is a constant, then

$$\mathrm{d}E = -\mu N \mathrm{d}s = -\mu N \sqrt{1 + (\phi')^2}\mathrm{d}x. \tag{1}$$

N has two parts in itself, one is the part against the vertical component of the gravity, the other is centripetal force. To make force analysis, we define θ the angle between the velocity of snowboarder and the ground, and ρ the radius of curvature. Then kinematical equation for the perpendicular direction is written as

$$N = mg\cos\theta + \frac{mv^2}{\rho}, \tag{2}$$

and ρ can be calculated as

$$\frac{1}{\rho} = \frac{\phi''}{(1 + (\phi')^2)^{\frac{3}{2}}}. \tag{3}$$

According to the law of conservation of energy, the reduction in energy is equal to the energy loss caused by friction, then

$$E(x) = \frac{1}{2}mv^2 + mg\phi(x), \tag{4}$$

where we assume the land surface is the zero point of the gravitational potential energy. Substitute Eq. (2),(3),(4) into Eq. (1), we get an ordinary differential equation for E,

$$\frac{\mathrm{d}}{\mathrm{d}x}E + \frac{2\mu\phi''}{1+(\phi')^2}E = -\mu mg[1 - \frac{2\phi\phi''}{1+(\phi')^2}]. \tag{5}$$

To get the initial condition for Eq. (5), notice that the collision happens at $(x_0, \phi(x_0))$, where the collision angle is $\frac{\pi}{2} + \theta_0$. The collision is assumed perfect inelastic, that is to say the slider loses his velocity perpendicular to the snow surface. Then we have

$$v_0 = \sqrt{\frac{2m(h-y_0)}{g}}\sin\theta, \tag{6}$$

and then the initial mechanical energy

$$E_0[\phi] = mgy_0 + mg(h-y_0)\frac{\phi'(x_0)^2}{1+\phi'(x_0)^2}. \tag{7}$$

Solution to Eq. (5) and (7) is written as

$$E[\phi, x] = e^{-2\mu\int_{x_0}^{x} F[\phi(s)]\mathrm{d}s}\big[-\mu mg\int_{x_0}^{x}(1-2\phi(t)F[\phi(t)])e^{2\mu\int_{x_0}^{t} F[\phi(s)]\mathrm{d}s}\mathrm{d}t + E_0[\phi]\big], \tag{8}$$

where functional F is defined as

$$F[\phi(x)] = \frac{\phi''(x)}{1+\phi'(x)^2}. \tag{9}$$

One part of Eq. (8) can be calculated analytically,

$$\int_{x_0}^{t}\frac{\phi''}{1+(\phi')^2}\mathrm{d}s = \arctan\phi'|_t - \arctan\phi'|_{x_0} = \theta(t) - \theta(x_0). \tag{10}$$

Then $E[\phi, W]$, the mechanical energy at the other end becomes

$$E[\phi, W] = -\mu mge^{-2\mu\theta(W)}\int_{x_0}^{W}(1-2\phi(t)F[\phi(t)])e^{2\mu\theta(t)}\mathrm{d}t + e^{-2\mu[\theta(W)-\theta(x_0)]}E_0[\phi]. \tag{11}$$

Larger $E[\phi, W]$ means more energy before the next jump, which is what we are pursuing.

Eq. (11) provides us a fundamental way to analyze the problem. However, $E[\phi, W]$ is a functional of $\phi(x)$, containing its zero, first, second derivatives and their integrals, hence a direct variational method can hardly be directly applied to it. In order to avoid a variational problem in the function space, we limit some feature of $\phi(x)$ and reduce the problem to a lower dimensional space.

4 Application in 2D Case

4.1 Quarter-Circle Type

From a circle series expansion point of view, we can use a circle to simulate a curve as the first-order approximation. Thus in this issue we firstly assume the trail consists of a vertical wall, a horizontal path and a quarter circle. This is simple but can provide us much information about the interactions among the relevant factors. In this case, no energy is added to the snowboarder, therefore the player may not reach the other edge of the course. So we only study process in (x_0, w), where w is the midpoint of the path. Our goal here is to analyze how the related factors, μ, h_i, x_0 and w, influence the energy loss.

Notice that on the quarter circle, we have

$$F[\phi(x)] = \frac{\phi''(x)}{1+\phi'(x)^2} = \frac{1}{R-y}, \tag{12}$$

where R is the radius. Then according to Eq. (11), we have

$$\begin{aligned} E(w) &= e^{-2\mu\theta(x_0)}E(x_0) - \mu mg(w-R) - mgR(1-e^{-2\mu\theta(x_0)}) \\ &- \frac{3\mu mg}{1+4\mu^2}[2\mu R - (x_0 - R + 2\mu(R-y_0))e^{-2\mu\theta(x_0)}], \end{aligned} \tag{13}$$

where

$$\theta(x) = -\arcsin(1-\frac{x}{R}). \tag{14}$$

Eq. (13) is a useful result to analyze the effect of w, x_0, h_i, and μ. We calculate the energy $E(w)$ for different radius to search for the best R, which we denote as R^*. For most parameters, $E(R)$ has a maximum inside the interval $[x_0, W]$, which is chosen as the R^*. What we are interested here is the relationship between

R^* and (W, h_i, x_0, μ). According to the halfpipe standard set by International Ski Federation [3], w is around 8.5m$\sim$ 9m for 18 FT pipe and 9.5m$\sim$ 10m for 22 FT pipe. x_0 and h_i is related to the trick and the slider himself. μ is related to the quality of the snow, the temperature and the intensity of pressure between the board and snow. Usually, this value is between 0.08 and 0.11 [4]. Based on Eq. (13), we analyze the influence of pipe width W, friction coefficient μ, initial height h_i and landing position x_0, respectively.

- Half width of the pipe w. According to Eq. (13), there is no w in $\partial E/\partial R$. Therefore, we expect that R^* has nothing to do with w. For $f = 9.0$m, $\mu = 0.1$ and $x_0 = 0.5$m, the result is in Fig. 3. R^* is a constant equals to 3.83m.

- Friction coefficient μ. We calculate R^* numerically to see of what role μ plays. The result is in Fig. 4. Our results indicate that for a smaller μ, a larger R^* is needed to reduce the energy loss.

- Initial height h_i. Initial height h_i is a parameter we couldn't assume beforehand. A good halfpipe should be specified to satisfy all kinds of needs. Therefore, h_i-R^* relationship is important in determining the shape. Our result in Fig. 5 shows that h_i and R^* have a rough linear relationship with gradient 0.16.

- Landing position x_0. Landing point x_0 determines the collision energy loss. Lower collision point and moderate slope means larger collision energy loss. A larger radius will provide higher collision point but suffers from steep collision surface, so R^* is a balance between these two factors. According to our numerical results in Fig. 6, when reducing x_0, R^* accelerates to decrease. While x_0 tends to 0, R^* also tends to zero. On the other hand, when x_0 is big enough, R^* hits the boundary h_0, which means no verticals.

To sum up, a quarter circle path gives a direct access to the problem. If we assume the energy loss is due to the friction and collision loss, regard the slider as a mass point, then the total energy loss can be calculated analytically and provide us lots of information of the interactions among all kinds of influential factors.

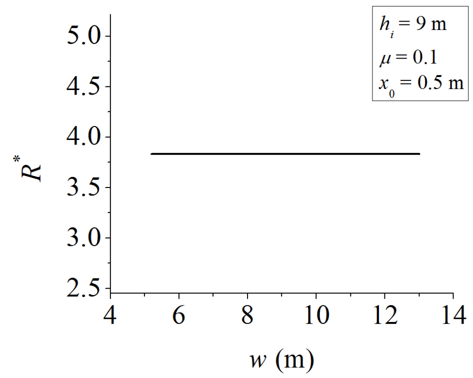


Fig 3: R^*-w curve

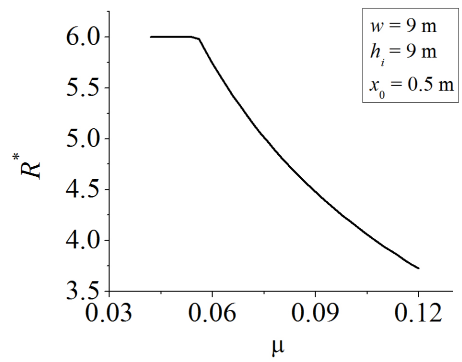


Fig 4: R^*-μ curve

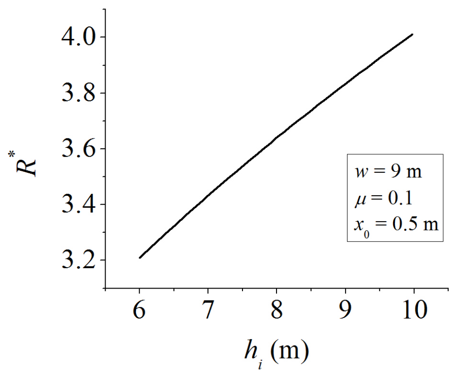


Fig 5: R^*-h_i curve

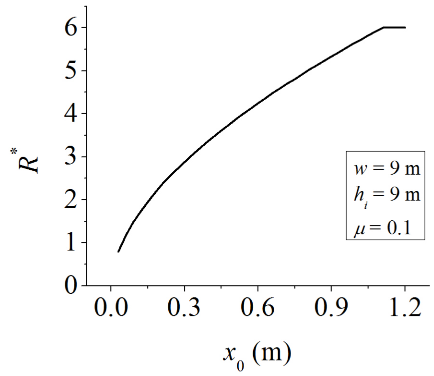


Fig 6: R^*-x_0 curve

Within the quarter circle style, we claim that for $h_i = 9.0$m, $x_0 = 0.5$m, $\mu = 0.1$ and $w = 9.0$m, the best radius is 3.831m.

However, a quarter circle path is too simple for the real halfpipe. In addition, according to our computation, when x_0 is large (> 0.5), E is approximately a constant if $R \in [3, h_0]$. This implies that R no longer affects so much on E, thus other factors may be dominant. Therefore, in order to pursue better shape, we turn to look for more complicated curve.

4.2 Quarter Ellipse Type

We now study the case in which the transition curve of the ramp is a quarter section of a ellipse with semi-major axis a and semi-minor axis b. For further calculation, here we specify some parameters defined before. According to the halfpipe size standard released by International Ski Federation[3], we set w equals to 9m, h_0 equals to 5.4m. According to the pressure density on the snow and the sliding velocity, we assume $\mu = 0.1$. h is set to be 9m, which is common for professional snowboarders. We assume $m = 75$kg, which brings 6.6kJ of the initial mechanical energy. It should be pointed out that these assumption is not essential for the following analysis.

We solve the Eq. (5) numerically using forward Euler's method[5], the step is chosen to be 10^{-4}m. a^* and b^*, the best values of a and b, are obtained using bisection method.

4.2.1 Snow Friction Only

Landing is a rapid but complex process. There will be deformation of board and snow, and temperature changes. For simplicity, we ignore the energy loss caused by collision first to see its effect. In this case, only sliding friction account for the energy loss. We fix b to find the best a and fix a to find the best b, respectively.

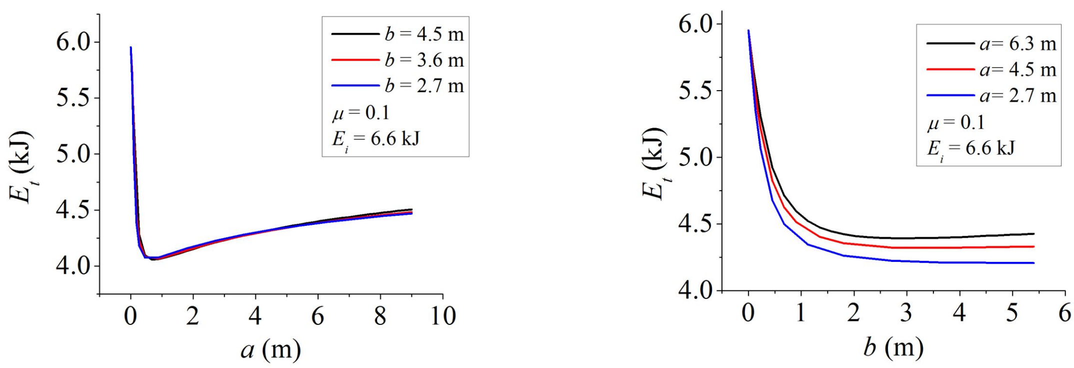


Fig 7: Left: E_t-a curve under different b; Right: E_t-b curve under different a. Only the snow friction is considered

Define the final energy at $x = w$ as E_t, which is a gauge to measure the quality of the course. It is shown in Fig. 7 that, E_t reaches its maximum when a tends to zero with b fixed or b tends to zero with a fixed. Therefore, we come to the conclusion that collision energy loss is essential in the whole process. Ignoring it will lead the trail to be a right angle, which is unreasonable.

4.2.2 Snow Friction and Collision

Now we take the collision into consideration. Here we fix the landing point $x_0 = 0.5$. By re-conducting the calculation before, we get Fig. 8. It is shown that for each b, E_t has a peak value in the interval $[x_0, w]$, and the peak shifts to the right when b becomes larger. On the other hand, for each a in Fig. 8, E_t ever increases as b gets larger. In this case, our conclusion is that the longer b is, the smaller the energy loss there will be, but the value for a is inside the interval $[x_0, w]$ and is determined by related parameters.

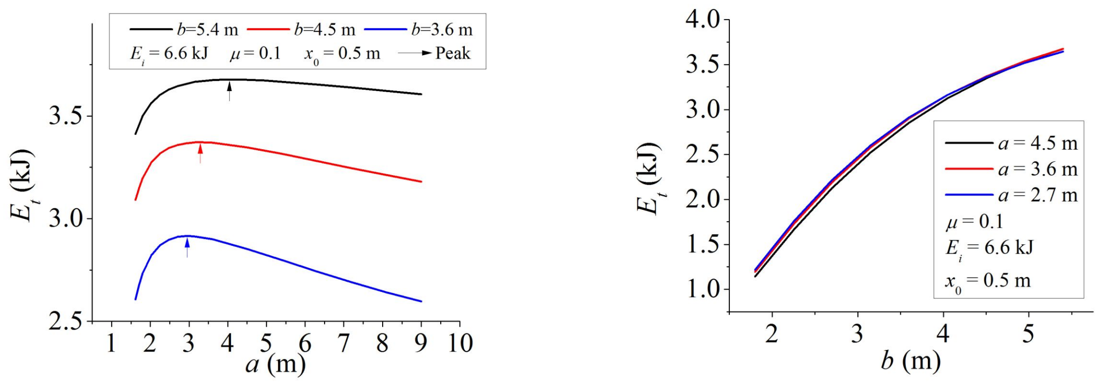


Fig 8: Left: E_t-a curve under different b; Right: E_t-b curve under different a. The snow friction and collision are considered

In this way, we vary a and compare the peak value for b, then a best (a^*, b^*) is obtained.

4.2.3 Snow Friction, Air Friction and Collision

Furthermore, we add air friction to make our model more practical and to analyze the role wind plays. In this case, we assume the air resistance is proportional to the velocity since the speed is not fast. The damping coefficient α is defined as resistance force applied on unit area under unit velocity, which is affected by the velocity of the moving object. In the low speed case, the change of α due to the velocity can be ignored, thus we assume a constant α in the following calculation.

To study the influence of wind friction, we vary α from 0Ns/m^3 to 1.5Ns/m^3 with other parameters fixed, and assume the windward area A is 0.5m^2.

In this case, the differential equation of E becomes

$$\frac{\mathrm{d}}{\mathrm{d}x}E + \frac{2\mu\phi''}{1+(\phi')^2}E = -\mu mg[1 - \frac{2\phi\phi''}{1+(\phi')^2}] - \alpha Av\sqrt{1+(\phi')^2}. \tag{15}$$

We calculate the influence on a^* and b^* by α. Our result in Fig. 9 shows that a^* shifts to the right a little bit as α increases, and b^* is still on the boundary. From now on, for simplicity, we use the average value of α under low speed, which, according to the Wind Scale Table [6], is about 0.75Ns/m^3.

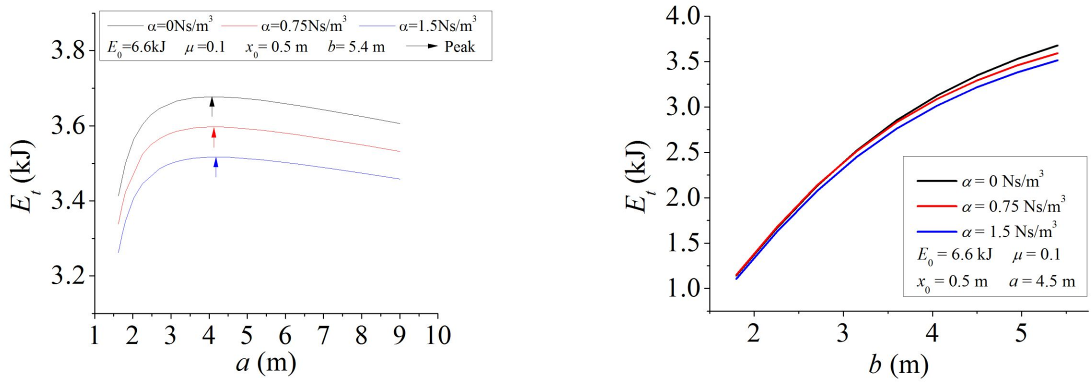


Fig 9: Left: E_t-a curve under different b; Right: E_t-b curve under different a. The snow friction, wind friction and collision are considered

In conclusion, for given $(\mu, x_0, h_i, \alpha, w)$, we can provide the best value of a and b. In the common case, b^* reaches to h_0 and a^* is between x_0 and w. The existence of wind resistance will alter the final energy E_t, on the other hand, the wind does not affect much of a^* and b^*, or the shape of the snow course. At the parameters $(h_0,\ h_i,\ x_0,\ \mu,\ \alpha) = (5.4,\ 9.0,\ 0.5,\ 0.1,\ 0.75)$, we claim that the best ellipse parameter is $(a^*,\ b^*) = (4.19,\ 5.4)$

4.3 Sensitivity

In the following section, we will analyze the sensitivity of shape-parameters a^* and b^*, according to friction coefficient μ, landing position x_0, initial height h_i, halfpipe height h_0, damping coefficient α and initial energy E_i, respectively. In order to compare the influence of these factors, we introduce a non-dimensional variable s, the sensitivity coefficient. Sensitivity coefficient of two quantities A and B is defined as

$$s = \frac{\partial A}{\partial B} / \frac{A}{B}\Big|_{B=B_0}, \tag{16}$$

where B_0 is a fixed point and A is a function of B.

Cause b^* always equals to the halfpipe height, we only estimate the sensitivity of a^* according to the six factors mentioned above.

- x_0. To our surprise, there is a minimum in the a^*-x_0 curve at around $x_0 =$ 0.45m. This is approximately the value of common landing point. Longer or shorter x_0 will both lead to larger a^*. The sensitivity coefficient s is 0.552 at $x_0 =$ 0.5m. The result is shown in Fig. 10
- h_i. A higher initial height means larger speed before collision, therefore we expect steeper collision angle or higher collision height, which lead to larger a^*. The result is shown in Fig. 11. The sensitivity coefficient s is -2.110 at $h_0 =$ 9.0m.
- α. As we talked in the last section, α effects little on a^*. The corresponding s is 0.011 at $\alpha =$ 0.75Ns/m^3. The result is shown in Fig. 12
- h_0. When h_0 changes, so does b_0. Our result in Fig. 13 shows that a^* is sensitive to h_0, and s=2.374.
- μ. Changes of μ will affect the energy loss significantly as shown in Fig. 14. For larger μ, a^* needs to increase to reduce the energy loss caused by friction. $s =$ 0.993 at $\mu =$ 0.1.
- E_i. In the case where no rotation involves, the effect of E_i is equivalent to h_i. The result is shown in Fig. 15, and $s =$ -2.110.

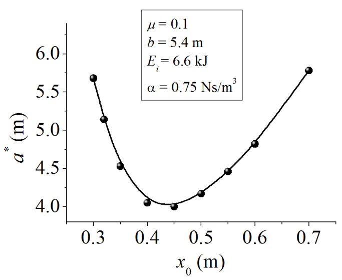


Fig 10: a^*-x_0 curve

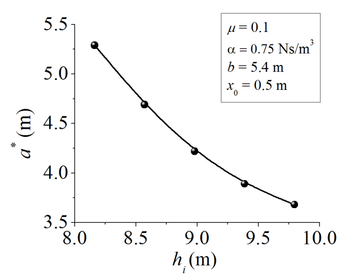


Fig 11: a^*-h_i curve

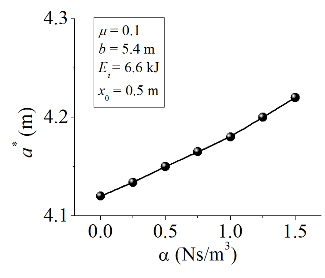


Fig 12: a^*-α curve

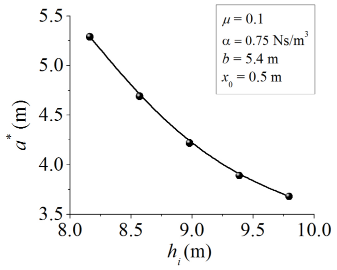


Fig 13: a^*-h_0 curve

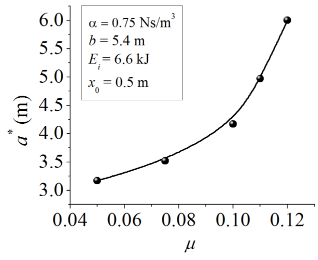


Fig 14: a^*-μ curve

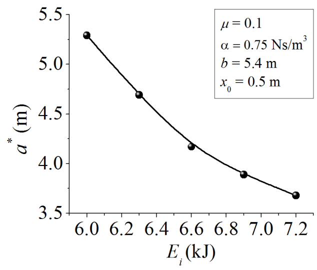


Fig 15: a^*-E_i curve

5 Application in 3D Case

Now we move our study to a more reasonable and complex case involving the third dimension of the halfpipe. In fact, the halfpipe does not lie horizontally, but is oriented downhill with a slope angle φ. In this way, the sliding is not a pure energy loss process, but can gain energy from the gravity. Therefore, the third dimension will alter the velocity of the snowboarder, which will bring changes to a^* and b^*.

Fig. 16 is our sketch for a 3-dimensional halfpipe. For a standard 18 FT halfpipe, the length of the halfpipe L is about 100-150m, and for professional snowboarders, there will be 6-7 tricks in a round. Therefore, in a whole trick, the player will travel about 20m to the downhill direction, which is defined as l.

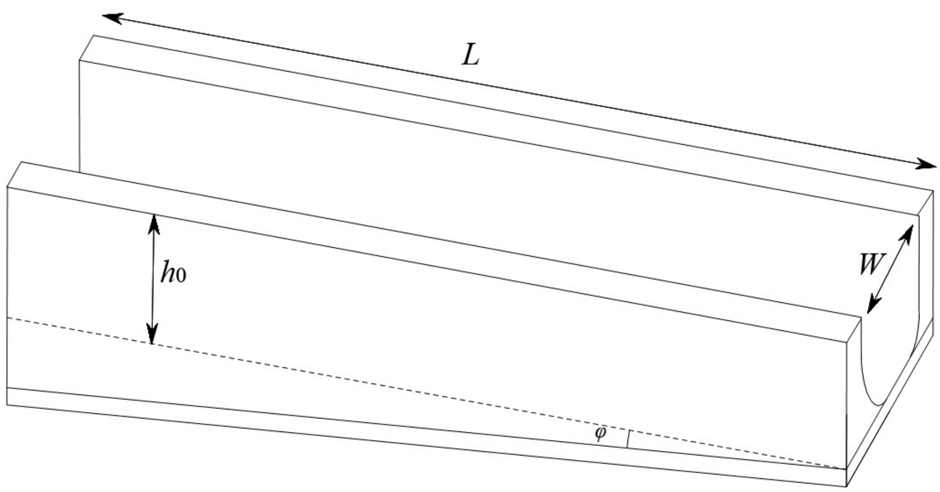


Fig 16: 3D halfpipe

We use (x', y') to describe the new course. It is the upper part of the curve edge trapezoid formed by a plane and the halfpipe. Under the coordinates transformation in Eq. (5), the problem changes to a 2-D case.

$$\begin{cases} x' = \sqrt{1 + \frac{l^2}{W^2}}x, \\ y' = y - \frac{x}{W} l \sin\varphi. \end{cases}$$

By adopting Eq. (11), we can work out the corresponding a^* and b^* similarly. a^* and b^* are calculated numerically according to different slope angle φ, and the result is in Fig. 17.

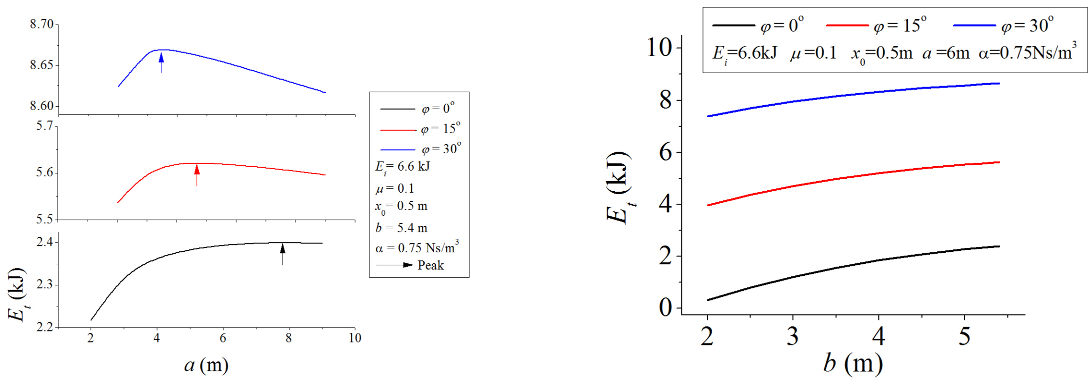


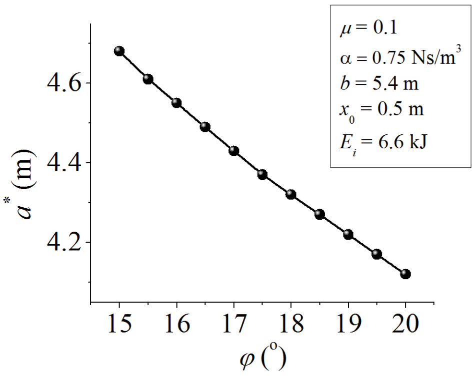
Fig 17: Left: E_t-a curve under different b; Right: E_t-b curve under different a. The snow friction and collision are considered

The conclusion we make in 2D case holds for the 3D case here. b^* reaches h_0 and a^* lies in $[x_0, w]$.

Now we begin to analyze the influence brought by slope angle φ. With other parameters fixed, we calculate the a^* and b^* according to φ, respectively.

It turns out that a steeper slope requires smaller a^*, which is shown in Fig. 18. Meanwhile, b^* is always equal to h_0 regardless of φ. The sensitivity coefficient of a^* according to φ is -0.4605 at $\varphi = 17.0^{\mathrm{o}}$

Now it's time for us to determine the suitable φ for a halfpipe. The calculation above only concerns half of the course, that is from the hanging to the nadir of the curve. Here we complete the whole curve for one trick to work out the best φ, denoted as φ^*.



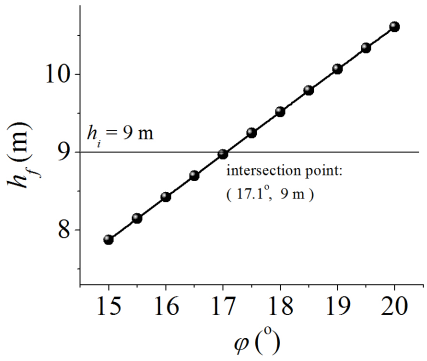


Fig 18: Left: E_t-a curve under different b; Right: E_t-b curve under different a. The snow friction and collision are considered

Fig 19: Left: E_t-a curve under different b; Right: E_t-b curve under different a. The snow friction and collision are considered

We claim that a similar hanging height above the deck on each side of the halfpipe is a sign for good φ. The reason is as the follows. In a round of snowboard contest, the player will perform 6-7 tricks on both sides of the halfpipe. Thus small φ will result in the condition that the player never takes air. Also, too large φ will lead to higher and higher hanging height, which is dangerous for the player. Therefore, we look for φ^* that makes the same hang height above the deck on both sides. The initial hang height above the ground h_i is still assumed to be 9.0, cause the halfpipe is symmetric, φ^* should make the final height above the ground h_f equals to $9.0m$. We calculate the $h_f - \varphi$ relationship in Fig. 19, fit the data by using cubic spline interpolation and finally get the point intersected by the $h_f - \varphi$ curve and the line $h_f = 9.0$m. Thus φ^* is captured, which is 17.1°.

The corresponding (a^*, b^*) for $\varphi^* = 17.1°$ is $(4.42\text{m}, 5.4\text{m})$.

6 Tailors and Tradeoffs

Above analysis is purely theoretical, based on some ideal assumptions. In the real game, the player is not a mass point without rotation, and the goal is not simply to reach the highest point. Practical situations are much more complicated and more factors should be taken into consideration. So we have to tailor the halfpipe to optimize various requirements.

6.1 Safety Analysis

No one wants to get hurt in games. The safety requirements should always be put in the first place. Take the design of edge angle as an example. In theoretical halfpipe the slope angle near edges, named as θ_e, should be close to 90^o. However, such large θ_e means less holding force from halfpipe to player, which will increase the possibility for player to fly away the track too early. At the same time, when the player falls onto the surface of the halfpipes edge from air, the shock is in proportional to $\cos\theta$. So small θ_e may increase the shock, hurting players or making them lose their balance. In conclusion, θ_e should neither be too large nor too small. Usually this is set from 83^o to 88^o [3].

The slope angle of the halfpipe, φ, is another factors related to safety. Larger φ means higher jump but also higher danger. Therefore, while adjusting φ in our design, we could only slightly increase it from the theoretical value 17.1^o to about 17.5^o. This is also the actual slope angle used in regular game nowadays.

Besides, to minimize the energy loss caused by friction, the bottom part should be abandoned (in other words, a should equals w). However, we maintain this part, to give the player more time to prepare the next jump.

6.2 The Construction

The shape of ellipse is not a big problem challenge in actual construction. The building of precipitous ramp may increase the difficulty, since the sharp cliff cannot hold the snow firmly on its surface. Therefore, decreasing θ_e from nearly 90^o to 85^o can also meet construction requirements.

6.3 The Twist

In regulation games, players are demanded to play more twists in the air. The angular velocity for twists is obtained by wriggle the waist and stomp the ramp. To help players perform more twists, we should provide them more time in the air, and offer them a more safe side wall to step on. So on one hand, we could increase φ to speed the player up within the safe range. On the other hand, as mentioned above, the edge angle should be a little smaller than 90^o.

7 Strengths and Weaknesses

7.1 Strengths

- We do not construct a complex model with all the variables together. Instead, we build a simple basic model first and then introduce corrections step by step. Therefore, it is easier to analyze the effect of different factors separately.
- We have taken many kinds of different factors into consideration and made all the analyses systematically and comprehensively.
- The halfpipe size and the slope angle obtained by our model fit the real data well. That's to say, our model is successful in application layer.
- A lot of figures are set to illustrate the relations between different variables visually. This is more accessible and easier to analyze than those complicated function expressions.
- We obtain the optimal size for elliptical half pipe, which is more general than circle curve and still easy and quick to construct in reality.
- We analyze the sensitivities of the results to different variables not only qualitatively, but also quantitatively, by introducing the sensitivity coefficients.

7.2 Weaknesses

- We ignore the influence of snowboarder's movement on the height of *vertical air*, which may reduce the computational accuracy. For example, twist and jump may affect the mechanical energy, stand up and down postures may affect the velocity.
- For simplicity, the direction of velocity is considered unchanging though out the movement, while in fact, one skilled snowboarder may change his moving direction to achieve some complex tricks, which may cause the moving path not in one plane .
- We use ellipse as our final approximation, which may not be the best curve to guarantee the highest *vertical air*.

8 Conclusion

In this paper, we study the shape of a snowboard course and its influential factors under energetic view. The main idea is to measure the "vertical air" by final energy. After making several assumptions, we build our basic model, which is simple but sufficient to offer a fundamental idea and reveal essential interactions among different factors. We make force analysis, set up kinematical equations and obtain the general form of final mechanical energy analytically.

After that, two specific types of transition curve are introduced to calculate the optimal design. In quarter-circle model, we obtain analytical solutions and find the best radius R^* of the circle. While in quarter-ellipse model, we use numerical simulation to get the optimal semi-major axis a^* and semi-minor axis b^*. Then we further develop the 2D quarter-ellipse model into a more actual and complex case, involving the third dimension of the halfpipe. Through numerical computation, we find the most suitable slope angle of the halfpipe is 17.1^o. Next, the sensitivities of different variables are analyzed both qualitatively and quantitatively. It helps us to compare the significance of these variables in building a best halfpipe.

At last, the tailors and tradeoffs to develop a "practical" course are discussed. After taking construction difficulty, players' safety and their performance into consideration, our result becomes more practical.

In conclusion, through systematic and comprehensive analysis of various factors, we find a reasonable shape of the snowboard course, with $a = 4.42$m, $b = 5.4$m and $\varphi = 17.5^o$. Our results fit the real date well.

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