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Problem Chosen

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2006 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Summary

Although air travel today is available to most people, difficulties to access still exist. A passenger with a disability may encounter inconvenience to change to a different airplane. Because airline provides wheelchairs and escorts to serve disabilities, it will generate costs. This paper is aimed to construct the economical scheduling algorithm about the movement of wheelchairs and escorts throughout a day, and find the minimal budget costs.

Firstly, combining the theory of greedy method, we divide the costs into five parts, such as the cost of wheelchairs' storing, the cost for making the escorts available and so on. For each part, we establish a cost model to estimate minimal cost. We discover the number of wheel chairs and escorts is the key to contribute the total costs.

Secondly, we determine the optimal position of pilling up wheel chairs via geometrical analysis. And we carry out real-time simulation the process of scheduling wheel chairs to find minimal number of wheel chairs and escorts.

Using the above optimal model, we find a feasible solution. For large airport, the minimal budget costs is 480,900 dollars/year, for medium airport, the minimal budget costs is 170,880 dollars/year, for small airport, the minimal budget costs is 60,540 dollars/year. The distinctive point in our model is that scheduling algorithm can work for different types of airports under high and low traffic loads.

Finally, we evaluate our models' strengths and weaknesses. Then, projections of potential costs and needs in the future with recommendations are available in the end.

Project M.C.S.D.:
Minimal Costs for Serving Disabilities

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Introduction

Air travel is capable to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to easily walk to a different flight's waiting area. Some airlines render services, which provide wheelchairs upon request, to help the handicapped people change to different flights. To ensure wheel chairs are available, passengers, who require help, should be encouraged to call their airline to reserve one before flight arrival or departure. Passengers may also request wheelchair assistance at airline ticket counters.

Airlines are under constant pressure to keep their costs down. Combining the theory of greedy method, we divide the costs into five parts: the cost of wheelchairs' purchase, maintenance and wearing out, the cost for making the escorts available, the cost of wheelchairs' storing, the cost for liability risk that wheel chairs left in high traffic areas when people try to move around them, and the cost of holding a plane which causes that potential customers may choose to avoid an airline and lead to fewer ticket sales.

Based on minimizing the above costs, our objectives are:

- to model the serving disabilities' process in airdrome.
- to give a recapitulative overview and analysis of serving disabilities' process.
- to determine where the escorts and wheel chairs should be and how they should move throughout each day.
- to bring forward the economical dispatch algorithm and illustrate program whether suits wide range of airports under a variety of circumstances or not.
- to give the detailed description of a scheduling algorithm.
- to determine all potential costs and balance their respective weights.
- to consider the costs for budgets planning in both the short term and long term.
- to consider potential costs and forecast disabilities' needs in the future with recommendation to meet future needs.

We propose many models to solve the problems.

Problem Approach

We break the overall problem down into several smaller pieces, solve the pieces separately, and put the pieces together to find the overall solution.

- Analyze the original of five parts costs mentioned above.
- Considering the above analyzing, we need establish respective models for five parts costs.
- Under the condition that satisfies the disabilities' needs and minimizes the total costs, we need determine the number of wheel chairs and escorts by carrying out optimal dispatch algorithm.

Background

General situation in airdrome

Airdrome's dedication to total quality management and continuous improvement activities has made it a leading supplier to the global aerospace industry. Commercial and military aircraft and space programs the world over utilizes the design, engineering and manufacturing expertise of Airdrome. The general picture of common airdrome is shown in **Figure1**.

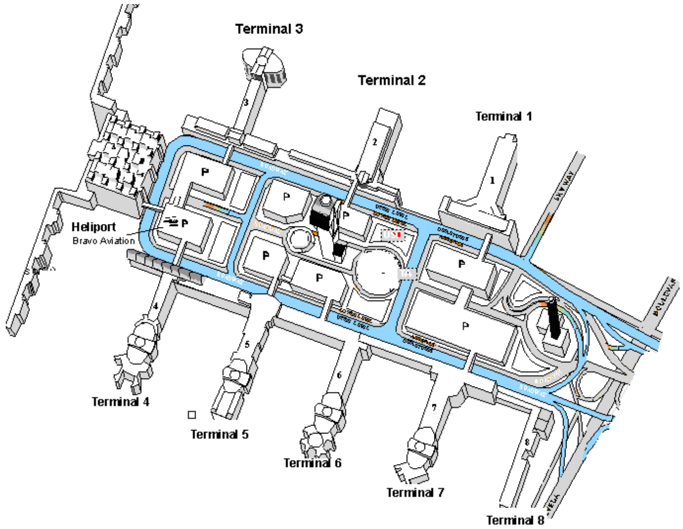

Fig.1 The general picture of airdrome

The internal structure in one of terminals

For general terminal, it has three parts. One is arrival level, another is concourse level, and the third is departure level. The picture (**Figure 2**) as below illustrates the internal structure.

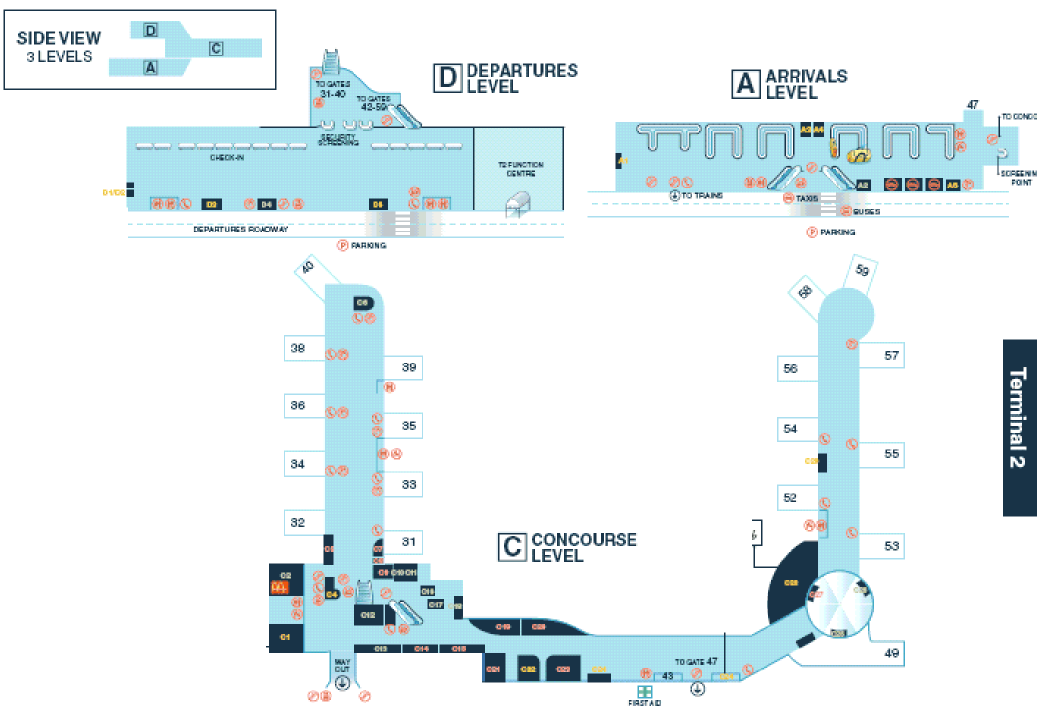

Fig.2 the picture of inner structure

The process of changing planes

Passengers travel some places through flying multiple airports. During each stop, traveler should deplane at the first level called arrival level. And then they walk to different flight's concourses on the second level to wait. After making safety- check-up, traveler goes to the departure level to board. The sketch map of boarding process is shown in **Figure 3**.

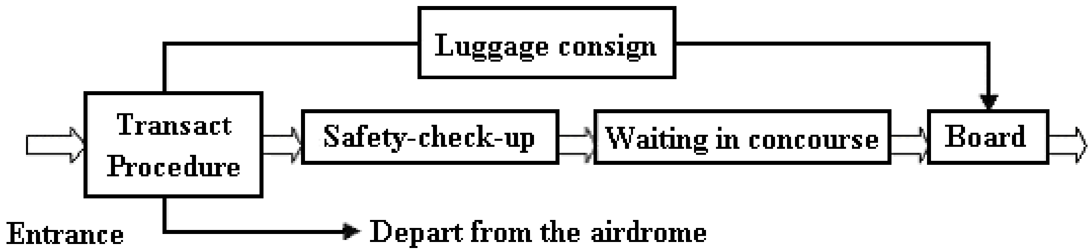

Fig.3 the sketch map of changing plane

Escorts

Escorts are employees of the airline. Their duty is to arrange wheel chairs to meet disabilities' needs. They can accompany disabilities to the boarding gate if the disabilities cannot travel independently. Thereby, whether escorts accompany disabilities or not, depends that whether or not disability requires accompany.

Assumptions & Hypotheses

- Holding a plane is caused by many factors, such as weather, security etc. We consider the factor only if some disabilities must wait for an escort and become late for their flight.
- There is no space limited for wheel chairs' storing. i.e. We can choose the space freely according to our model.
- Disabilities, who require help, must notify airline at least 1 hour in advance. [8]
- All the disabilities need wheel chairs, and 50% of them need escort's accompany.
- Look upon all the staffs that serve disabilities as escorts.
- The open time of airport is from 6:30a.m. to 10:30p.m.[12]
- Airplane has two types, one is big with holding 110~140 passengers, and the other is medium with holding 90~110 passengers.
- Every escort works 48 hours per week. [12]
- The size of a wheel chair occupy 0.8m×0.8m[2]
- The price of a wheel chair is $300[2]
- The wage of a escort annually is $20,000
- The rent-fee of storing space per square meter annually is $5,500.[11]

Notions

Table 1. Symbols used

Symbol	Unit	Definition

Variables

f	dollar	the total costs
f_1	dollar	the cost of wheelchairs' purchase, maintenance and wearing out
f_2	dollar	the cost for making the escorts available
f_3	dollar	the cost of wheelchairs' storing
f_4	dollar	the cost for liability risk
f_5	dollar	the cost of holding a plane
n	chair	the number of wheel chairs
n_0	people	the number of escorts right to serve all the disabilities
x	percentage	the proportion that the percentage of the delaying time account for average time of flight
λ	unitless	parameter of an exponential distribution
n_c	people	the number of customers in a flight
n_d	people	the number of disabilities in a flight
v_1	m/h	velocity which escort and wheel chair moves with disability
v_2	m/h	velocity which escort and wheel chair moves without disability

Constants

m_0	dollar	every chair has left residual value ultimately on average
$\overline{m}$	dollar	the average fee of maintaining a wheel chair
y_m	year	the life length of a wheel chair on average
w	dollar	the wage of a escort annually
t_{wpw}	hour	working-time per week every escort
t_{opd}	hour	the open-time of airport per day
r	dollar	the rent-fee per square meter annually for storing space
s_0	m^2	the area a wheel chair occupy
	percentage	the special proportion to make all the customers avoid a

x_m		airline
L_{Ar}	meter	the distance vector between Ar and boarding gates
L_{Co}	meter	the distance vector between Co and arrival gates
L_1	meter	the distance matrix between arrival gates and boarding gates
L_2	meter	the distance matrix between boarding gates and arrival gates

Design the model
Costs Estimation Model
Cost Structure

- **Direct Cost**
- **Wheel Chairs**

Considering that wheel chairs will wear out and are expensive and require maintenance, we need to determine how many numbers of chairs used and adopt which depreciation method. Suppose there are n wheel chairs and m dollars per wheel chair. If every chair has residual value m_0 dollars ultimately, we adopt the conventional y_m-year straight-line method, so annually the depreciation would track at about

$$(m - m_0)/y_m$$

Each year, the average fee of maintaining a wheel chair is $\overline{m}$

Therefore, this part of cost f_1 is

$$f_1 = n \cdot (m + \overline{m} - m_0)/y_m \quad (1)$$

- **Escorts**

It is reasonable that some escorts are employed to manage the wheel chairs and serve customers with disability. Escorts with wheel chairs must be constantly moved around the airport so that they are available to the disabilities. Of course the escorts aren't volunteers and must be paid. But airlines cannot charge the disabilities for the escort's fare. So airlines should pay the fare for escorts. Suppose their average wage is w dollars annually, and every escort works t hours per day. The open time is t_0 hours per day when the airplanes operate.

Further more, we assume that there are n_0 escorts who are just right to serve all the disabilities in some interval. Suppose that there are seven days in a week, and one month is made up of four weeks and twelve months constitute a year (actually, there is no influence.).

Therefore, this part of costs f_2 is

$$f_2 = \frac{w}{t_{wpw} \cdot 4 \cdot 12} \cdot n_0 \cdot t_{opd} \cdot 7 \cdot 4 \cdot 12$$

$$= \frac{w \cdot n_0 \cdot 7 \cdot t_{opd}}{t_{wpw}} \qquad (2)$$

where t_{wpw} is working-time per week every escort (48*h/week*)

t_{opd} is the open-time of airport per day (12*h/day*)

- **Store**

As all we know, the wheel chairs must be stored when they are at leisure. And at most airdrome, space is expensive and severely finite. Considering the size of space is depended on the number of wheel chairs, we minimize the size of space by minimizing the number of wheelchairs. The rent-fee is *r* dollars per square meter annually. A wheel chair occupy s_0 square meter.

Therefore, this part of cost f_3 is

$$f_3 = r \cdot s_0 \cdot n \qquad (3)$$

But in fact, it is impossible that the space just holds all the wheel chairs, and there must be some additional space left.

➢ **Indirect Cost**

- **Liability risk**

If wheel chairs are piled up on the high traffic areas, it can probably hurt some customers, which causes high costs. So we should make our effort to take some proper measures, to avoid the liability risk. However, no matter which measure we take, this part of cost can't be avoided completely. It is reasonable to insure it against risk.

Nevertheless, we denote the cost caused by liability risk f_4.

- **Hold a plane**

Without considering the factor of weather, the cost f_5 of holding a plane if some disabilities must wait for an escort and becomes late for their flight. Then it can affect the airline's average flight delay, which can lead to fewer ticket sales as potential customers may choose to avoid an airline.

Under ideal situation, suppose that when the percentage of the delaying time accounting for average time of flight (we define the proportion as *x*) attains some value, all the customers will avoid this airline.

✧ Linear Model

We define the percentage of actual ticket sales volume accounting for ticket sales volume when no airplane delays as *y*. Considering only the delaying influence caused by disabilities, we suppose that airplane doesn't have any delay, the ticket sales won't be influenced at all, i.e. *y* =100%. If the percentage of the delaying time accounting for average time of flight (we define the proportion as *x*) attains some value x_m, all the customers will avoid this airline, then *y* = 0%.

Supposing that *y* decreasing with *x* is linear, we give the equation:

$$y = 1 - x / x_m \tag{4}$$

And sketch map as follow:

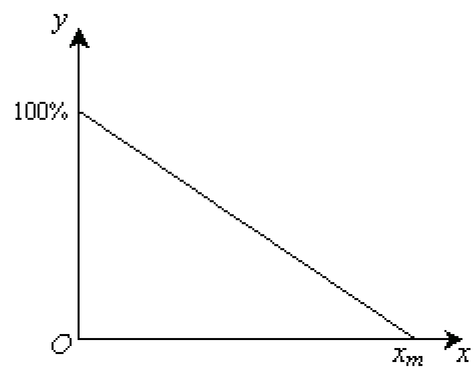

Fig.4 The sketch map of Straight-line Model

✧ Nonlinear Model

Straight-line Model is very ideal situation, which can't reflect the fact, so we need refine the model to reflect limited decrease. According to the basic common sense, initially, the decrease average time of holding a plane can't influence ticket sales obviously, so the curve is not sensitive with the change of *x*, and when *x* trends to x_m, the curve is also not sensitive with the change of *x*. Now the Differential Equation gives

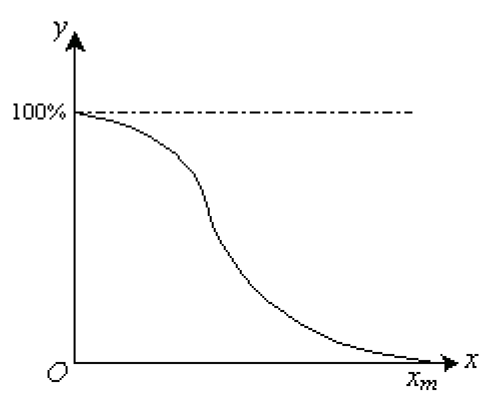

Fig.5 The sketch map of refining the Model

$$\frac{dy}{dx} = kx \tag{5}$$

$$k = r(x - x_m), \qquad r > 0 \tag{6}$$

where r is a constant. Substitute (6) into Equation (5) leads to

$$\frac{dy}{dx} = r(x - x_m)x \tag{7}$$

or

$$dy = r(x - x_m)xdx \tag{8}$$

Again we assume the initial condition $y(0) = 1$ and $y(x_m) = 0$

which integrates to

$$\int dy = \int r(x - x_m)xdx \tag{9}$$

or

$$y = \frac{1}{3}rx^3 - \frac{1}{2}rx_m x^2 + C \tag{10}$$

for some arbitrary constant C. Using the initial condition,

$$C = 1 \quad \text{and} \quad r = \frac{6}{x_m^3}.$$

We can obtain

$$y = \frac{2}{x_m^3}x^3 - \frac{3}{x_m^2}x^2 + 1 \tag{11}$$

Analyzing the above-mentioned, we calculate this part cost f_5 is

$$f_5 = f_0 \cdot (1 - y) \tag{12}$$

where f_0 is annual ticket sales income which is a constant.

Aggregates costs
The total costs

$$f = f_1 + f_2 + f_3 + f_4 + f_5 \tag{13}$$

Our Model's goal is

$$\min \ f$$

According to the theory of greedy method, we consider the situation shown as follow:

Analysis of Structure and Weight of Cost

As this paper described, the costs contain

- f_1: the cost of wheelchairs' purchase, maintenance and wearing out
- f_2: the cost for making the escorts available
- f_3: the cost of wheelchairs' storing
- f_4: the cost for liability risk that wheel chairs left in high traffic areas when people try to move around them
- f_5: the cost of holding a plane which causes that potential customers may choose to avoid an airline and lead to fewer ticket sales.

Among these five types of costs, f_4 and f_5 are most big costs. The former is caused by accidental injure and even life loss, which is difficult to measure. The latter is caused by holding plane, the biggest costs, will influence the ticket sales.

By comparison, f_1 and f_2 are the lower costs. But obviously, f_3, f_5 are dependent on f_1 and f_2. If f_1 and f_2 increase, f_3 will increase along, but f_5 will decrease. In other words, f_5 is more sensitive than f_1 and f_2. It's wonderful to reduce f_5 dramatically via enhancing properly f_1 and f_2.

In the short term, if cutting down the number of wheel chairs and escorts, the costs f_1, f_2 and f_3 would decrease, and f_5 is not sensitive with the change. It seems that the total costs would decrease.

Actually, in the long term, after some period of time, the cost f_5 is sure to increase according to the function $f_5 = f_0 \cdot (\frac{3}{x_m^2} x^2 - \frac{2}{x_m^3} x^3)$

Detailed Analysis of Costs

- **Where Wheel Chairs Are Stored**

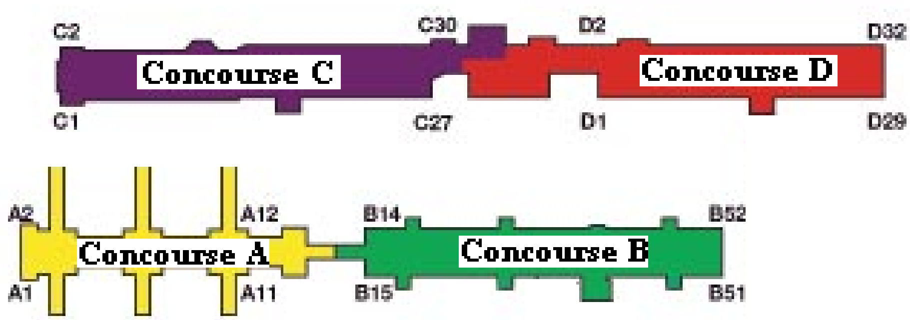

Fig. 6 The real figure of concourses in some airport

When the number of wheel chairs is constant, no doubt that pilling up wheel chairs in proper position will shorten the time of wheel chairs' scheduling. The efficiency will increase, which reduce the costs indirectly. Thereby, the proper position determined is also the key.

We abstract the practical map (Figure.6) into simplified one.

- **Two Types of Distribution of Gates**

✧ **Only one side**

Suppose boarding or arrival gates distribute on only one side in a level.

Theorem 1 For Rectangle *ACDE*, there must exist Point *O* which superposes Point B, which can make the distance of $|OA|+|OB|+|OC|$ shortest.

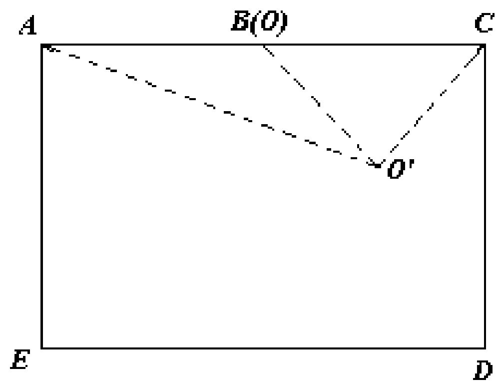

Fig.7 Rectangle *ACDE*

Proof. Suppose Point *O* is not the optimal point which can make the distance of $|OA|+|OB|+|OC|$ is the shortest, then we choose Point *O'* randomly, which is other than *O*.

Obviously,

$$|O'A|+|O'C|>|AC|$$

then

$$|O'A|+|O'B|+|O'C|>|OA|+|OB|+|OC|$$

Therefore, Point *O* is optimal.□

If we place at B, obvious B is in high traffic areas, which represents a liability risk as people try to move around them. Thereby, Point *O* is optimal theoretically, but it can probably cause liability risk. It can hurt some customers, which causes high costs.

In order to solve the problem, we consider place in some lower traffic areas, such as Point D or E. But the scheduling time will increase.

✧ **Two sides and even multi-sides**

Suppose boarding or arrival gates distribute on two sides or multi-sides on a level.

Theorem 2 For Rectangle *ACDF*, there must exist Point *O* in the center of *ACDF*, which can make the distance of $|OA|+|OB|+|OC|+|OD|+|OE|+|OF|$ short- est.

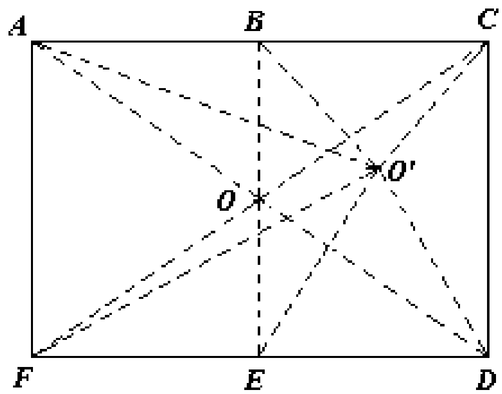

Fig.8 Rectangle *ACDF*

Proof. As the same way, suppose Point *O* is not the optimal point which can make the distance of $|OA|+|OB|+|OC|+|OD|+|OE|+|OF|$ is the shortest, then we choose Point *O'* randomly, which is other than *O*.

Obviously,

$$|O'A|+|O'D|>|AD|$$

$$|O'B|+|O'E|>|BE|$$

$$|O'C|+|O'F|>|CF|$$

i.e. $|O'A|+|O'B|+|O'C|+|O'D|+|O'E|+|O'F|$

$$>|OA|+|OB|+|OC|+|OD|+|OE|+|OF|.$$

Therefore, Point *O* is optimal.□

Based on the above analysis, we should store wheel chairs in the center of the space.

- **Scheduling Model**

- **Information system**

This part is core, which is “CPU” of the whole scheduling process.

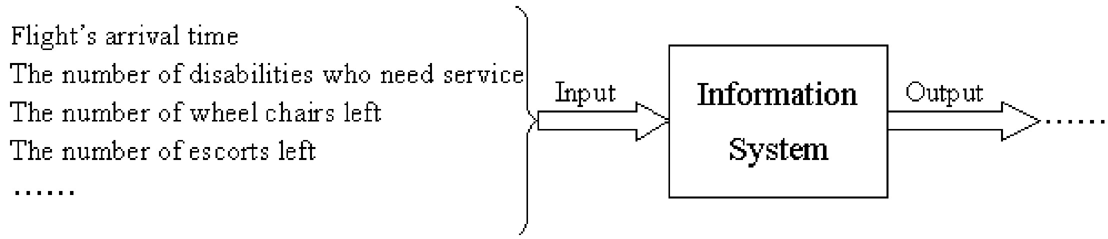

Fig.9 The information system

- **Epsilon Airlines' plane Arrival**

In some practical scheduling, we can know the time of airlines' arrival. But in simulation, we must generate some random numbers regularly to simulate the real situation.

Suppose that the number of events that airplanes arrive in any time interval of length *t* has a Poisson distribution with parameter αt (where α, the rate of the arrival process, is the expected number of airplanes arriving in 1 unit of time) and that numbers of occurrences in non-overlapping intervals are independent of one another. Then the distribution of elapsed time between the occurrence of two successive arrivals is exponential with $\lambda = \alpha$.

✧ Exponential Distribution

This problem is a large number of arriving-time, which has an exponential distribution with parameter λ. The density function is

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & , t > 0 \\ 0 & , t \le 0 \end{cases} \tag{14}$$

Where, the expectation is $1/\lambda$.

The time-interval of airplanes arriving one by one are $T_1, T_2, \Lambda, T_i, \Lambda$ respectively, which are independent on each other, and obey an exponential distribution with parameter λ.

✧ The estimation of λ

In the period of time, if total numbers of airplanes arrived is n_p, and the time interval between the *i*th and the (*i*+1)th airplane arriving is t_i, then we can make a cursory estimate for λ

$$E(T_i) = \frac{1}{\hat{\lambda}} = \frac{\sum_{i=1}^{n_p} t_i}{n_p} \tag{15}$$

- **Forecast The Number of Passengers With Disability**

In some practical scheduling, we can know numbers of disabilities who need service. But under simulation, similarly, we must generate some random numbers regularly.

Usually, there may be some disabilities in a flight, which we assume is $\omega\%$ of the whole customers.

Then we obtain

$$n_d = n_c \cdot \omega\% \tag{16}$$

where n_c is the number of customers in a flight

n_d is the number of disabilities in a flight

Besides, we can't predict the number of customers who need escorts. The number ξ is influenced by many unforeseen random variables, according to **Central Limit Theorem**, ξ obeys the normal distribution $N(\xi;\mu,\sigma)$.

- **The estimation of μ and σ**

According to 3σ-principle,

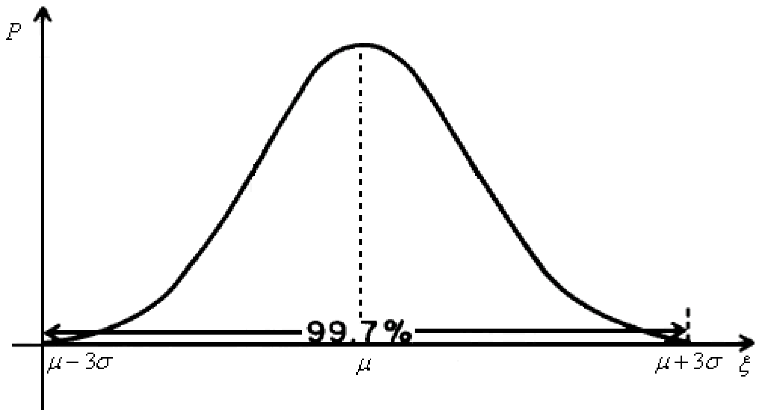

Fig.10 The normal curve

$$P(|\xi-\mu|\le 3\sigma)\approx 0.9974\rightarrow 1$$

We assume the initial condition

$$\begin{cases}\mu+3\sigma=n_d\\ \mu-3\sigma=0\end{cases} \tag{17}$$

Solve this equations set,

$$\begin{cases}\mu=n_d/2\\ \sigma=n_d/6\end{cases}$$

Rewrite, ξ obeys the normal distribution $N(\xi;n_d/2,n_d/6)$.

- **Determine distance**

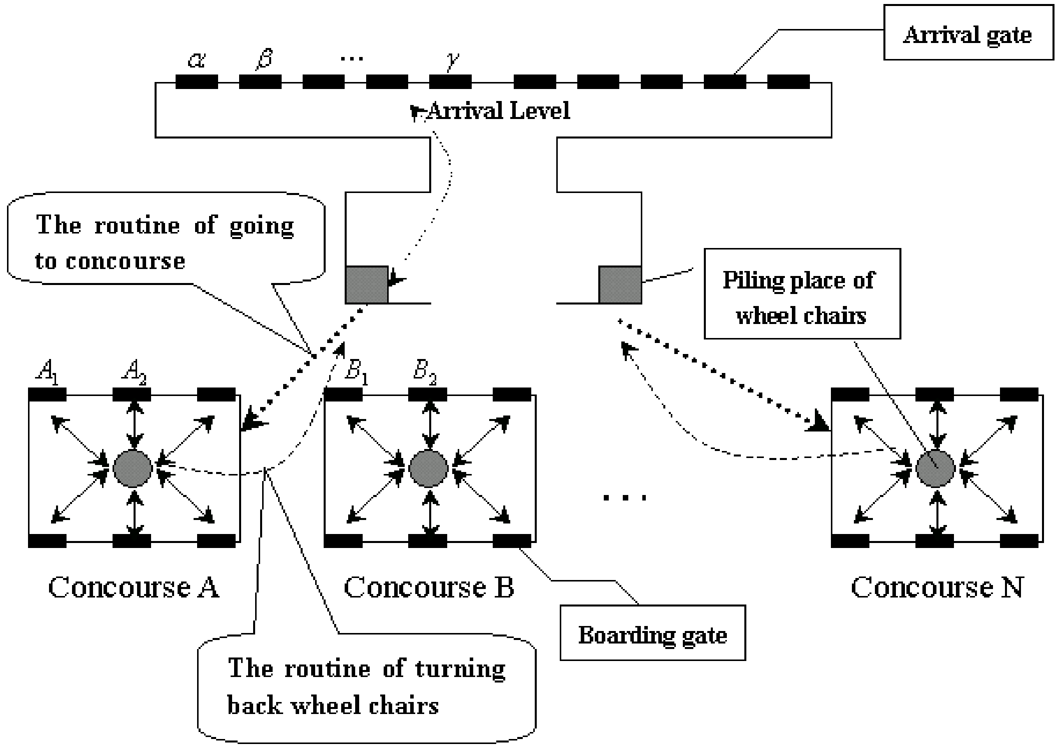

Fig.11 The sketch map of airport

There are many concourses A, B, C, D,⋯⋯, every concourse has many boarding gates, such as A1, A2, A3,⋯⋯, B1, B2, B3,⋯⋯and so on. At the same time, there are many arrival gates α, β, γ,⋯⋯. The position of storing on arrival level is Ar, and the position of storing on concourse level is Co.
We denote the distance vector between Ar and boarding gates

$$L_{Ar}=\left(|A_rA_1|,|A_rA_2|,\cdots,|A_rB_1|,|A_rB_2|,\cdots\right)^T$$

the distance vector between Co and arrival gates

$$L_{Co}=\left(|C_o\alpha|,|C_o\beta|,|C_o\gamma|,\cdots\right)^T$$

the distance matrix between arrival gates and boarding gates

$$L_1=\begin{pmatrix} |\alpha A_1| & |\alpha A_2| & \cdots & |\alpha B_1| & |\alpha B_2| & \cdots \\ |\beta A_1| & |\beta A_2| & \cdots & |\beta B_1| & |\beta B_2| & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ |\gamma A_1| & |\gamma A_2| & \cdots & |\gamma B_1| & |\gamma B_2| & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

the distance matrix between boarding gates and arrival gates

$$L_2=\begin{pmatrix} |A_1\alpha| & |A_2\alpha| & \cdots & |B_1\alpha| & |B_2\alpha| & \cdots \\ |A_1\beta| & |A_2\beta| & \cdots & |B_1\beta| & |B_2\beta| & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ |A_1\gamma| & |A_2\gamma| & \cdots & |B_1\gamma| & |B_2\gamma| & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

Numerically,

$$L_1 = L_2 .$$

Further more, we must definition two velocities

v_1： velocity which escort and wheel chair moves with disability

v_2： velocity which escort and wheel chair moves without disability

Obviously,

$$v_1 < v_2 .$$

Thus, we transfer distance into time as follows
the time vector from Ar to boarding gates

$$T_{Ar} = \left(\frac{|A_r A_1|}{v_2}, \frac{|A_r A_2|}{v_2}, \cdots, \frac{|A_r B_1|}{v_2}, \frac{|A_r B_2|}{v_2}, \cdots \right)^T$$

the time vector from Co to arrival gates

$$T_{Co} = \left(\frac{|C_o \alpha|}{v_2}, \frac{|C_o \beta|}{v_2}, \frac{|C_o \gamma|}{v_2}, \cdots \right)^T$$

the time matrix from arrival gates to boarding gates

$$T_1 = \begin{pmatrix} \frac{|\alpha A_1|}{v_1} & \frac{|\alpha A_2|}{v_1} & \cdots & \frac{|\alpha B_1|}{v_1} & \frac{|\alpha B_2|}{v_1} & \cdots \\ \frac{|\beta A_1|}{v_1} & \frac{|\beta A_2|}{v_1} & \cdots & \frac{|\beta B_1|}{v_1} & \frac{|\beta B_2|}{v_1} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ \frac{|\gamma A_1|}{v_1} & \frac{|\gamma A_2|}{v_1} & \cdots & \frac{|\gamma B_1|}{v_1} & \frac{|\gamma B_2|}{v_1} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

the time matrix from boarding gates to arrival gates

$$T_2 = \begin{pmatrix} \frac{|A_1 \alpha|}{v_2} & \frac{|A_2 \alpha|}{v_2} & \cdots & \frac{|B_1 \alpha|}{v_2} & \frac{|B_2 \alpha|}{v_2} & \cdots \\ \frac{|A_1 \beta|}{v_2} & \frac{|A_2 \beta|}{v_2} & \cdots & \frac{|B_1 \beta|}{v_2} & \frac{|B_2 \beta|}{v_2} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ \frac{|A_1 \gamma|}{v_2} & \frac{|A_2 \gamma|}{v_2} & \cdots & \frac{|B_1 \gamma|}{v_2} & \frac{|B_2 \gamma|}{v_2} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

Scheduling Process

Since we want to confirm the proper number of wheel chairs n and the number of escorts n_0, there may be many solutions satisfied. But when we consider the costs of wheel chairs, the rent-fee of their storing space and escorts' wage, there is only one optimal solution.

Some disabilities need escorts' accompany, but others not. So, there are quantities of wheel chairs left on the concourse level. Thus, we must arrange some staffs to transfer the wheel chairs on the concourse level to the arrival level in time. In fact, in order to improve the efficiency, we can bring some advanced devices to help complete this work.

- **Algorithm Detailed Idea**

On Arrival level and Concourse Level locates space for piling up wheel chairs. Our algorithm is aimed to determine how to schedule the escorts and wheel chairs so that every disability can get wheel-chair service.

When every disability who reserve in advance arrives at arrival gates, he or she must obtain a wheel chair, but he or she has right to choose whether to need escort or not. In fact, at some specific time, he or she doesn't obtain service immediately. Only if serving him or her at some time domain, thus it meets requirement.

Once on wheel chair, he or she goes to concourse directly. Of course, when one goes to concourse, another disability may be served. Besides, we schedule free wheel chairs from concourse level to arrival level at some frequency.

In this scheduling process, we need make some supplements as follows:

Escorts return arrival level after escorting disability to concourse;

The free wheel chairs are piled at appointed position;

The number of wheel chairs from concourse level to arrival level one time is constant.

In a word, the key of algorithm is to find the proper number of wheel chairs n and the number of escorts n_0, so that the corresponding cost is minimal and disabilities' requirement is satisfied.

- **Figure presents the intuitive summary of how our model works**.

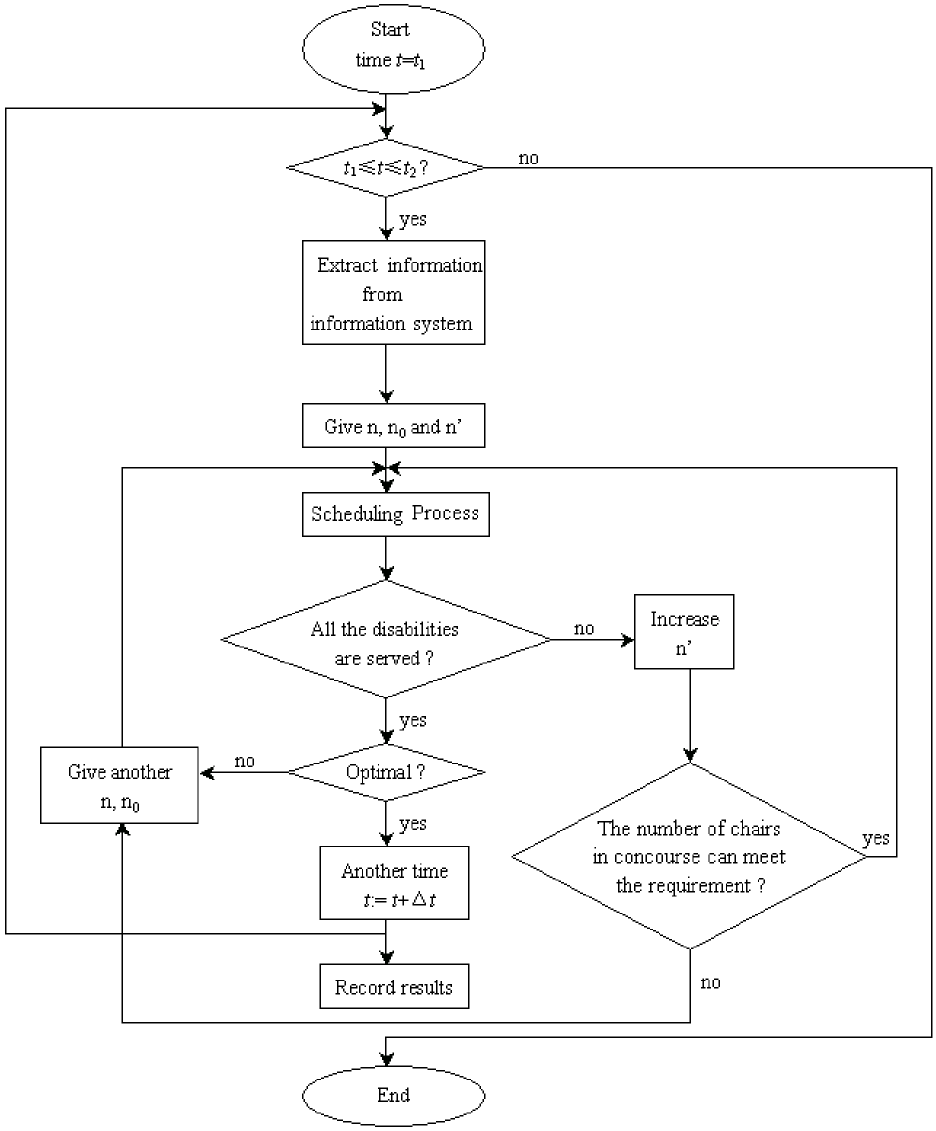

Fig.12 The schematic depicting the steps occurring in the Scheduling Model

Where n is the number of wheel chairs

n_0 is the number of escorts

n' is times that the given number of wheel chairs on the concourse level transferred to the arrival level at unit interval

Model Results

- **Real-time simulation the process of flights' arriving**

According to the actual data of SYDNEY AIRPORT [9], we estimate the parameter λ =*42 min* for the time-interval of airplanes arrival obeying an exponential distribution.

✧ The simulation gives the number of disabilities arrival at specific time for different types of airdromes (large airport, medium airport and small airport). Figures of the simulation are shown in **Figure.13~15**

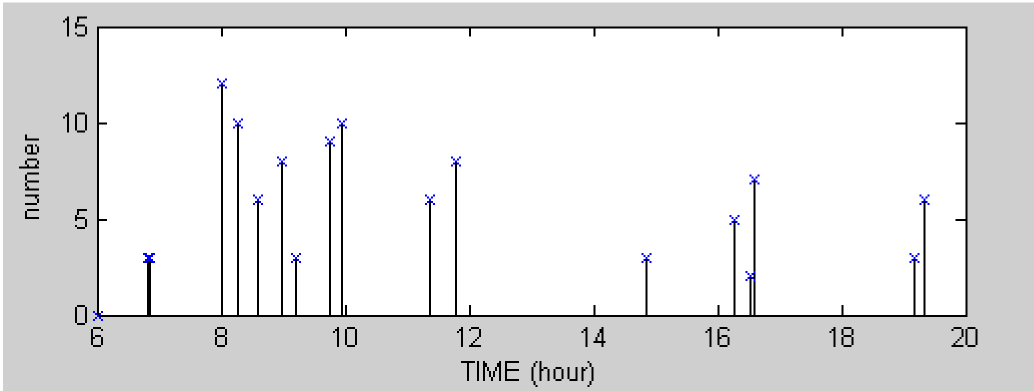

Fig.13 The figure of the number of disabilities arrival for small airport

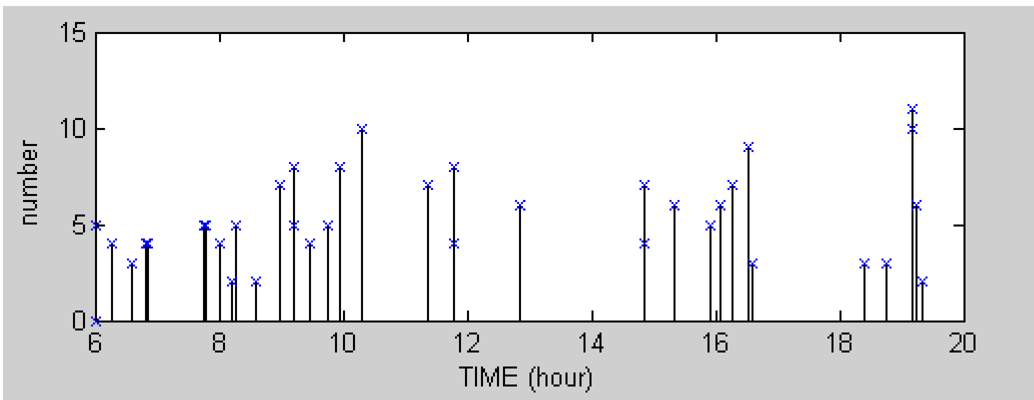

Fig.14 The figure of the number of disabilities arrival for medium airport

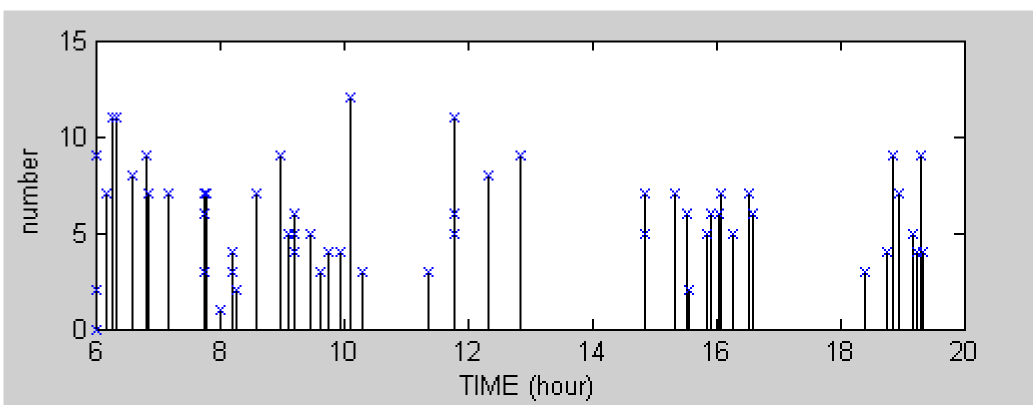

Fig.15 The figure of the number of disabilities arrival for large airport

✧ Analyzing the figure of the number of disabilities, we obtain the distribution of disabilities flow on the different sections of time-interval. The figures of distribution for different types of airdromes are shown as follows:

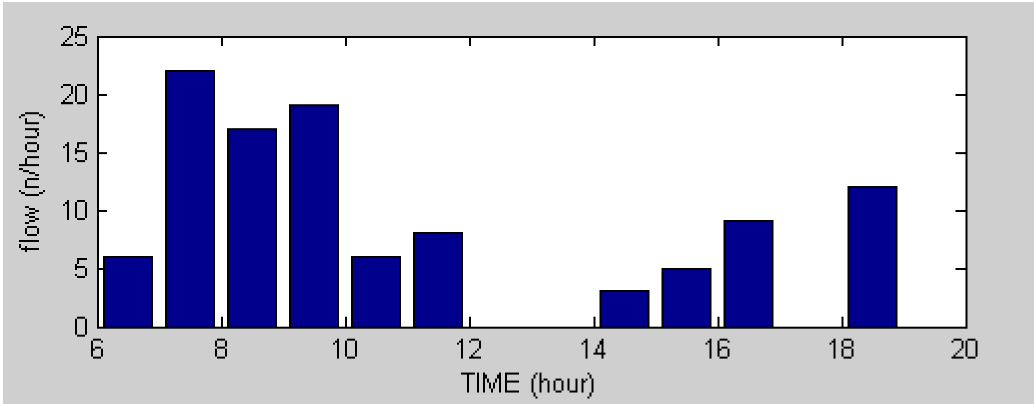

Fig.16 The figure of the distribution of disabilities flow for small airport

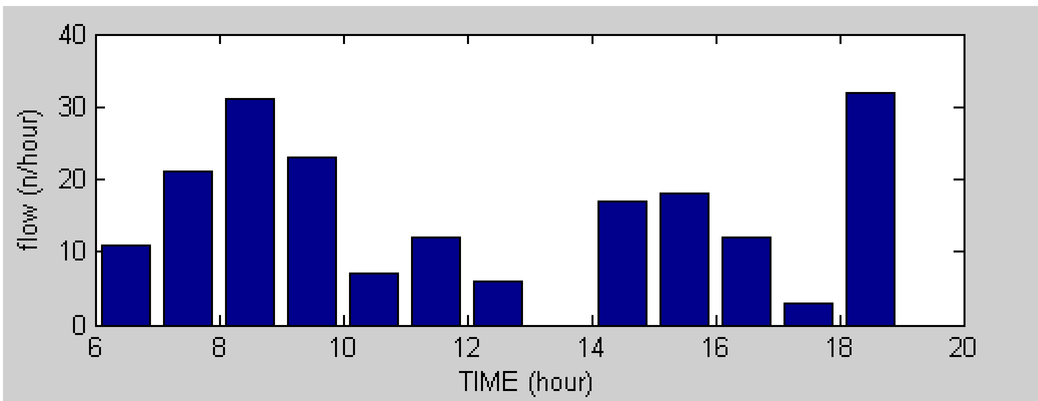

Fig.17 The figure of the distribution of disabilities flow for medium airport

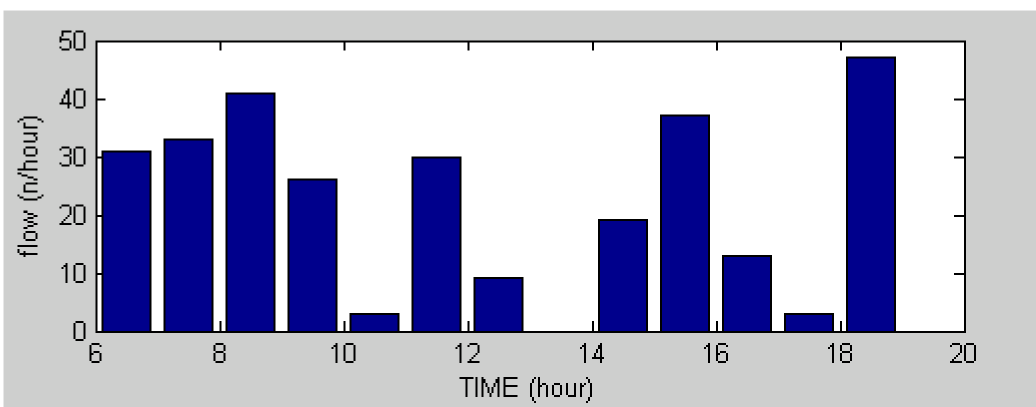

Fig.18 The figure of the distribution of disabilities flow for large airport

From the above-figures, we can obtain the disabilities flow at peak time and at low time.

- **Simulating the entire working dispatch in a day**

For different types of airdromes, by scheduling algorithm for wheel chairs and escorts we obtain the number of wheel chairs and escorts in different time-interval during a day. The table is shown as below:

Table2. The optimal number of wheel chairs and escorts in different time-interval for different types of airdromes

Time (h)	Small Airport		Medium Airport		Large Airport	
	Wheelchairs	Escorts	Wheelchairs	Escorts	Wheelchairs	Escorts
6:00~7:00	4	4	13	11	69	57
7:00~8:00	13	11	24	19	74	51
8:00~9:00	10	9	35	30	92	55
9:00~10:00	11	9	26	22	58	36
10:00~11:00	4	3	8	4	7	6
11:00~12:00	5	5	14	11	67	35
12:00~13:00	0	0	7	6	20	14
13:00~14:00	0	0	0	0	0	0
14:00~15:00	2	3	19	16	43	23
15:00~16:00	3	2	20	16	83	48
16:00~17:00	5	3	14	11	29	26
17:00~18:00	0	0	4	2	7	1
18:00~19:00	7	6	36	20	105	68
19:00~20:00	0	0	0	0	0	0

- **Budget Costs**

From different types of airdromes perspective, the budget costs is shown as follows:

Table 3.

Costs for different types of airports

	f_1 (dollars/year)	f_2 (dollars/year)	f_3 (dollars/year)	f_4 (dollars/year)	f_5 (dollars/year)	f (dollars/year)
Small airport	780	14, 000	45, 760	0	0	60, 540
Medium airport	2, 160	42, 000	126, 720	0	0	170, 880
Large airport	6, 300	105, 000	369, 600	0	0	480, 900

Why our cost is the lowest?

Firstly, in our scheduling process, we simulate the real-time scheduling, to find the indispensable number of escorts and wheel chairs. Thus, we can almost avoid holding a plane. That is to say, event { f_5 > 0} is a small probability event.

Secondly, if hurting some customers, which causes high costs. In our work, we consider place in some lower traffic areas to avoid liability risk, so f_4 is very small.

Thus, f_1, f_2, f_3 make mainly up of the total costs, which are indispensable. By the real-time scheduling algorithm, we can find the optimal number of escorts and wheel chairs. i.e. f_1, f_2, f_3 are optimal.

Projects In the Future

In the future, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid. According to United Nation projections, by the year 2025, there are over 50 percent of people over 60 years of age have a disability in Western Australia. Therefore, we can convert older people to disabilities according to specific ratio to the parameters.

We conclude providing the needs for disabilities by airlines in the future:

- Airlines provide guides for people with disabilities
- Airlines provide medical for the disabilities. And the equipment that may not be brought on board is an oxygen supply. Passengers may use the aircraft's supply and a reasonable charge may be assessed.
- Airlines provide boarding assistance to passengers with disabilities by providing a mechanical lift or other device.
- Airlines must allow passengers with disabilities to store canes and other assistive devices close to their seats and cannot count this equipment toward the limit on the number of carry-on items and space in the airplane.
- On request, airlines must make available to any disabilities the following information about the aircraft:

✧ The location of seats with movable aisle armrests
✧ Any limitations on the airplane's ability to accommodate passengers with disabilities
✧ Any limitations on the availability of storage facilities for assistive devices
✧ Whether the airplane has an accessible lavatory

According to above-needs items, we give some suggestions to meet future needs. From Epsilon airlines perspective, they should adopt reasonable scheduling effective facilities and staffs, which are mentioned above in our models, to keep their costs down. And considering each part the potential costs which are generated by airlines' providing, we similarly borrow ideals from theory of greedy method to minimize every part costs. By minimizing separate costs of each part, we obtain the minimal total costs and scheduling the process of serving disabilities. It can meet future needs.

Strengths

✧ The model does a good job to simulate the real-time process of scheduling wheel chairs after airplane arriving.
✧ The scheduling algorithm can work a wide range of airports under a variety of circumstances. And the program is flexible enough to work for any size airdromes; it not specific to the individual routes.
✧ We adopt very skilled methods to obtain airplane arriving time-interval distribution; sequentially we obtain people flow distribution.
✧ The optimization algorithm runs very quickly.

Weaknesses

✧ The greatest weakness of this model is that it assumes that disabilities who ask for wheel chairs must notify the airline in advance.
✧ We only construct the scheduling algorithm to approximately estimate the costs and the number of wheel chairs and escorts.

Improvements in the Model

Our models need plentiful statistic data. With precise size of airport, the number of passengers, the rent-fee and so on, we could apply them into practical situation.

With sufficient information, we could use these models to create a simulation of a scheduling process. Future more, we could compute more practical costs.

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Appendix

```
clear all;
n=input('the type of the airfield:');
t_arr(1,:)=[50 2 69 15 19 24 14 33 12 85 25 183 85 15 5 155 0 10];
t_arr(2,:)=[16 20 70 2 24 60 0 16 50 90 64 120 28 35 10 140 20 30];
t_arr(3,:)=[10 10 50 35 1 26 54 32 29 100 34 190 3 18 10 170 6 20];
%Time interval of airplanes at different airports
for j=1:3
```

```
    tt(j,1)=6;
    for kk=2:19
        tt(j,kk)=tt(j,kk-1)+t_arr(j,kk-1)/60;
    end
end
    if n==1
        T(1,:)=tt(1,:);
    elseif n==2
T(1,:)=[tt(1,:),tt(2,:)];
else
    T(1,:)=[tt(1,:),tt(2,:),tt(3,:)];
end
T(1,:)=sort(T(1,:));
    for k=2:length(T(1,:))
        r=randperm(100);
        if find(r==1)<35 % The probability of big-planes
    ko=randperm(30);
    T(2,k)=0;
    for nm=1:(ko(20)+110);
  ko1=randperm(20); % The probability of the people who need wheelchairs
        if ko1(4)==1
            T(2,k)=T(2,k)+1;
        end
    end
else
    ko=randperm(20);
    T(2,k)=0;
     for nm=1:(ko(20)+90);
 ko1=randperm(20);% The probability of the people who need wheelchairs
        if ko1(4)==1
            T(2,k)=T(2,k)+1;
        end
    end
end
end
T(2,1)=0;
subplot(2,1,1)
bar(T(1,:),T(2,:))
hold on;
plot(T(1,:),T(2,:),'x')
xlabel('TIME (hour)');   ylabel('number ');
% The first line of the matrixT shows the arrival time of the planes
% The second line of T shows the number of disabilities at the very time
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ddt=1;ts_n=1;
for ts=6.5:ddt:20
 sum=0;
    tr=find(T(1,:)>=ts&T(1,:)<ts+ddt);
    if length(tr)==0
         Q_av(ts_n)=0;
    else
         for jk=1:length(tr)
sum=sum+T(2,tr(jk));
end
Q_av(ts_n)=sum/ddt;
end
ts_n=ts_n+1;
end
subplot(2,1,2)
ts2=6.5:ddt:20;
bar(ts2,Q_av)
xlabel('TIME (hour)');   ylabel('flow (n/hour)');
%%%%%%%%%%%%% Flow of disabilities at different time quantum
v1=0.5*3600;v2=0.8*3600;x=5;
if n==1
    l1=500;
else
    l1=(n-1)*2*500;
end
    for j=1:length(Q_av)
    m1=0;
    if Q_av(j)~=0
for hj=1:ceil(Q_av(j))
    xc=randperm(2);
    if xc(2)==2
        m1=m1+1;
    end
end
end
n_wheels(j)=ceil(2*l1*Q_av(j)/v1);
n_escorts(j)=ceil(2*m1*l1/v1+2*Q_av(j)*v1/(v2*x));
end   %%%%%%%% Simplified scheduling
n_wheels
n_escorts %%%%%%%% Show the results
```