Modification of the DSN Radio Frequency Angular Tropospheric Refraction Model

A. L. Berman
TDA Engineering Office

The previously derived DSN Radio Frequency Angular Tropospheric Refraction Model contained an assumption which was subsequently seen to be at a variance with the theoretical basis of angular refraction. The modification necessary to correct the model is minor in that the value of a constant is changed. The accuracy of the modified model is now considered to be:

Elevation angle range	*Maximum error*
90 deg ⩾ el ⩾ 5 deg	*0.003 deg*
5 deg ⩾ el ⩾ -3 deg	*0.010 deg*

I. Modification of Refraction Model

In References 1 and 2, an assumption was made that is at a variance with the theoretical basis[1] of angular refraction. It was assumed in translating from optical frequency to radio frequency that (as in range refraction):

$$\text{refraction} \propto \int N(h)\, dh$$

where:

$N(h)$ = total refractivity at radio frequencies

h = height

[1] Private communication with V. J. Slabinski, COMSAT Corporation.

so that:

$$R_{RF}(P,T,Z,RH) \approx R_{OP}(P,T,Z) \left\{1 + \frac{\int NW(h)\, dh}{\int ND(h)\, dh}\right\}$$

where:

$R_{OP} = R_{OP}(P,T,Z)$ = optical refraction model

$R_{RF} = R_{RF}(P,T,Z,RH)$ = radio frequency refraction model

P = pressure

T = temperature

Z = zenith angle

RH = relative humidity

$N(h) = ND(h) + NW(h)$

$ND(h)$ = dry, or optical component, of refractivity

$NW(h)$ = wet component of refractivity

However, in fact, one has for angular refraction (in *contrast* to range refraction):

$$\text{refraction} \propto \int dN\,(h)$$

so that:

$$R_{RF}(P,T,Z,RH) \approx R_{OP}(P,T,Z)\left\{1 + \frac{NW_s}{ND_s}\right\}$$

Because it was empirically determined that:

$$\frac{\int_0^{\infty} NW(h)\,dh}{\int_0^{\infty} ND(h)\,dh} \approx 0.3224\left(\frac{NW_s}{ND_s}\right)$$

s = parameter surface value

the correction of the model requires a change in only one constant.

The full modified model is:

$$R = F_p F_t F_w \left(\exp\left\{\frac{\sum_{j=0}^{8} K_{j+3}\,[U(Z)]^j}{1 + \Delta_3(Z)}\right\} - K_{12}\right)$$

$$F_p = \left(\frac{P}{P_0}\left\{1 - \frac{\Delta_1(P,Z)}{1 + \Delta_3(Z)}\right\}\right)$$

$$F_t = \left(\frac{T_0}{T}\left\{1 - \frac{\Delta_2(T,Z)}{1 + \Delta_3(Z)}\right\}\right)$$

$$F_w = \left(1 + \frac{W_0 RH}{TP}\left\{\exp\left[\frac{W_1 T - W_2}{T - W_3}\right]\right\}\right)$$

$$\Delta_1(P,Z) = (P - P_0)\ \{\exp\,[A_1(Z - A_2)]\}$$

$$\Delta_2(T,Z) = (T - T_0)\ \{\exp\,[B_1(Z - B_2)]\}$$

$$\Delta_3(Z) = (Z - C_0)\ \{\exp\,[C_1(Z - C_2)]\}$$

where

R = refraction, s

Z = actual zenith angle, deg

EL = elevation angle

EL = 90 deg − Z

$U(Z) = \left\{\frac{Z - K_1}{K_2}\right\}$

K_1 = 46.625

K_2 = 45.375

K_3 = 4.1572

K_4 = 1.4468

K_5 = 0.25391

K_6 = 2.2716

K_7 = −1.3465

K_8 = −4.3877

K_9 = 3.1484

K_{10} = 4.5201

K_{11} = −1.8982

K_{12} = 0.89000

P = pressure, mm Hg

P_0 = 760.00 mm Hg

A_1 = 0.40816

A_2 = 112.30

T = temperature, K

T_0 = 273.00 K

B_1 = 0.12820

B_2 = 142.88

C_0 = 91.870

C_1 = 0.80000

C_2 = 99.344

RH = Relative humidity (100% = 1.0)

$W_0 = 2.2 \times 10^4$

$W_1 = 17.149$

$W_2 = 4684.1$

$W_3 = 38.450$

Note that only the value of W_0 has changed, from

$$W_0 = 7.1 \times 10^3$$

to:

$$W_0 = \frac{7.1 \times 10^3}{0.3224} = 2.2 \times 10^4$$

II. Evaluation of Modified Model Accuracy

The major source of inaccuracy was a 1%, 1σ uncertainty due to the approximation of the ratio of integrated wet refractivity to integrated dry refractivity by 0.3224 times the ratio of surface wet refractivity to surface dry refractivity. Thus the modification of the model eliminates the known 1% uncertainty in the wet refractivity term, as well as a systematic, and (previously) unknown error which ranged from 2 to 20%. The accuracy of the modified radio frequency angular tropospheric refraction model is now considered to be of the same order as the optical angular tropospheric refraction model presented in Table 2 of Ref. 2, but enlarged by 50% to account for the increased size of radio frequency refraction when compared to optical refraction:

Zenith angle range	Maximum error, deg
0 deg $\leqslant$ zenith $\leqslant$ 85 deg	0.003 deg
85 deg $\leqslant$ zenith $\leqslant$ 93 deg	0.010 deg

References

1. Berman, A. L., and Rockwell, S. T., "A Proposal for a New Radio Frequency Angular Tropospheric Refraction Model For Use Within the DSN," in *The Deep Space Network Progress Report 42-25,* pp. 142-153. Jet Propulsion Laboratory, Pasadena, California, February 15, 1975.

2. Berman, A. L., and Rockwell, S. T., *New Optical and Radio Frequency Angular Tropospheric Refraction Models for Deep Space Applications,* Technical Report 32-1601, Jet Propulsion Laboratory, Pasadena, California, November 1, 1975.