**Summary of the MacCamy-Fuchs Model for Wave Diffraction Loads**

**Theoretical Background**

The linear potential-flow solution of wave diffraction by a vertical surface-piercing circular cylinder is presented first. Consider a bottom-mounted surface-piercing circular cylinder of radius centered at . The potential-flow solution of the diffraction problem with a regular long-crested incident wave in the -direction of amplitude and angular frequency is given by

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where is the scattered potential with contributions from the incident wave potential and the diffracted potential . The unit-amplitude incident potential is of the form

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where is the wave number. The flow velocity and acceleration in the -direction (the direction of wave propagation) from the unobstructed incident waves are

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At , the center of the cylinder, the flow acceleration is

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The unit-amplitude scattered potential on the surface of the cylinder in a cylindrical coordinate system is given by

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where when and 2 when . is the derivative of the Hankel function of the first kind. The diffraction load in direction is given by

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and the unit-amplitude wave diffraction load is

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where is the th component of the body surface normal pointing out of the fluid. For the transverse load in the -direction, for a circular cylinder. Only the cosine mode () of Eq. (6) has a net contribution to . The sectional transverse load, , is given by

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The exact sectional diffraction load is given by

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It can be shown that Eq. (10) is equivalent to Eq. (7) of MacCamy and Fuchs [1]. Assuming the sectional diffraction load is of the form of the acceleration term in the Morison equation, we have

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where is the effective mass coefficient. Matching Eq. (10) and Eq. (11), we can derive the following expression for the effective mass coefficient:

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The magnitude and phase of are shown in Figure 1.



**Figure 1**. Magnitude and phase of the exact effective mass coefficient, .

When , the phase of is close to zero; the transverse diffraction load is mostly in phase with the flow acceleration. The diffraction load is small when ; therefore, we can approximate the diffraction load as being in phase with the flow acceleration, thus rendering a real number:

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where

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and and are the derivatives of the Bessel functions of the first and second kind, respectively.

In irregular waves, we have

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where is the flow acceleration associated with the incident wave component at angular frequency . The effective mass coefficient is a function of the product of the wave number associated with the angular frequency and the column radius , i.e., the nondimensional column radius.

**Implementation in OpenFAST**

In OpenFAST, the user typically specifies a constant added mass coefficient, , and a constant pressure coefficient (associated with the pressure gradient in an unsteady flow field), (default to 1), which leads to a constant effective mass coefficient equal to . With, the MacCamy-Fuchs diffraction model, the effective mass coefficient based on the user input will be replaced by given by Eq. (13). Therefore, it is no longer necessary for the user to specify , which is only used when computing the fluid-inertia forces proportional to the fluid acceleration. Note that the user should still specify , which is used to compute the added-mass force proportional to the acceleration of the structure.

In the future implementation of HydroDyn, the flow acceleration will be evaluated at the body-exact position. Since the body position is generally not known a priori, the fluid-inertia forces cannot be evaluated beforehand and will have to be computed during the simulation using the instantaneous flow-acceleration field. To conform to this new framework, it is convenient to absorb into the flow acceleration to obtain the modified flow acceleration, , which can be precomputed by SeaState:

|  |  |  |
| --- | --- | --- |
|  |  | (16) |
|  |  | (17) |

Since also depends on the member radius (or diameter), we potentially need to compute and store a separate grid of for each member with MacCamy-Fuchs correction enabled because they can have different diameters. This will likely result in excessive memory use; therefore, in the OpenFAST implementation, we will require all members with the MacCamy-Fuchs model enabled to have the same diameter. In this fashion, we will only need to compute and store one additional grid for the scaled flow acceleration, , in addition to the grid for the unscaled flow acceleration, , which will still be required to evaluate the fluid-inertia forces on members without the MacCamy-Fuchs model selected and to evaluate axial fluid-inertia forces at the end joints of members.

Strictly speaking, the MacCamy-Fuchs correction is only valid for deep-drafted vertical surface-piercing circular cylinders of constant diameter. We will try to limit the members eligible for the MacCamy-Fuchs correction to this category. See the next section for details.

For a member with constant diameter, the exact expression for the transverse fluid-inertia force used in the HydroDyn is [2] (Section 7.5.1.1.1)

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

With the MacCamy-Fuchs correction, we simply need to replace the term, , with

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

The above vector expression is also readily compatible with directional wave spreading.

**Modifications to the SeaState/HydroDyn Input File and HydroDyn Behavior**

In the SeaState input file, a new input parameter *MCFD* is added. The diameter of the members to use the MacCamy-Fuchs diffraction model should be specified here. This information is needed for SeaState to generate the grid for based on Eq. (19). Naturally, only one value can be specified for *MCFD*. SeaState will skip the evaluation of if *MCFD* ≤ 0, in which case the MacCamy-Fuchs diffraction model cannot be used with any member in the HydroDyn input file.

To minimize the changes to the HydroDyn input file, the user will be asked to supply the keyword “MCF” instead of a value to all entries associated with the members for which the MacCamy-Fuchs diffraction model is to be used. More specifically, the entries that need to be set to “MCF” include

1. *SimplCp* and *SimplCpMG* if *MCoefMod* = 1,
2. *DpthCp* and *DpthCpMG* if *MCoefMod* = 2, and
3. *MemberCp1*, *MemberCp2*, *MemberCpMG1*, and *MemberCpMG2* if *MCoefMod* = 3.

All other hydrodynamic coefficients, including the added mass coefficient, , can be specified as usual.

To ensure that the MacCamy-Fuchs diffraction solution is approximately applicable, for each member with the MacCamy-Fuchs model enabled, HydroDyn will

1. check if the inclination of the member is within 10 degrees from vertical,
2. check if it is surface piercing with one end joint of the member above SWL and the other below,
3. check if the member is deep drafted with the initial draft being at least as large as the radius, and
4. check if the member has a constant diameter (including the thickness of the marine growth if any) equal to *MCFD* in the SeaState input file.

If any of the above condition is not met, HydroDyn will abort and provide a suitable error message. Finally, HydroDyn will simply ignore the “MCF” keywork if used when *PropPot* = TRUE and skip the above checks.

**Restrictions and Compatibilities**

The MacCamy-Fuchs diffraction model is not used when *WaveMod* = 0 and incompatible with *WaveMod* = 6 (user-defined full wave kinematics). If WaveMod = 0 or 6, *MCFD* in the SeaState input file must be less than or equal to zero. The correction also supports no or any of the available wave-stretching methods. Finally, because the MacCamy-Fuchs model is derived based on the linear wave theory, second-order fluid acceleration (sum and different frequencies) will be ignored when computing the transverse fluid-inertia forces for members with the MacCamy-Fuchs model enabled. All other hydrodynamic loads are still computed from the total first- plus second-order wave kinematics if selected.

**References**

[1] MacCamy, R.C., Fuchs, R.A., *Wave Forces on Piles: A Diffraction Theory*, Technical Memorandum No. 69, US Army Corps of Engineers, Beach Erosion Board, 1954.

[2] Jonkman, J.M., Robertson, A.N., Hayman, G.J., *HydroDyn User’s Guide and Theory Manual* (draft), National Renewable Energy Laboratory: Golden, CO, USA, 2015. Available from: <https://www.nrel.gov/wind/nwtc/assets/downloads/HydroDyn/HydroDyn_Manual.pdf>.