Finite Element Analysis of Rectangular Slab with Calcpad

|  |  |
| --- | --- |
| **Input data**  Slab dimensions - m, m  Thickness - m  Load - kN/m²  Modulus of elasticity - MPa  Poisson`s ratio - |  |

Finite element mesh

We will use rectangular finite element with DOFs

Number of elements along *a* and *b* - ,

Total number of elements -

Total number of joints -

Element dimensions - ,

Supported joints count -

Joint coordinates

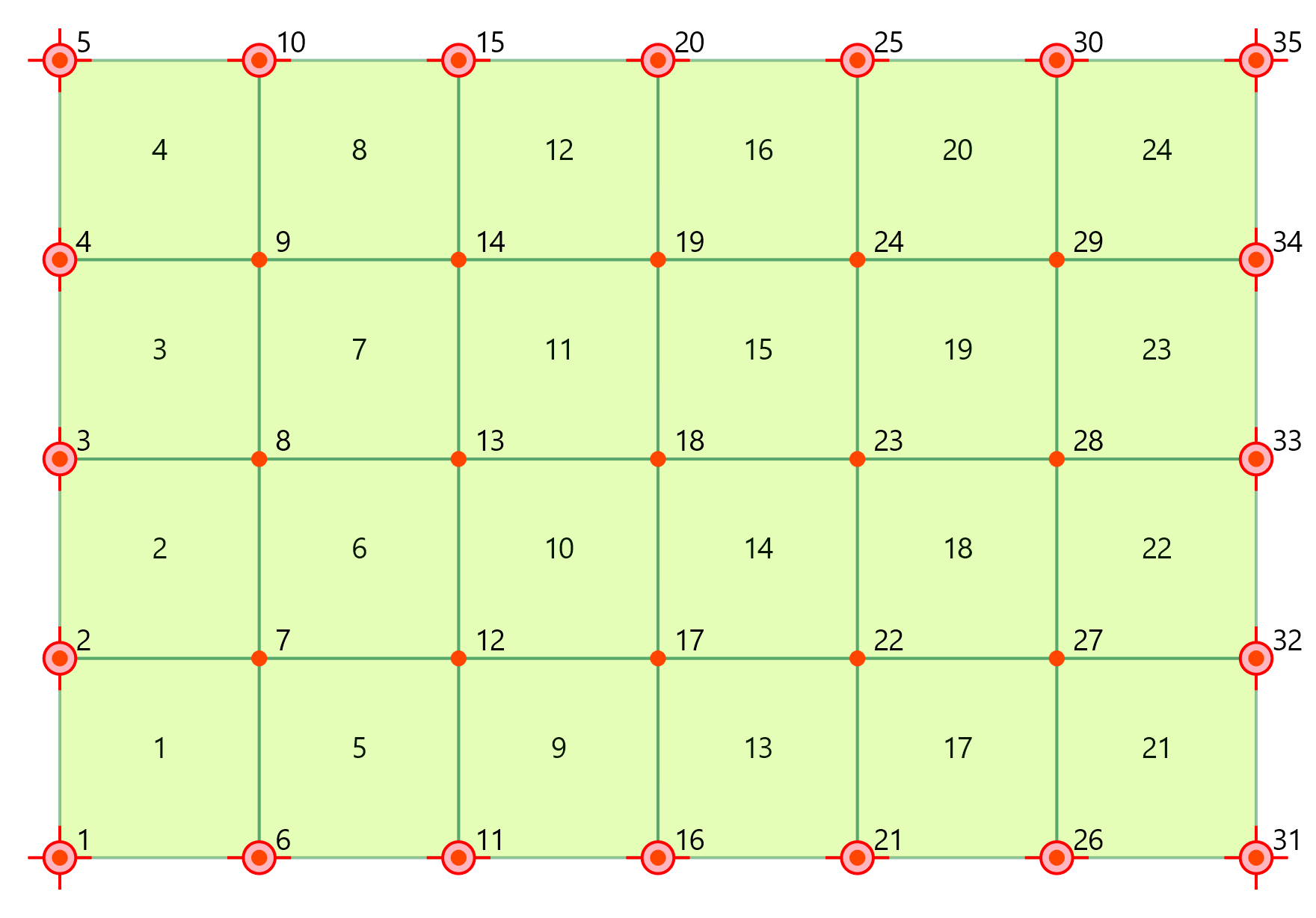
m

m

Numbers of elements joints

Supported joints

Coordinates of elements centers



Finite element formulation

**Shape functions**

**Along dimension *a***

|  |  |  |
| --- | --- | --- |
| Base functions | First derivatives | Second derivatives |

**Along dimension *b***

|  |  |  |
| --- | --- | --- |
| Base functions | First derivatives | Second derivatives |
| For vertical displacements  *w* | For rotations *θ*ₓ | For rotations *θ*ᵧ |

A diagram of a triangle with lines and points

Description automatically generatedFor twist ψ

Shape functions vector

**Constitutive matrix** (stress - strain relationship)

**Strain-displacement matrix**

,

The elements of the stiffness matrix will be calculated by using the equation

**Element stiffness matrix**

(above the main diagonal only)

Element load vector

Solution

Global stiffness matrix

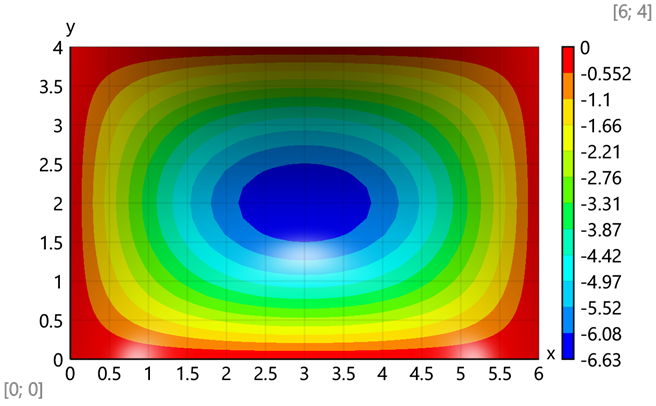
Global load vector

Solution of the system of equations

Results

Joint displacements

mm



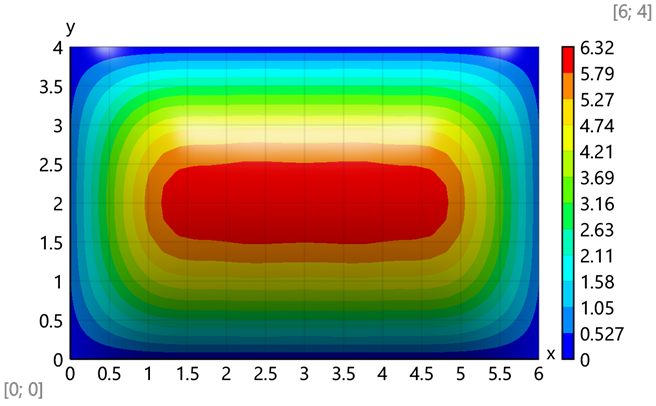
Bending moments

Results for element 15 and joint 18:

Average bending moments at joints, kNm/m -

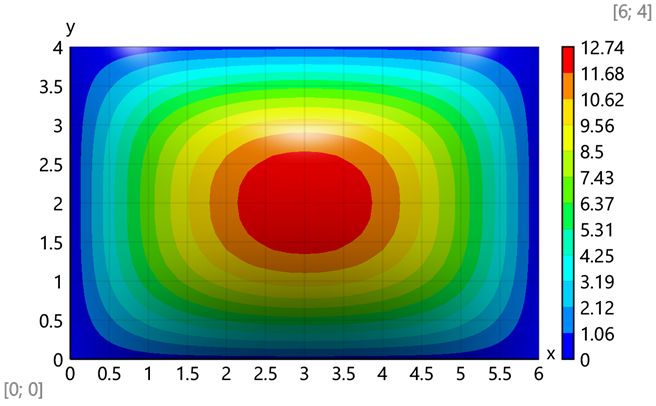
Bending moments for the plate

Bending moments - *Mx*



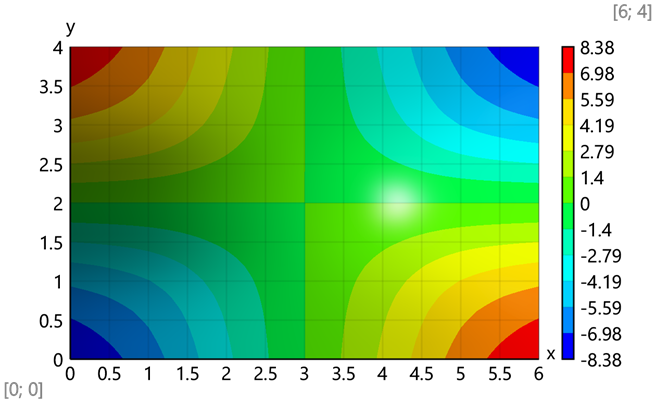
Maximal value - kNm/m

Bending moments - *M*y



Maximal value - kNm/m

Bending moments - *M*xy



Maximal value - kNm/m

**Solution with SAP 2000 structural analysis software**

Input data

A red grid with yellow dots and numbers

Description automatically generated

S T A T I C L O A D C A S E S

STATIC CASE SELF WT

CASE TYPE FACTOR

LOAD1 DEAD 0.0000

J O I N T D A T A

JOINT GLOBAL-X GLOBAL-Y GLOBAL-Z RESTRAINTS ANGLE-A ANGLE-B ANGLE-C

1 -3.00000 -2.00000 0.00000 0 0 1 1 1 0 0.000 0.000 0.000

2 -3.00000 -1.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

3 -3.00000 0.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

4 -3.00000 1.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

5 -3.00000 2.00000 0.00000 0 0 1 1 1 0 0.000 0.000 0.000

6 -2.00000 -2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

7 -2.00000 -1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

8 -2.00000 0.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

9 -2.00000 1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

10 -2.00000 2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

11 -1.00000 -2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

12 -1.00000 -1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

13 -1.00000 0.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

14 -1.00000 1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

15 -1.00000 2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

16 0.00000 -2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

17 0.00000 -1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

18 0.00000 0.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

19 0.00000 1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

20 0.00000 2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

21 1.00000 -2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

22 1.00000 -1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

23 1.00000 0.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

24 1.00000 1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

25 1.00000 2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

26 2.00000 -2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

27 2.00000 -1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

28 2.00000 0.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

29 2.00000 1.00000 0.00000 0 0 0 0 0 0 0.000 0.000 0.000

30 2.00000 2.00000 0.00000 0 0 1 0 1 0 0.000 0.000 0.000

31 3.00000 -2.00000 0.00000 0 0 1 1 1 0 0.000 0.000 0.000

32 3.00000 -1.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

33 3.00000 0.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

34 3.00000 1.00000 0.00000 0 0 1 1 0 0 0.000 0.000 0.000

35 3.00000 2.00000 0.00000 0 0 1 1 1 0 0.000 0.000 0.000

S H E L L E L E M E N T D A T A

SHELL JNT-1 JNT-2 JNT-3 JNT-4 SECTION ANGLE AREA

1 1 6 2 7 SSEC1 0.000 1.000

2 2 7 3 8 SSEC1 0.000 1.000

3 3 8 4 9 SSEC1 0.000 1.000

4 4 9 5 10 SSEC1 0.000 1.000

5 6 11 7 12 SSEC1 0.000 1.000

6 7 12 8 13 SSEC1 0.000 1.000

7 8 13 9 14 SSEC1 0.000 1.000

8 9 14 10 15 SSEC1 0.000 1.000

9 11 16 12 17 SSEC1 0.000 1.000

10 12 17 13 18 SSEC1 0.000 1.000

11 13 18 14 19 SSEC1 0.000 1.000

12 14 19 15 20 SSEC1 0.000 1.000

13 16 21 17 22 SSEC1 0.000 1.000

14 17 22 18 23 SSEC1 0.000 1.000

15 18 23 19 24 SSEC1 0.000 1.000

16 19 24 20 25 SSEC1 0.000 1.000

17 21 26 22 27 SSEC1 0.000 1.000

18 22 27 23 28 SSEC1 0.000 1.000

19 23 28 24 29 SSEC1 0.000 1.000

20 24 29 25 30 SSEC1 0.000 1.000

21 26 31 27 32 SSEC1 0.000 1.000

22 27 32 28 33 SSEC1 0.000 1.000

23 28 33 29 34 SSEC1 0.000 1.000

24 29 34 30 35 SSEC1 0.000 1.000

S H E L L U N I F O R M L O A D S Load Case LOAD1

SHELL DIRECTION VALUE

1 GLOBAL-Z -10.0000

2 GLOBAL-Z -10.0000

3 GLOBAL-Z -10.0000

4 GLOBAL-Z -10.0000

5 GLOBAL-Z -10.0000

6 GLOBAL-Z -10.0000

7 GLOBAL-Z -10.0000

8 GLOBAL-Z -10.0000

9 GLOBAL-Z -10.0000

10 GLOBAL-Z -10.0000

11 GLOBAL-Z -10.0000

12 GLOBAL-Z -10.0000

13 GLOBAL-Z -10.0000

14 GLOBAL-Z -10.0000

15 GLOBAL-Z -10.0000

16 GLOBAL-Z -10.0000

17 GLOBAL-Z -10.0000

18 GLOBAL-Z -10.0000

19 GLOBAL-Z -10.0000

20 GLOBAL-Z -10.0000

21 GLOBAL-Z -10.0000

22 GLOBAL-Z -10.0000

23 GLOBAL-Z -10.0000

24 GLOBAL-Z -10.0000

Results

Displacements, mm

A screenshot of a computer generated image

Description automatically generated

Bending moments - M11, kNm/m

A diagram of a graph

Description automatically generated with medium confidenceA screenshot of a computer

Description automatically generated

Bending moments - M22, kNm/m

A colorful chart with numbers

Description automatically generated with medium confidenceA screenshot of a computer

Description automatically generated

Bending moments - M12, kNm/m

A colorful grid with numbers

Description automatically generatedA screenshot of a computer

Description automatically generated

J O I N T D I S P L A C E M E N T S

JOINT LOAD U3 R1 R2

1 LOAD1 0.0000 0.0000 0.0000

2 LOAD1 0.0000 0.0000 2.685E-03

3 LOAD1 0.0000 0.0000 3.743E-03

4 LOAD1 0.0000 0.0000 2.685E-03

5 LOAD1 0.0000 0.0000 0.0000

6 LOAD1 0.0000 -2.797E-03 0.0000

7 LOAD1 -2.509E-03 -1.927E-03 2.151E-03

8 LOAD1 -3.503E-03 0.0000 3.016E-03

9 LOAD1 -2.509E-03 1.927E-03 2.151E-03

10 LOAD1 0.0000 2.797E-03 0.0000

11 LOAD1 0.0000 -4.588E-03 0.0000

12 LOAD1 -4.122E-03 -3.184E-03 1.070E-03

13 LOAD1 -5.777E-03 0.0000 1.514E-03

14 LOAD1 -4.122E-03 3.184E-03 1.070E-03

15 LOAD1 0.0000 4.588E-03 0.0000

16 LOAD1 0.0000 -5.176E-03 0.0000

17 LOAD1 -4.653E-03 -3.600E-03 0.0000

18 LOAD1 **-6.529E-03** 0.0000 0.0000

19 LOAD1 -4.653E-03 3.600E-03 0.0000

20 LOAD1 0.0000 5.176E-03 0.0000

21 LOAD1 0.0000 -4.588E-03 0.0000

22 LOAD1 -4.122E-03 -3.184E-03 -1.070E-03

23 LOAD1 -5.777E-03 0.0000 -1.514E-03

24 LOAD1 -4.122E-03 3.184E-03 -1.070E-03

25 LOAD1 0.0000 4.588E-03 0.0000

26 LOAD1 0.0000 -2.797E-03 0.0000

27 LOAD1 -2.509E-03 -1.927E-03 -2.151E-03

28 LOAD1 -3.503E-03 0.0000 -3.016E-03

29 LOAD1 -2.509E-03 1.927E-03 -2.151E-03

30 LOAD1 0.0000 2.797E-03 0.0000

31 LOAD1 0.0000 0.0000 0.0000

32 LOAD1 0.0000 0.0000 -2.685E-03

33 LOAD1 0.0000 0.0000 -3.743E-03

34 LOAD1 0.0000 0.0000 -2.685E-03

35 LOAD1 0.0000 0.0000 0.0000

S H E L L E L E M E N T R E S U L T A N T S

SHELL LOAD JOINT M11 M22 M12 V13 V23

1 LOAD1

1 0.00 0.00 **-7.25** -1.10 -6.699E-01

6 -6.603E-03 -4.402E-02 -6.58 -1.10 -6.44

2 -4.402E-02 -6.603E-03 -6.15 -5.16 -6.699E-01

7 4.01 5.72 -5.47 -5.16 -6.44

2 LOAD1

2 -4.402E-02 -6.603E-03 -3.87 -6.49 -2.463E-01

7 4.01 5.68 -3.62 -6.49 -1.52

3 -3.938E-02 -5.907E-03 -1.42 -7.81 -2.463E-01

8 5.32 6.96 -1.18 -7.81 -1.52

3 LOAD1

3 -3.938E-02 -5.907E-03 1.42 -7.81 2.463E-01

8 5.32 6.96 1.18 -7.81 1.52

4 -4.402E-02 -6.603E-03 3.87 -6.49 2.463E-01

9 4.01 5.68 3.62 -6.49 1.52

4 LOAD1

4 -4.402E-02 -6.603E-03 6.15 -5.16 6.699E-01

9 4.01 5.72 5.47 -5.16 6.44

5 0.00 0.00 7.25 -1.10 6.699E-01

10 -6.603E-03 -4.402E-02 6.58 -1.10 6.44

5 LOAD1

6 -6.603E-03 -4.402E-02 -5.11 -6.776E-01 -7.13

11 -5.610E-03 -3.740E-02 -3.74 -6.776E-01 -10.31

7 3.98 5.72 -4.44 -1.22 -7.13

12 4.52 8.90 -3.07 -1.22 -10.31

6 LOAD1

7 3.97 5.68 -2.74 -2.14 -1.81

12 4.51 8.85 -2.21 -2.14 -3.02

8 5.27 6.95 -1.15 -2.56 -1.81

13 6.23 11.33 -6.135E-01 -2.56 -3.02

7 LOAD1

8 5.27 6.95 1.15 -2.56 1.81

13 6.23 11.33 6.135E-01 -2.56 3.02

9 3.97 5.68 2.74 -2.14 1.81

14 4.51 8.85 2.21 -2.14 3.02

8 LOAD1

9 3.98 5.72 4.44 -1.22 7.13

14 4.52 8.90 3.07 -1.22 10.31

10 -6.603E-03 -4.402E-02 5.11 -6.776E-01 7.13

15 -5.610E-03 -3.740E-02 3.74 -6.776E-01 10.31

9 LOAD1

11 -5.610E-03 -3.740E-02 -2.13 -2.181E-01 -10.30

16 -5.365E-03 -3.577E-02 -7.767E-01 -2.181E-01 -11.30

12 4.53 8.91 -1.92 -2.197E-01 -10.30

17 4.53 9.91 -5.589E-01 -2.197E-01 -11.30

10 LOAD1

12 4.52 8.85 -1.10 -5.303E-01 -3.04

17 4.53 9.86 -5.411E-01 -5.303E-01 -3.47

13 6.22 11.33 -5.763E-01 -5.303E-01 -3.04

18 **6.22 12.76** -1.303E-02 -5.303E-01 -3.47

11 LOAD1

13 6.22 11.33 5.763E-01 -5.303E-01 3.04

18 **6.22 12.76** 1.303E-02 -5.303E-01 3.47

14 4.52 8.85 1.10 -5.303E-01 3.04

19 4.53 9.86 5.411E-01 -5.303E-01 3.47

12 LOAD1

14 4.53 8.91 1.92 -2.197E-01 10.30

19 4.53 9.91 5.589E-01 -2.197E-01 11.30

15 -5.610E-03 -3.740E-02 2.13 -2.181E-01 10.30

20 -5.365E-03 -3.577E-02 7.767E-01 -2.181E-01 11.30

13 LOAD1

16 -5.365E-03 -3.577E-02 7.767E-01 2.181E-01 -11.30

21 -5.610E-03 -3.740E-02 2.13 2.181E-01 -10.30

17 4.53 9.91 5.589E-01 2.197E-01 -11.30

22 4.53 8.91 1.92 2.197E-01 -10.30

14 LOAD1

17 4.53 9.86 5.411E-01 5.303E-01 -3.47

22 4.52 8.85 1.10 5.303E-01 -3.04

18 **6.22 12.76** 1.303E-02 5.303E-01 -3.47

23 6.22 11.33 5.763E-01 5.303E-01 -3.04

15 LOAD1

18 **6.22** **12.76** -1.303E-02 5.303E-01 3.47

23 6.22 11.33 -5.763E-01 5.303E-01 3.04

19 4.53 9.86 -5.411E-01 5.303E-01 3.47

24 4.52 8.85 -1.10 5.303E-01 3.04

16 LOAD1

19 4.53 9.91 -5.589E-01 2.197E-01 11.30

24 4.53 8.91 -1.92 2.197E-01 10.30

20 -5.365E-03 -3.577E-02 -7.767E-01 2.181E-01 11.30

25 -5.610E-03 -3.740E-02 -2.13 2.181E-01 10.30

17 LOAD1

21 -5.610E-03 -3.740E-02 3.74 6.776E-01 -10.31

26 -6.603E-03 -4.402E-02 5.11 6.776E-01 -7.13

22 4.52 8.90 3.07 1.22 -10.31

27 3.98 5.72 4.44 1.22 -7.13

18 LOAD1

22 4.51 8.85 2.21 2.14 -3.02

27 3.97 5.68 2.74 2.14 -1.81

23 6.23 11.33 6.135E-01 2.56 -3.02

28 5.27 6.95 1.15 2.56 -1.81

19 LOAD1

23 6.23 11.33 -6.135E-01 2.56 3.02

28 5.27 6.95 -1.15 2.56 1.81

24 4.51 8.85 -2.21 2.14 3.02

29 3.97 5.68 -2.74 2.14 1.81

20 LOAD1

24 4.52 8.90 -3.07 1.22 10.31

29 3.98 5.72 -4.44 1.22 7.13

25 -5.610E-03 -3.740E-02 -3.74 6.776E-01 10.31

30 -6.603E-03 -4.402E-02 -5.11 6.776E-01 7.13

21 LOAD1

26 -6.603E-03 -4.402E-02 6.58 1.10 -6.44

31 0.00 0.00 7.25 1.10 -6.699E-01

27 4.01 5.72 5.47 5.16 -6.44

32 -4.402E-02 -6.603E-03 6.15 5.16 -6.699E-01

22 LOAD1

27 4.01 5.68 3.62 6.49 -1.52

32 -4.402E-02 -6.603E-03 3.87 6.49 -2.463E-01

28 5.32 6.96 1.18 7.81 -1.52

33 -3.938E-02 -5.907E-03 1.42 7.81 -2.463E-01

23 LOAD1

28 5.32 6.96 -1.18 7.81 1.52

33 -3.938E-02 -5.907E-03 -1.42 7.81 2.463E-01

29 4.01 5.68 -3.62 6.49 1.52

34 -4.402E-02 -6.603E-03 -3.87 6.49 2.463E-01

24 LOAD1

29 4.01 5.72 -5.47 5.16 6.44

34 -4.402E-02 -6.603E-03 -6.15 5.16 6.699E-01

30 -6.603E-03 -4.402E-02 -6.58 1.10 6.44

35 0.00 0.00 -7.25 1.10 6.699E-01

**Analytical solution**

Auxiliary functions

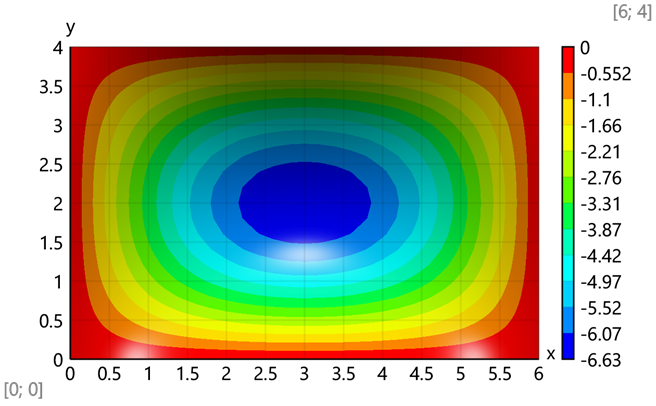
Deflections

Bending moments

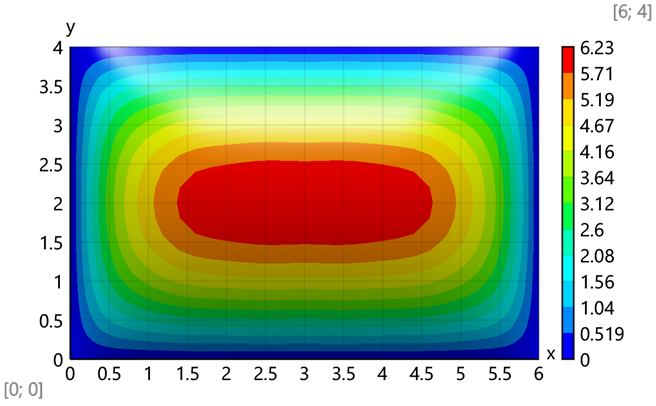
Principal bending moments

Results

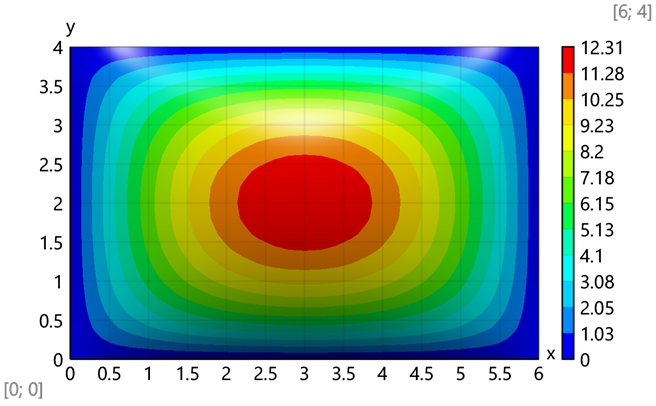
Deflections, mm



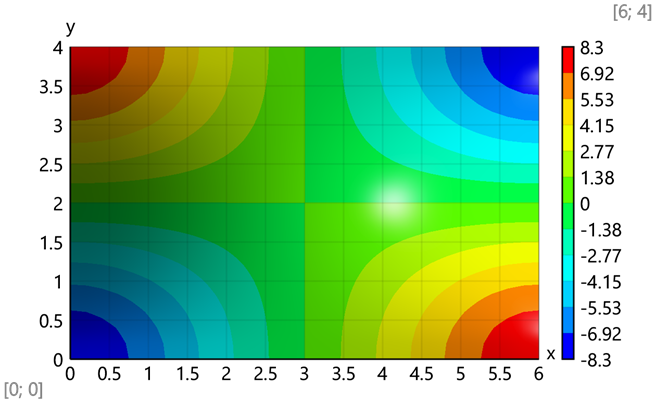
Bending moments - *Mx*, kNm/m



Bending moments - *M*y, kNm/m



Bending moments - *Mx*y, kNm/m



Maximum value -

Comparison of the results

|  |  |  |  |
| --- | --- | --- | --- |
|  | Analytical | FEA Calcpad | FEA SAP 2000 |
| w, mm | 6,627 | 6,629 | 6,529 |
| M*x*, kNm/m | 6,231 | 6,275 | 6,22 |
| My, kNm/m | 12,315 | 12,744 | 12,76 |
| M*x*y, kNm/m | 8,329 | 8,378 | 7,25 |

Difference, %

|  |  |  |  |
| --- | --- | --- | --- |
|  | Analytical | FEA Calcpad | FEA SAP 2000 |
| w, mm | 0,00% | 0,03% | -1,48% |
| M*x*, kNm/m | 0,00% | 0,71% | -0,18% |
| My, kNm/m | 0,00% | 3,48% | 3,61% |
| M*x*y, kNm/m | 0,00% | 0,59% | -12,95% |