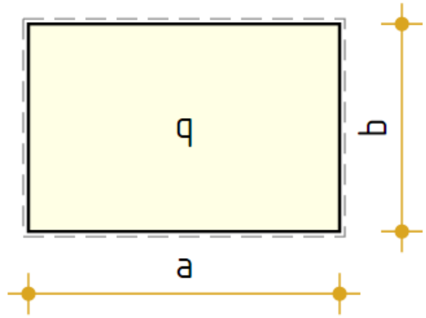
Finite Element Analysis of Rectangular Slab with Calcpad

Input data



Slab dimensions - $a = 6$ m, $b = 4$ m

Thickness - $t = 0.1$ m

Load - $q = 10$ kN/m²

Modulus of elasticity - $E = 35000$ MPa

Poisson`s ratio - $\nu = 0.15$

Finite element mesh

We will use rectangular finite element with $n = 16$ DOFs

Number of elements along *a* and *b* - $n_a = 6$, $n_b = 4$

Total number of elements - $n_e = n_a \cdot n_b = 6 \cdot 4 = 24$

Total number of joints - $n_j = (n_a + 1) \cdot (n_b + 1) = (6 + 1) \cdot (4 + 1) = 35$

Element dimensions - $a_1 = \frac{a}{n_a} = \frac{6}{6} = 1$, $b_1 = \frac{b}{n_b} = \frac{4}{4} = 1$

Supported joints count - $n_s = 2 \cdot (n_a + n_b) = 2 \cdot (6 + 4) = 20$

Joint coordinates

$\vec{x}_j = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 3\ \ 3\ \ 3\ \ 3\ \ 3\ \ \ldots\ \ 6]$ m

$\vec{y}_j = [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ \ldots\ \ 4]$ m

Numbers of elements joints

$$\mathbf{transp}(e_j) =$$

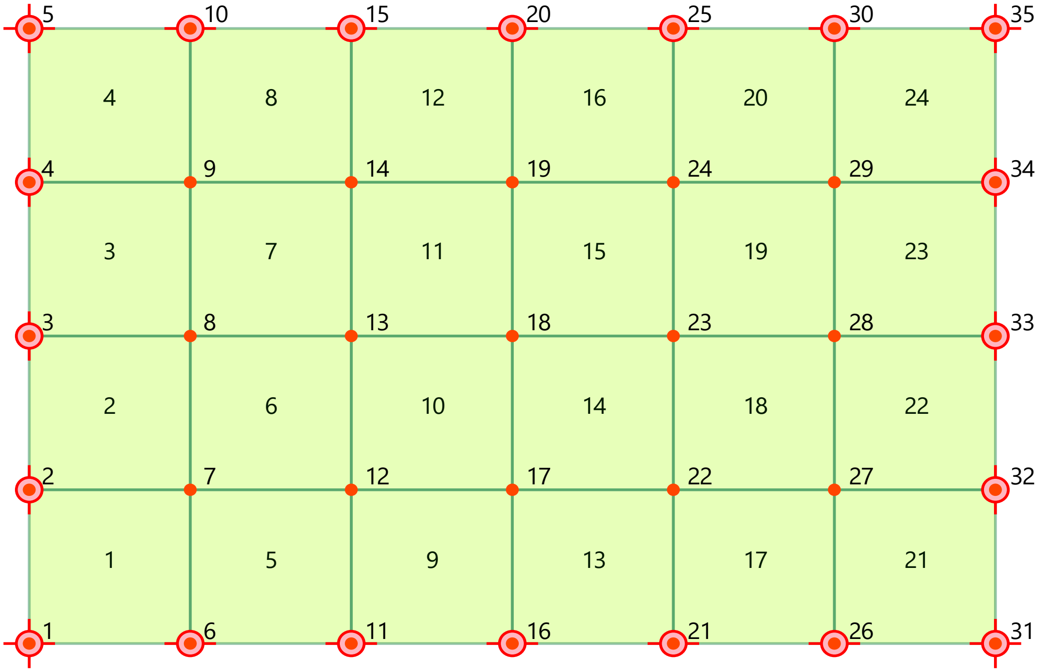
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 6 & 7 & 8 & 9 & 11 & 12 & 13 & 14 & 16 & 17 & 18 & 19 & 21 & 22 & 23 & 24 & \cdots & 29 \\ 6 & 7 & 8 & 9 & 11 & 12 & 13 & 14 & 16 & 17 & 18 & 19 & 21 & 22 & 23 & 24 & 26 & 27 & 28 & 29 & \cdots & 34 \\ 7 & 8 & 9 & 10 & 12 & 13 & 14 & 15 & 17 & 18 & 19 & 20 & 22 & 23 & 24 & 25 & 27 & 28 & 29 & 30 & \cdots & 35 \\ 2 & 3 & 4 & 5 & 7 & 8 & 9 & 10 & 12 & 13 & 14 & 15 & 17 & 18 & 19 & 20 & 22 & 23 & 24 & 25 & \cdots & 30 \end{bmatrix}$$

Supported joints

$\vec{s}_j = [1\ \ 6\ \ 11\ \ 16\ \ 21\ \ 26\ \ 31\ \ 5\ \ 10\ \ 15\ \ 20\ \ 25\ \ 30\ \ 35\ \ 2\ \ 3\ \ 4\ \ 32\ \ 33\ \ 34]$

Coordinates of elements centers

$$j_e(e) = \mathbf{row}(e_j; e),\ \ x_c(e) = \frac{\mathbf{sum}\left(\mathbf{extract}\left(\vec{x}_j; j_e(e)\right)\right)}{4},\ \ y_c(e) = \frac{\mathbf{sum}\left(\mathbf{extract}\left(\vec{y}_j; j_e(e)\right)\right)}{4}$$



Finite element formulation

Shape functions

Along dimension *a*

Base functions

$$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$$

$$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot \left(1 - \xi \cdot (2 - \xi)\right)$$

$$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$$

$$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$$

First derivatives

$$\Phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$$

$$\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$$

$$\Phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$$

$$\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$$

Second derivatives

$$\Phi''_{1a}(\xi) = -\frac{6}{{a_1}^2} \cdot (1 - 2 \cdot \xi)$$

$$\Phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$$

$$\Phi''_{3a}(\xi) = \frac{6}{{a_1}^2} \cdot (1 - 2 \cdot \xi)$$

$$\Phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$$

Along dimension *b*

Base functions

$$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$$

$$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot \left(1 - \eta \cdot (2 - \eta)\right)$$

$$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$$

$$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$$

First derivatives

$$\Phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$$

$$\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$$

$$\Phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$$

$$\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$$

Second derivatives

$$\Phi''_{1b}(\eta) = -\frac{6}{{b_1}^2} \cdot (1 - 2 \cdot \eta)$$

$$\Phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$$

$$\Phi''_{3b}(\eta) = \frac{6}{{b_1}^2} \cdot (1 - 2 \cdot \eta)$$

$$\Phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$$

For vertical displacements

$wN_{1,w}(\xi;\eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{2,w}(\xi;\eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{3,w}(\xi;\eta) = \Phi_{3a}(\xi) \cdot \Phi_{3b}(\eta)$

$N_{4,w}(\xi;\eta) = \Phi_{1a}(\xi) \cdot \Phi_{3b}(\eta)$

For rotations ϑ_x

$N_{1,\theta_x}(\xi;\eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{2,\theta_x}(\xi;\eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{3,\theta_x}(\xi;\eta) = \Phi_{4a}(\xi) \cdot \Phi_{3b}(\eta)$

$N_{4,\theta_x}(\xi;\eta) = \Phi_{2a}(\xi) \cdot \Phi_{3b}(\eta)$

For rotations ϑ_y

$N_{1,\theta_y}(\xi;\eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,\theta_y}(\xi;\eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{3,\theta_y}(\xi;\eta) = \Phi_{3a}(\xi) \cdot \Phi_{4b}(\eta)$

$N_{4,\theta_y}(\xi;\eta) = \Phi_{1a}(\xi) \cdot \Phi_{4b}(\eta)$

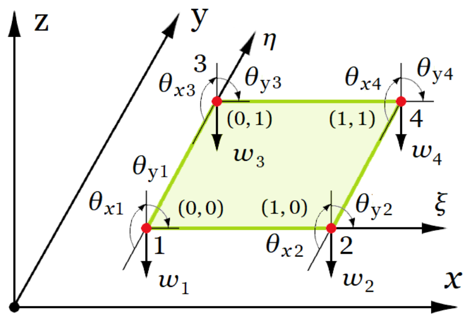
For twist ψ

$N_{1,\psi}(\xi;\eta) = \Phi_{2a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,\psi}(\xi;\eta) = \Phi_{4a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{3,\psi}(\xi;\eta) = \Phi_{4a}(\xi) \cdot \Phi_{4b}(\eta)$

$N_{4,\psi}(\xi;\eta) = \Phi_{2a}(\xi) \cdot \Phi_{4b}(\eta)$



Shape functions vector

$$
\begin{aligned}
N(i;\xi;\eta) \ = \ \textbf{take}\big(i; & N_{1,w}(\xi;\eta); N_{1,\theta_x}(\xi;\eta); N_{1,\theta_y}(\xi;\eta); N_{1,\psi}(\xi;\eta);\\
& N_{2,w}(\xi;\eta); N_{2,\theta_x}(\xi;\eta); N_{2,\theta_y}(\xi;\eta); N_{2,\psi}(\xi;\eta);\\
& N_{3,w}(\xi;\eta); N_{3,\theta_x}(\xi;\eta); N_{3,\theta_y}(\xi;\eta); N_{3,\psi}(\xi;\eta);\\
& N_{4,w}(\xi;\eta); N_{4,\theta_x}(\xi;\eta); N_{4,\theta_y}(\xi;\eta); N_{4,\psi}(\xi;\eta)\big)
\end{aligned}
$$

Constitutive matrix (stress - strain relationship)

$$
D = \frac{E \cdot t^3}{12 \cdot (1-\nu^2)} \cdot \left[1; \nu; 0 \mid \nu; 1; 0 \mid 0; 0; \frac{1-\nu}{2}\right] =
$$

$$
\frac{35000 \cdot 0.1^3}{12 \cdot (1-0.15^2)} \cdot \left[1; 0.15; 0 \mid 0.15; 1; 0 \mid 0; 0; \frac{1-0.15}{2}\right] = \begin{bmatrix} 2.98 & 0.448 & 0 \\ 0.448 & 2.98 & 0 \\ 0 & 0 & 1.27 \end{bmatrix}
$$

Strain-displacement matrix

$$
\begin{aligned}
B_1(j;\xi;\eta) = \textbf{take}\big(j; & \Phi''_{1a}(\xi) \cdot \Phi_{1b}(\eta); \Phi''_{2a}(\xi) \cdot \Phi_{1b}(\eta); \Phi''_{1a}(\xi) \cdot \Phi_{2b}(\eta); \Phi''_{2a}(\xi) \cdot \Phi_{2b}(\eta); \Phi''_{3a}(\xi)\\
& \cdot \Phi_{1b}(\eta); \Phi''_{4a}(\xi) \cdot \Phi_{1b}(\eta); \Phi''_{3a}(\xi) \cdot \Phi_{2b}(\eta); \Phi''_{4a}(\xi) \cdot \Phi_{2b}(\eta); \Phi''_{3a}(\xi)\\
& \cdot \Phi_{3b}(\eta); \Phi''_{4a}(\xi) \cdot \Phi_{3b}(\eta); \Phi''_{3a}(\xi) \cdot \Phi_{4b}(\eta); \Phi''_{4a}(\xi) \cdot \Phi_{4b}(\eta); \Phi''_{1a}(\xi)\\
& \cdot \Phi_{3b}(\eta); \Phi''_{2a}(\xi) \cdot \Phi_{3b}(\eta); \Phi''_{1a}(\xi) \cdot \Phi_{4b}(\eta); \Phi''_{2a}(\xi) \cdot \Phi_{4b}(\eta)\big)
\end{aligned}
$$

$$
\begin{aligned}
B_2(j;\xi;\eta) = \textbf{take}\big(j; & \Phi_{1a}(\xi) \cdot \Phi''_{1b}(\eta); \Phi_{2a}(\xi) \cdot \Phi''_{1b}(\eta); \Phi_{1a}(\xi) \cdot \Phi''_{2b}(\eta); \Phi_{2a}(\xi) \cdot \Phi''_{2b}(\eta); \Phi_{3a}(\xi)\\
& \cdot \Phi''_{1b}(\eta); \Phi_{4a}(\xi) \cdot \Phi''_{1b}(\eta); \Phi_{3a}(\xi) \cdot \Phi''_{2b}(\eta); \Phi_{4a}(\xi) \cdot \Phi''_{2b}(\eta); \Phi_{3a}(\xi)\\
& \cdot \Phi''_{3b}(\eta); \Phi_{4a}(\xi) \cdot \Phi''_{3b}(\eta); \Phi_{3a}(\xi) \cdot \Phi''_{4b}(\eta); \Phi_{4a}(\xi) \cdot \Phi''_{4b}(\eta); \Phi_{1a}(\xi)\\
& \cdot \Phi''_{3b}(\eta); \Phi_{2a}(\xi) \cdot \Phi''_{3b}(\eta); \Phi_{1a}(\xi) \cdot \Phi''_{4b}(\eta); \Phi_{2a}(\xi) \cdot \Phi''_{4b}(\eta)\big)
\end{aligned}
$$

$$
\begin{aligned}
B_3(j;\xi;\eta) = 2\ \textbf{take}\big(j; & \Phi'_{1a}(\xi) \cdot \Phi'_{1b}(\eta); \Phi'_{2a}(\xi) \cdot \Phi'_{1b}(\eta); \Phi'_{1a}(\xi) \cdot \Phi'_{2b}(\eta); \Phi'_{2a}(\xi)\\
& \cdot \Phi'_{2b}(\eta); \Phi'_{3a}(\xi) \cdot \Phi'_{1b}(\eta); \Phi'_{4a}(\xi) \cdot \Phi'_{1b}(\eta); \Phi'_{3a}(\xi) \cdot \Phi'_{2b}(\eta); \Phi'_{4a}(\xi)\\
& \cdot \Phi'_{2b}(\eta); \Phi'_{3a}(\xi) \cdot \Phi'_{3b}(\eta); \Phi'_{4a}(\xi) \cdot \Phi'_{3b}(\eta); \Phi'_{3a}(\xi) \cdot \Phi'_{4b}(\eta); \Phi'_{4a}(\xi)\\
& \cdot \Phi'_{4b}(\eta); \Phi'_{1a}(\xi) \cdot \Phi'_{3b}(\eta); \Phi'_{2a}(\xi) \cdot \Phi'_{3b}(\eta); \Phi'_{1a}(\xi) \cdot \Phi'_{4b}(\eta); \Phi'_{2a}(\xi) \cdot \Phi'_{4b}(\eta)\big)
\end{aligned}
$$

$B(j;\xi;\eta) = [B_1(j;\xi;\eta); B_2(j;\xi;\eta); B_3(j;\xi;\eta)]$

$x_1(e) = \vec{x}_{j_{.e_{j_{.e,1}}}}$, $y_1(e) = \vec{y}_{j_{.e_{j_{.e,1}}}}$

The elements of the stiffness matrix will be calculated by using the equation

$$K_{e,ij} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 B_i(\xi;\eta)^T \cdot D \cdot B_j(\xi;\eta)\, d\xi \;\, d\eta$$

Element stiffness matrix

(above the main diagonal only)

$BTDB_e(i;j;\xi;\eta) = \mathbf{transp}\big(B(i;\xi;\eta)\big) \cdot D \cdot B(j;\xi;\eta)$

$$K_e(i;j) = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 BTDB_e(i;j;\xi;\eta)\, d\eta \;\, d\xi$$

$\$\mathrm{Repeat}\Big\{\$\mathrm{Repeat}\Big\{K_{e_{.i,j}} = K_e(i;j); j = i\ldots n\Big\}; i = 1\ldots n\Big\} = 0.333$

$K_e =$

$$\begin{bmatrix}
35.19 & 9.78 & 9.78 & 2.02 & -17.29 & 6.26 & -0.827 & 0.488 & -0.614 & 2.69 & 2.69 & -1.05 & -17.29 & -0.827 & 6.26 & 0.488 \\
0 & 5.73 & 2.47 & 0.935 & -6.26 & 1.72 & -0.488 & 0.15 & -2.69 & 1.26 & 1.05 & -0.332 & -0.827 & 0.239 & 0.488 & -0.119 \\
0 & 0 & 5.73 & 0.935 & -0.827 & 0.488 & 0.239 & -0.119 & -2.69 & 1.05 & 1.26 & -0.332 & -6.26 & -0.488 & 1.72 & 0.15 \\
0 & 0 & 0 & 0.333 & -0.488 & 0.15 & 0.119 & -0.0549 & -1.05 & 0.332 & 0.332 & -0.0786 & -0.488 & 0.119 & 0.15 & -0.0549 \\
0 & 0 & 0 & 0 & 35.19 & -9.78 & 9.78 & -2.02 & -17.29 & 0.827 & 6.26 & -0.488 & -0.614 & -2.69 & 2.69 & 1.05 \\
0 & 0 & 0 & 0 & 0 & 5.73 & -2.47 & 0.935 & 0.827 & 0.239 & -0.488 & -0.119 & 2.69 & 1.26 & -1.05 & -0.332 \\
0 & 0 & 0 & 0 & 0 & 0 & 5.73 & -0.935 & -6.26 & 0.488 & 1.72 & -0.15 & -2.69 & -1.05 & 1.26 & 0.332 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.333 & 0.488 & 0.119 & -0.15 & -0.0549 & 1.05 & 0.332 & -0.332 & -0.0786 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35.19 & -9.78 & -9.78 & 2.02 & -17.29 & -6.26 & 0.827 & 0.488 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.73 & 2.47 & -0.935 & 6.26 & 1.72 & -0.488 & -0.15 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.73 & -0.935 & 0.827 & 0.488 & 0.239 & 0.119 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.333 & -0.488 & -0.15 & -0.119 & -0.0549 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35.19 & 9.78 & -9.78 & -2.02 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.73 & -2.47 & -0.935 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.73 & 0.935 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.333
\end{bmatrix}$$

Element load vector

$$F_{e,i} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 N_i(\xi;\eta)^T \cdot q\, d\xi \;\, d\eta$$

$\vec{F_e} = [2.5\;\; 0.417\;\; 0.417\;\; 0.07\;\; 2.5\;\; -0.417\;\; 0.417\;\; -0.07\;\; 2.5\;\; -0.417\;\; -0.417\;\; 0.07\;\; 2.5\;\; 0.417\;\; -0.417\;\; -0.07]\ \mathrm{kN}$

Solution

Global stiffness matrix $K =$

$$\begin{bmatrix}
1\times10^{20} & 9.78 & 9.78 & 2.02 & -17.29 & -0.827 & 6.26 & 0.488 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
9.78 & 1\times10^{20} & 2.47 & 0.935 & -0.827 & 0.239 & 0.488 & -0.119 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
9.78 & 2.47 & 1\times10^{20} & 0.935 & -6.26 & -0.488 & 1.72 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
2.02 & 0.935 & 0.935 & 0.333 & -0.488 & 0.119 & 0.15 & -0.0549 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
-17.29 & -0.827 & -6.26 & -0.488 & 1\times10^{20} & 19.56 & 0 & 0 & -17.29 & -0.827 & 6.26 & 0.488 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
-0.827 & 0.239 & -0.488 & 0.119 & 19.56 & 11.46 & 0 & 0 & -0.827 & 0.239 & 0.488 & -0.119 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
6.26 & 0.488 & 1.72 & 0.15 & 0 & 0 & 1\times10^{20} & 1.87 & -6.26 & -0.488 & 1.72 & 0.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0.488 & -0.119 & 0.15 & -0.0549 & 0 & 0 & 1.87 & 0.667 & -0.488 & 0.119 & 0.15 & -0.0549 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & -17.29 & -0.827 & -6.26 & -0.488 & 1\times10^{20} & 19.56 & 0 & 0 & -17.29 & -0.827 & 6.26 & 0.488 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & -0.827 & 0.239 & -0.488 & 0.119 & 19.56 & 11.46 & 0 & 0 & -0.827 & 0.239 & 0.488 & -0.119 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 6.26 & 0.488 & 1.72 & 0.15 & 0 & 0 & 1\times10^{20} & 1.87 & -6.26 & -0.488 & 1.72 & 0.15 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0.488 & -0.119 & 0.15 & -0.0549 & 0 & 0 & 1.87 & 0.667 & -0.488 & 0.119 & 0.15 & -0.0549 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -17.29 & -0.827 & -6.26 & -0.488 & 1\times10^{20} & 19.56 & 0 & 0 & -17.29 & -0.827 & 6.26 & 0.488 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.827 & 0.239 & -0.488 & 0.119 & 19.56 & 11.46 & 0 & 0 & -0.827 & 0.239 & 0.488 & -0.119 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.26 & 0.488 & 1.72 & 0.15 & 0 & 0 & 1\times10^{20} & 1.87 & -6.26 & -0.488 & 1.72 & 0.15 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.488 & -0.119 & 0.15 & -0.0549 & 0 & 0 & 1.87 & 0.667 & -0.488 & 0.119 & 0.15 & -0.0549 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -17.29 & -0.827 & -6.26 & -0.488 & 1\times10^{20} & 9.78 & -9.78 & -2.02 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.827 & 0.239 & -0.488 & 0.119 & 9.78 & 1\times10^{20} & -2.47 & -0.935 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.26 & 0.488 & 1.72 & 0.15 & -9.78 & -2.47 & 1\times10^{20} & 0.935 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.488 & -0.119 & 0.15 & -0.0549 & -2.02 & -0.935 & 0.935 & 0.333 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0.333
\end{bmatrix}$$

Global load vector

$$\vec{F}=[2.5\ \ 0.417\ \ 0.417\ \ 0.0694\ \ 5\ \ 0.833\ \ 0\ \ 0\ \ 5\ \ 0.833\ \ 0\ \ 0\ \ 5\ \ 0.833\ \ 0\ \ 0\ \ 2.5\ \ 0.417\ \ -0.417\ \ -0.0694\ \ \ldots\ \ 0.0694]\ kN$$

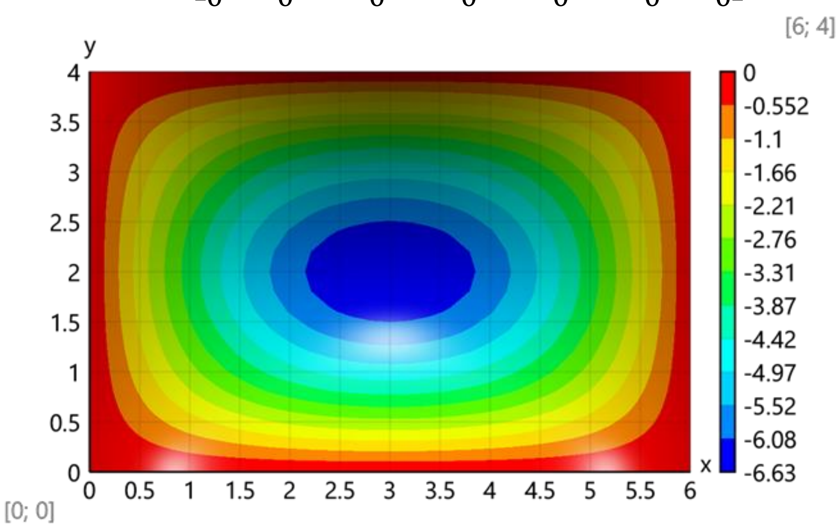
Solution of the system of equations

$$\vec{Z} = \mathbf{clsolve}(K; \vec{F}) = [0\ \ 0\ \ 0\ \ 3.3\ \ 0\ \ 2.84\ \ 0\ \ 2.09\ \ 0\ \ 3.91\ \ 0\ \ 0\ \ 0\ \ 2.84\ \ 0\ \ -2.09\ \ 0\ \ 0\ \ 0\ \ -3.3\ \ \ldots\ \ 3.3]\ mm$$

Results

Joint displacements

$$\mathbf{transp}(W_z) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.58 & 4.21 & 4.75 & 4.21 & 2.58 & 0 \\ 0 & 3.59 & 5.87 & 6.63 & 5.87 & 3.59 & 0 \\ 0 & 2.58 & 4.21 & 4.75 & 4.21 & 2.58 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{mm}$$



Maximal value - $w\left(\frac{a}{2};\frac{b}{2}\right) = w\left(\frac{6}{2};\frac{4}{2}\right) = 6.63\ mm$

Bending moments

$$Z_j(j) = \textbf{slice}\left(\vec{Z};\ k_1 \cdot (j-1)+1;\ k_1 \cdot j\right)$$

$$Z_e(e) = \left[Z_j\left(e_{j\,.e,1}\right); Z_j\left(e_{j\,.e,2}\right); Z_j\left(e_{j\,.e,3}\right); Z_j\left(e_{j\,.e,4}\right)\right]$$

Results for element 15 and joint 18:

$$\vec{Z}_e = Z_e(e) = Z_e(15) = [6.63\ \ 0\ \ 0\ \ 0\ \ 5.87\ \ \text{-}1.52\ \ 0\ \ 0\ \ 4.21\ \ \text{-}1.08\ \ \text{-}3.2\ \ 0.84\ \ 4.75\ \ 0\ \ \text{-}3.62\ \ 0]\ mm$$

$$M_e(x;y) = \text{-}D \cdot B\left(\frac{x}{a_1};\frac{y}{b_1}\right) \cdot Z_e(e)$$

$$\vec{M}_e = M_e(0;0) = [6.28\ \ 12.74\ \ 0]\ kNm/m$$

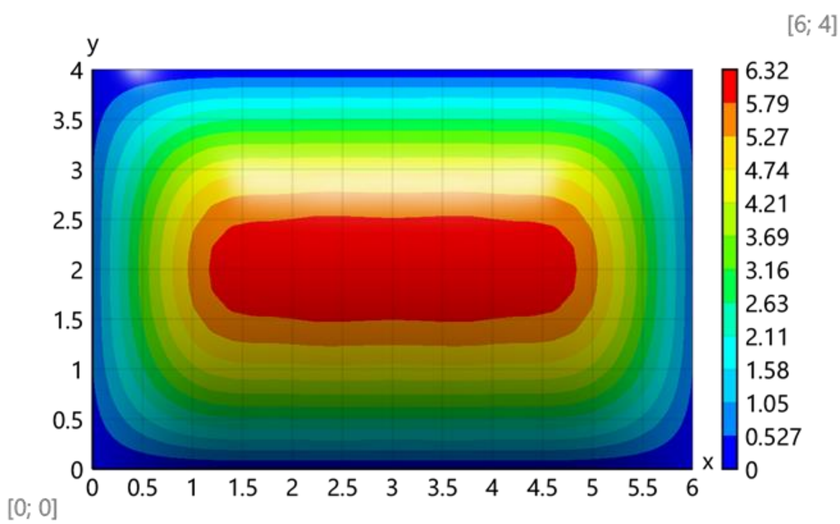
Average bending moments at joints, kNm/m - M_j =

$$\begin{bmatrix} 0 & 0.528 & 0.594 & 0.528 & 0 & 0.0845 & 4.11 & 5.45 & 4.11 & 0.0845 & 0.101 & 4.6 & 6.25 & 4.6 & 0.101 & 0.105 & 4.6 & 6.28 & 4.6 & 0.105 & \cdots & 0 \\ 0 & 0.0792 & 0.0892 & 0.0792 & 0 & 0.563 & 5.84 & 7.04 & 5.84 & 0.563 & 0.675 & 9.08 & 11.33 & 9.08 & 0.675 & 0.702 & 10.1 & 12.74 & 10.1 & 0.702 & \cdots & 0 \\ \text{-}8.38 & \text{-}5.31 & 0 & 5.31 & 8.38 & \text{-}6.18 & \text{-}4.22 & 0 & 4.22 & 6.18 & \text{-}3.05 & \text{-}2.13 & 0 & 2.13 & 3.05 & 0 & 0 & 0 & 0 & 0 & \cdots & \text{-}8.38 \end{bmatrix}$$

Bending moments for the plate

Bending moments - M_x

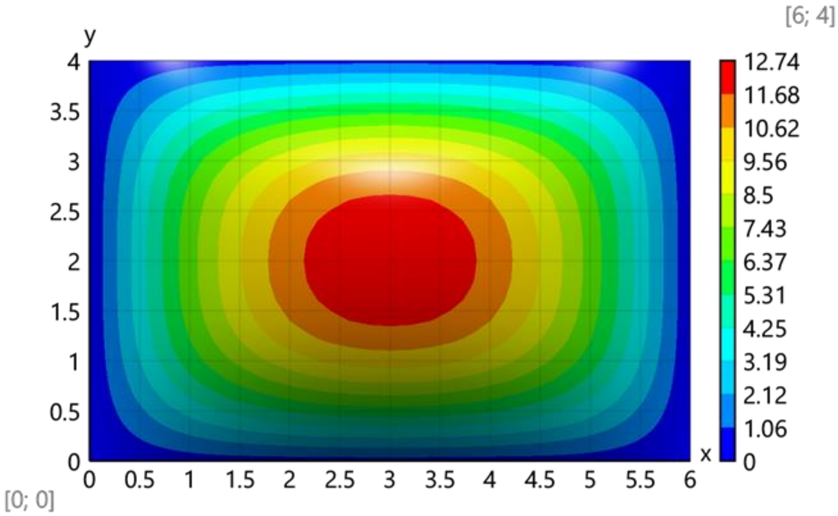
$$\textbf{transp}(Mx) = \begin{bmatrix} 0 & 0.0845 & 0.101 & 0.105 & 0.101 & 0.0845 & 0 \\ 0.528 & 4.11 & 4.6 & 4.6 & 4.6 & 4.11 & 0.528 \\ 0.594 & 5.45 & 6.25 & 6.28 & 6.25 & 5.45 & 0.594 \\ 0.528 & 4.11 & 4.6 & 4.6 & 4.6 & 4.11 & 0.528 \\ 0 & 0.0845 & 0.101 & 0.105 & 0.101 & 0.0845 & 0 \end{bmatrix} kNm/m$$



Maximal value - $M_x\left(\frac{a}{2};\frac{b}{2}\right) = M_x\left(\frac{6}{2};\frac{4}{2}\right) = 6.28$ kNm/m

Bending moments - M_y

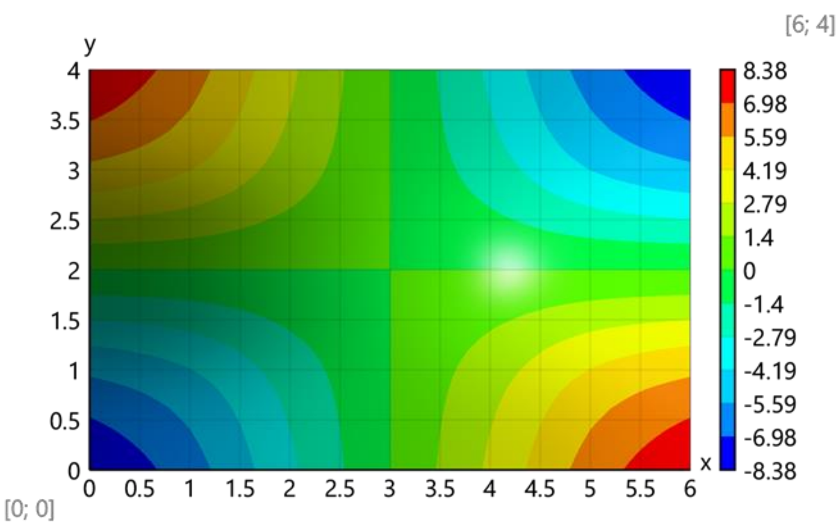
$$\mathbf{transp}(My) = \begin{bmatrix} 0 & 0.563 & 0.675 & 0.702 & 0.675 & 0.563 & 0 \\ 0.0792 & 5.84 & 9.08 & 10.1 & 9.08 & 5.84 & 0.0792 \\ 0.0892 & 7.04 & 11.33 & 12.74 & 11.33 & 7.04 & 0.0892 \\ 0.0792 & 5.84 & 9.08 & 10.1 & 9.08 & 5.84 & 0.0792 \\ 0 & 0.563 & 0.675 & 0.702 & 0.675 & 0.563 & 0 \end{bmatrix} kNm/m$$



Maximal value - $M_y\left(\frac{a}{2};\frac{b}{2}\right) = M_y\left(\frac{6}{2};\frac{4}{2}\right) = 12.74$ kNm/m

Bending moments - M_{xy}

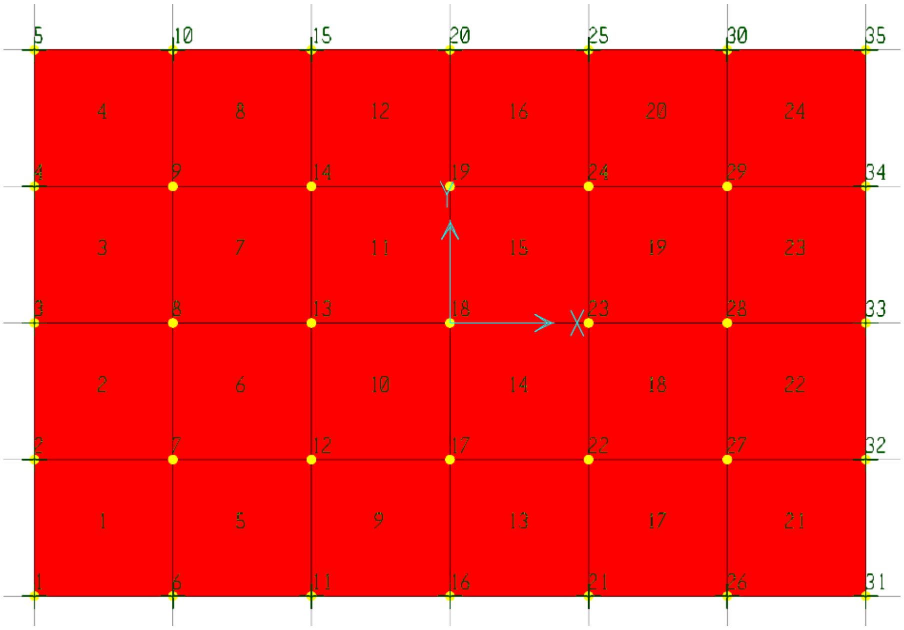
$$\mathbf{transp}(Mxy) = \begin{bmatrix} -8.38 & -6.18 & -3.05 & 0 & 3.05 & 6.18 & 8.38 \\ -5.31 & -4.22 & -2.13 & 0 & 2.13 & 4.22 & 5.31 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5.31 & 4.22 & 2.13 & 0 & -2.13 & -4.22 & -5.31 \\ 8.38 & 6.18 & 3.05 & 0 & -3.05 & -6.18 & -8.38 \end{bmatrix} kNm/m$$



Maximal value - $M_{xy}(0; 0) = -8.38$ kNm/m

Solution with SAP 2000 structural analysis software

Input data



S T A T I C   L O A D   C A S E S

STATIC CASE	CASE TYPE	SELF WT FACTOR
LOAD1	DEAD	0.0000

J O I N T   D A T A

JOINT	GLOBAL-X	GLOBAL-Y	GLOBAL-Z	RESTRAINTS	ANGLE-A	ANGLE-B	ANGLE-C
1	-3.00000	-2.00000	0.00000	0 0 1 1 1 0	0.000	0.000	0.000
2	-3.00000	-1.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
3	-3.00000	0.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
4	-3.00000	1.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
5	-3.00000	2.00000	0.00000	0 0 1 1 1 0	0.000	0.000	0.000
6	-2.00000	-2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
7	-2.00000	-1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
8	-2.00000	0.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
9	-2.00000	1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
10	-2.00000	2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
11	-1.00000	-2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
12	-1.00000	-1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
13	-1.00000	0.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
14	-1.00000	1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
15	-1.00000	2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
16	0.00000	-2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
17	0.00000	-1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
18	0.00000	0.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
19	0.00000	1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
20	0.00000	2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
21	1.00000	-2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
22	1.00000	-1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
23	1.00000	0.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
24	1.00000	1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
25	1.00000	2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000

26	2.00000	-2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
27	2.00000	-1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
28	2.00000	0.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
29	2.00000	1.00000	0.00000	0 0 0 0 0 0	0.000	0.000	0.000
30	2.00000	2.00000	0.00000	0 0 1 0 1 0	0.000	0.000	0.000
31	3.00000	-2.00000	0.00000	0 0 1 1 1 0	0.000	0.000	0.000
32	3.00000	-1.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
33	3.00000	0.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
34	3.00000	1.00000	0.00000	0 0 1 1 0 0	0.000	0.000	0.000
35	3.00000	2.00000	0.00000	0 0 1 1 1 0	0.000	0.000	0.000

S H E L L   E L E M E N T   D A T A

SHELL	JNT-1	JNT-2	JNT-3	JNT-4	SECTION	ANGLE	AREA
1	1	6	2	7	SSEC1	0.000	1.000
2	2	7	3	8	SSEC1	0.000	1.000
3	3	8	4	9	SSEC1	0.000	1.000
4	4	9	5	10	SSEC1	0.000	1.000
5	6	11	7	12	SSEC1	0.000	1.000
6	7	12	8	13	SSEC1	0.000	1.000
7	8	13	9	14	SSEC1	0.000	1.000
8	9	14	10	15	SSEC1	0.000	1.000
9	11	16	12	17	SSEC1	0.000	1.000
10	12	17	13	18	SSEC1	0.000	1.000
11	13	18	14	19	SSEC1	0.000	1.000
12	14	19	15	20	SSEC1	0.000	1.000
13	16	21	17	22	SSEC1	0.000	1.000
14	17	22	18	23	SSEC1	0.000	1.000
15	18	23	19	24	SSEC1	0.000	1.000
16	19	24	20	25	SSEC1	0.000	1.000
17	21	26	22	27	SSEC1	0.000	1.000
18	22	27	23	28	SSEC1	0.000	1.000
19	23	28	24	29	SSEC1	0.000	1.000
20	24	29	25	30	SSEC1	0.000	1.000
21	26	31	27	32	SSEC1	0.000	1.000
22	27	32	28	33	SSEC1	0.000	1.000
23	28	33	29	34	SSEC1	0.000	1.000
24	29	34	30	35	SSEC1	0.000	1.000

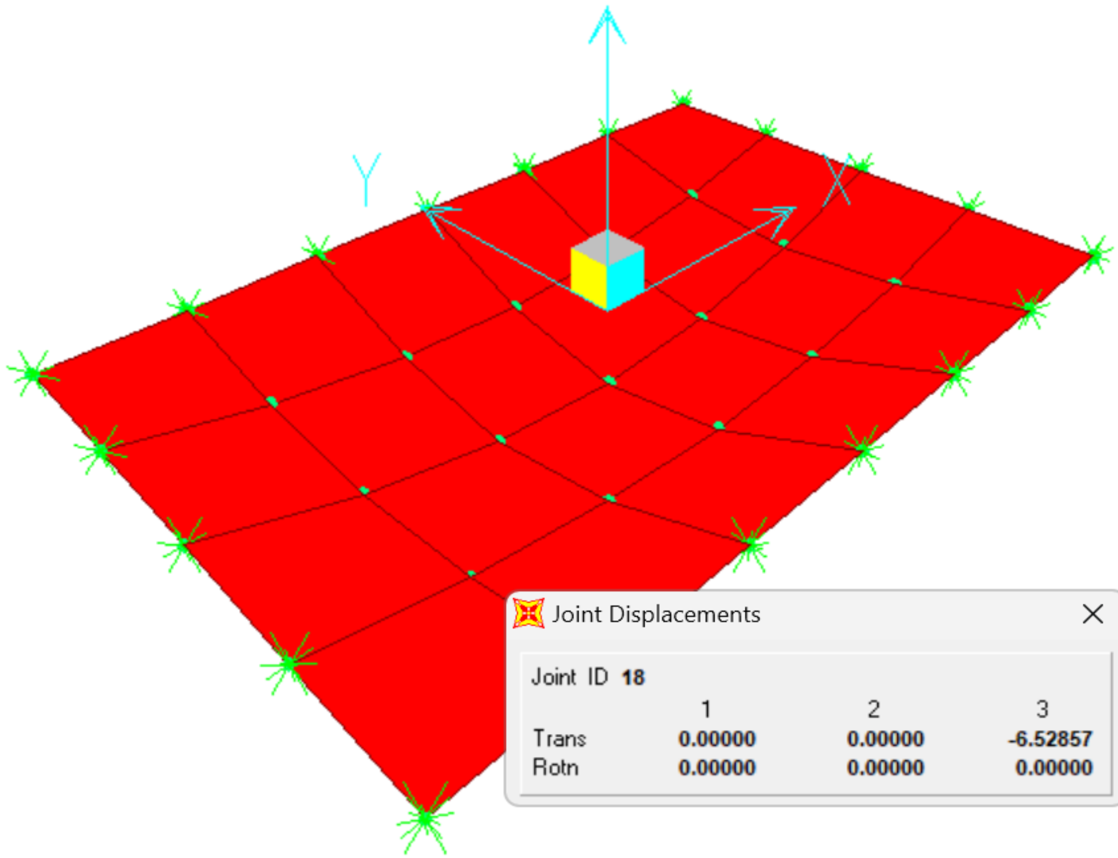
S H E L L   U N I F O R M   L O A D S   Load Case LOAD1

SHELL	DIRECTION	VALUE
1	GLOBAL-Z	-10.0000
2	GLOBAL-Z	-10.0000
3	GLOBAL-Z	-10.0000
4	GLOBAL-Z	-10.0000
5	GLOBAL-Z	-10.0000
6	GLOBAL-Z	-10.0000
7	GLOBAL-Z	-10.0000
8	GLOBAL-Z	-10.0000
9	GLOBAL-Z	-10.0000
10	GLOBAL-Z	-10.0000
11	GLOBAL-Z	-10.0000
12	GLOBAL-Z	-10.0000
13	GLOBAL-Z	-10.0000
14	GLOBAL-Z	-10.0000
15	GLOBAL-Z	-10.0000
16	GLOBAL-Z	-10.0000
17	GLOBAL-Z	-10.0000
18	GLOBAL-Z	-10.0000

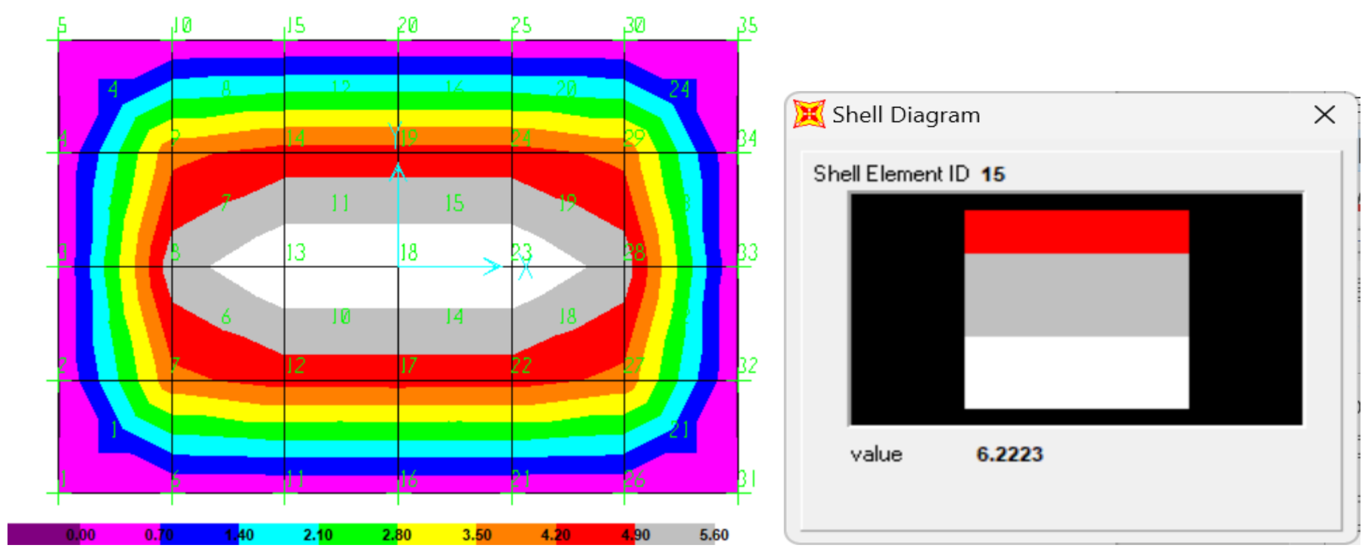
```
19      GLOBAL-Z      -10.0000
20      GLOBAL-Z      -10.0000
21      GLOBAL-Z      -10.0000
22      GLOBAL-Z      -10.0000
23      GLOBAL-Z      -10.0000
24      GLOBAL-Z      -10.0000
```

Results

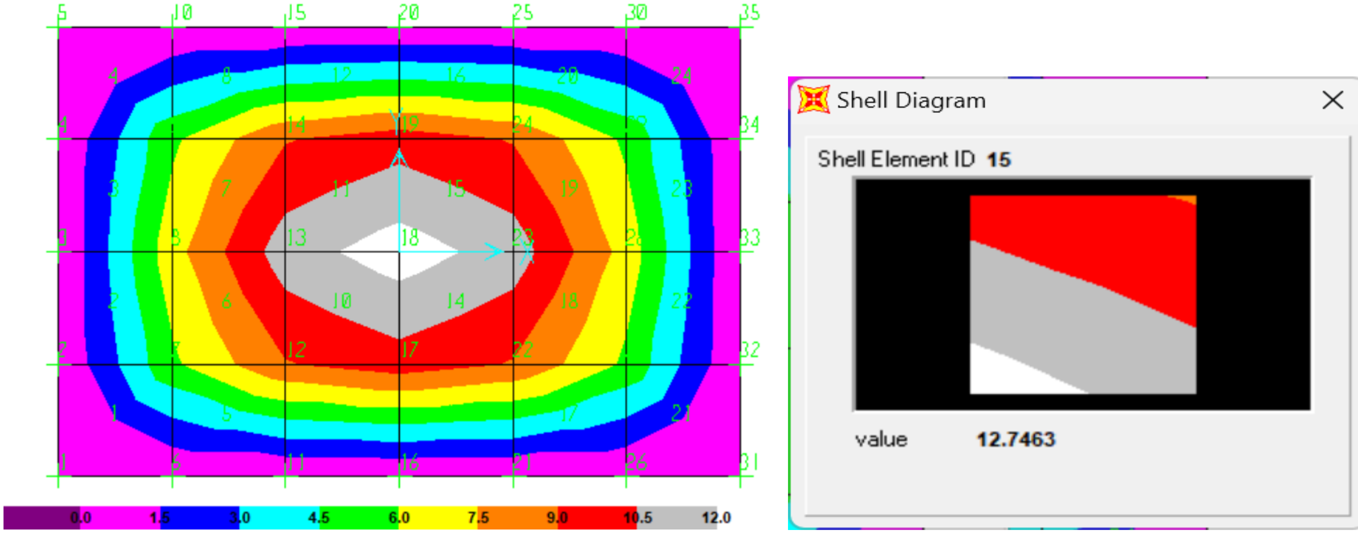
Displacements, mm



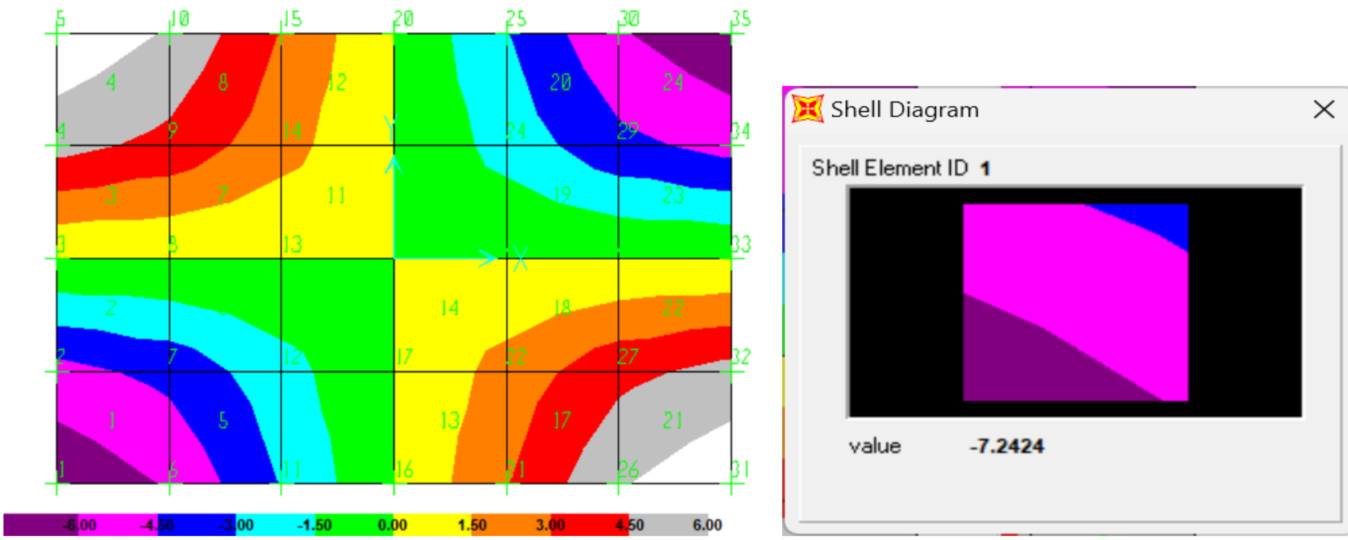
Bending moments - M11, kNm/m



Bending moments - M22, kNm/m



Bending moments - M12, kNm/m



J O I N T   D I S P L A C E M E N T S

JOINT	LOAD	U3	R1	R2
1	LOAD1	0.0000	0.0000	0.0000
2	LOAD1	0.0000	0.0000	2.685E-03
3	LOAD1	0.0000	0.0000	3.743E-03
4	LOAD1	0.0000	0.0000	2.685E-03
5	LOAD1	0.0000	0.0000	0.0000
6	LOAD1	0.0000	-2.797E-03	0.0000
7	LOAD1	-2.509E-03	-1.927E-03	2.151E-03
8	LOAD1	-3.503E-03	0.0000	3.016E-03
9	LOAD1	-2.509E-03	1.927E-03	2.151E-03
10	LOAD1	0.0000	2.797E-03	0.0000
11	LOAD1	0.0000	-4.588E-03	0.0000
12	LOAD1	-4.122E-03	-3.184E-03	1.070E-03
13	LOAD1	-5.777E-03	0.0000	1.514E-03
14	LOAD1	-4.122E-03	3.184E-03	1.070E-03
15	LOAD1	0.0000	4.588E-03	0.0000
16	LOAD1	0.0000	-5.176E-03	0.0000
17	LOAD1	-4.653E-03	-3.600E-03	0.0000

18	LOAD1	-6.529E-03	0.0000	0.0000
19	LOAD1	-4.653E-03	3.600E-03	0.0000
20	LOAD1	0.0000	5.176E-03	0.0000
21	LOAD1	0.0000	-4.588E-03	0.0000
22	LOAD1	-4.122E-03	-3.184E-03	-1.070E-03
23	LOAD1	-5.777E-03	0.0000	-1.514E-03
24	LOAD1	-4.122E-03	3.184E-03	-1.070E-03
25	LOAD1	0.0000	4.588E-03	0.0000
26	LOAD1	0.0000	-2.797E-03	0.0000
27	LOAD1	-2.509E-03	-1.927E-03	-2.151E-03
28	LOAD1	-3.503E-03	0.0000	-3.016E-03
29	LOAD1	-2.509E-03	1.927E-03	-2.151E-03
30	LOAD1	0.0000	2.797E-03	0.0000
31	LOAD1	0.0000	0.0000	0.0000
32	LOAD1	0.0000	0.0000	-2.685E-03
33	LOAD1	0.0000	0.0000	-3.743E-03
34	LOAD1	0.0000	0.0000	-2.685E-03
35	LOAD1	0.0000	0.0000	0.0000

S H E L L   E L E M E N T   R E S U L T A N T S

SHELL	LOAD	JOINT	M11	M22	M12	V13	V23
1	LOAD1						
		1	0.00	0.00	-7.25	-1.10	-6.699E-01
		6	-6.603E-03	-4.402E-02	-6.58	-1.10	-6.44
		2	-4.402E-02	-6.603E-03	-6.15	-5.16	-6.699E-01
		7	4.01	5.72	-5.47	-5.16	-6.44
2	LOAD1						
		2	-4.402E-02	-6.603E-03	-3.87	-6.49	-2.463E-01
		7	4.01	5.68	-3.62	-6.49	-1.52
		3	-3.938E-02	-5.907E-03	-1.42	-7.81	-2.463E-01
		8	5.32	6.96	-1.18	-7.81	-1.52
3	LOAD1						
		3	-3.938E-02	-5.907E-03	1.42	-7.81	2.463E-01
		8	5.32	6.96	1.18	-7.81	1.52
		4	-4.402E-02	-6.603E-03	3.87	-6.49	2.463E-01
		9	4.01	5.68	3.62	-6.49	1.52
4	LOAD1						
		4	-4.402E-02	-6.603E-03	6.15	-5.16	6.699E-01
		9	4.01	5.72	5.47	-5.16	6.44
		5	0.00	0.00	7.25	-1.10	6.699E-01
		10	-6.603E-03	-4.402E-02	6.58	-1.10	6.44
5	LOAD1						
		6	-6.603E-03	-4.402E-02	-5.11	-6.776E-01	-7.13
		11	-5.610E-03	-3.740E-02	-3.74	-6.776E-01	-10.31
		7	3.98	5.72	-4.44	-1.22	-7.13
		12	4.52	8.90	-3.07	-1.22	-10.31
6	LOAD1						
		7	3.97	5.68	-2.74	-2.14	-1.81
		12	4.51	8.85	-2.21	-2.14	-3.02
		8	5.27	6.95	-1.15	-2.56	-1.81
		13	6.23	11.33	-6.135E-01	-2.56	-3.02
7	LOAD1						
		8	5.27	6.95	1.15	-2.56	1.81
		13	6.23	11.33	6.135E-01	-2.56	3.02
		9	3.97	5.68	2.74	-2.14	1.81
		14	4.51	8.85	2.21	-2.14	3.02
8	LOAD1						
		9	3.98	5.72	4.44	-1.22	7.13

		14	4.52	8.90	3.07	-1.22	10.31
		10	-6.603E-03	-4.402E-02	5.11	-6.776E-01	7.13
		15	-5.610E-03	-3.740E-02	3.74	-6.776E-01	10.31
9	LOAD1						
		11	-5.610E-03	-3.740E-02	-2.13	-2.181E-01	-10.30
		16	-5.365E-03	-3.577E-02	-7.767E-01	-2.181E-01	-11.30
		12	4.53	8.91	-1.92	-2.197E-01	-10.30
		17	4.53	9.91	-5.589E-01	-2.197E-01	-11.30
10	LOAD1						
		12	4.52	8.85	-1.10	-5.303E-01	-3.04
		17	4.53	9.86	-5.411E-01	-5.303E-01	-3.47
		13	6.22	11.33	-5.763E-01	-5.303E-01	-3.04
		18	**6.22**	**12.76**	-1.303E-02	-5.303E-01	-3.47
11	LOAD1						
		13	6.22	11.33	5.763E-01	-5.303E-01	3.04
		18	**6.22**	**12.76**	1.303E-02	-5.303E-01	3.47
		14	4.52	8.85	1.10	-5.303E-01	3.04
		19	4.53	9.86	5.411E-01	-5.303E-01	3.47
12	LOAD1						
		14	4.53	8.91	1.92	-2.197E-01	10.30
		19	4.53	9.91	5.589E-01	-2.197E-01	11.30
		15	-5.610E-03	-3.740E-02	2.13	-2.181E-01	10.30
		20	-5.365E-03	-3.577E-02	7.767E-01	-2.181E-01	11.30
13	LOAD1						
		16	-5.365E-03	-3.577E-02	7.767E-01	2.181E-01	-11.30
		21	-5.610E-03	-3.740E-02	2.13	2.181E-01	-10.30
		17	4.53	9.91	5.589E-01	2.197E-01	-11.30
		22	4.53	8.91	1.92	2.197E-01	-10.30
14	LOAD1						
		17	4.53	9.86	5.411E-01	5.303E-01	-3.47
		22	4.52	8.85	1.10	5.303E-01	-3.04
		18	**6.22**	**12.76**	1.303E-02	5.303E-01	-3.47
		23	6.22	11.33	5.763E-01	5.303E-01	-3.04
15	LOAD1						
		18	**6.22**	**12.76**	-1.303E-02	5.303E-01	3.47
		23	6.22	11.33	-5.763E-01	5.303E-01	3.04
		19	4.53	9.86	-5.411E-01	5.303E-01	3.47
		24	4.52	8.85	-1.10	5.303E-01	3.04
16	LOAD1						
		19	4.53	9.91	-5.589E-01	2.197E-01	11.30
		24	4.53	8.91	-1.92	2.197E-01	10.30
		20	-5.365E-03	-3.577E-02	-7.767E-01	2.181E-01	11.30
		25	-5.610E-03	-3.740E-02	-2.13	2.181E-01	10.30
17	LOAD1						
		21	-5.610E-03	-3.740E-02	3.74	6.776E-01	-10.31
		26	-6.603E-03	-4.402E-02	5.11	6.776E-01	-7.13
		22	4.52	8.90	3.07	1.22	-10.31
		27	3.98	5.72	4.44	1.22	-7.13
18	LOAD1						
		22	4.51	8.85	2.21	2.14	-3.02
		27	3.97	5.68	2.74	2.14	-1.81
		23	6.23	11.33	6.135E-01	2.56	-3.02
		28	5.27	6.95	1.15	2.56	-1.81
19	LOAD1						
		23	6.23	11.33	-6.135E-01	2.56	3.02
		28	5.27	6.95	-1.15	2.56	1.81
		24	4.51	8.85	-2.21	2.14	3.02
		29	3.97	5.68	-2.74	2.14	1.81
20	LOAD1						

		24	4.52	8.90	-3.07	1.22	10.31
		29	3.98	5.72	-4.44	1.22	7.13
		25	-5.610E-03	-3.740E-02	-3.74	6.776E-01	10.31
		30	-6.603E-03	-4.402E-02	-5.11	6.776E-01	7.13
21	LOAD1						
		26	-6.603E-03	-4.402E-02	6.58	1.10	-6.44
		31	0.00	0.00	7.25	1.10	-6.699E-01
		27	4.01	5.72	5.47	5.16	-6.44
		32	-4.402E-02	-6.603E-03	6.15	5.16	-6.699E-01
22	LOAD1						
		27	4.01	5.68	3.62	6.49	-1.52
		32	-4.402E-02	-6.603E-03	3.87	6.49	-2.463E-01
		28	5.32	6.96	1.18	7.81	-1.52
		33	-3.938E-02	-5.907E-03	1.42	7.81	-2.463E-01
23	LOAD1						
		28	5.32	6.96	-1.18	7.81	1.52
		33	-3.938E-02	-5.907E-03	-1.42	7.81	2.463E-01
		29	4.01	5.68	-3.62	6.49	1.52
		34	-4.402E-02	-6.603E-03	-3.87	6.49	2.463E-01
24	LOAD1						
		29	4.01	5.72	-5.47	5.16	6.44
		34	-4.402E-02	-6.603E-03	-6.15	5.16	6.699E-01
		30	-6.603E-03	-4.402E-02	-6.58	1.10	6.44
		35	0.00	0.00	-7.25	1.10	6.699E-01

Analytical solution

Cylindrical stiffness - $D = \dfrac{E \cdot t^3}{12 \cdot (1 - \nu^2)} = \dfrac{35000\,\mathrm{MPa} \cdot (0.1\,\mathrm{m})^3}{12 \cdot (1 - 0.15^2)} = 2983.8\,\mathrm{kNm}$

$$\alpha = \frac{a}{b} = \frac{6\,\mathrm{m}}{4\,\mathrm{m}} = 1.5, \quad \alpha_2 = \alpha^2 = 1.5^2 = 2.25$$

$$q_0 = \frac{16 \cdot q}{\pi^2} = \frac{16 \cdot 10\,\mathrm{kN/\,m^2}}{3.14^2} = 16.21\,\mathrm{kN/\,m^2}$$

Auxiliary functions

$$k(n) = 2 \cdot n + 1, \quad k_2(n) = 4 \cdot n \cdot (n + 1) + 1$$

$$A(m;n) = k_2(m) + \alpha_2 \cdot k_2(n) \quad => \quad A_1(m;n) = \frac{1}{A(m;n)^2}$$

$$B(m;n) = k_2(m) + \nu \cdot \alpha_2 \cdot k_2(n) \quad => \quad B_1(m;n) = \frac{B(m;n)}{A(m;n)^2}$$

$$C(m;n) = \nu \cdot k_2(m) + \alpha_2 \cdot k_2(n) \quad => \quad C_1(m;n) = \frac{C(m;n)}{A(m;n)^2}$$

$$S_a(m;x) = \frac{\sin\left(\dfrac{k(m) \cdot \pi}{a} \cdot x\right)}{k(m)}, \quad S_b(n;y) = \frac{\sin\left(\dfrac{k(n) \cdot \pi}{b} \cdot y\right)}{k(n)}$$

Deflections

$$w(x;y) = \frac{q_0 \cdot \left(\frac{a}{\pi}\right)^4}{D} \cdot \sum_{m=0}^{N} S_a(m;x) \cdot \sum_{n=0}^{N} A_1(m;n) \cdot S_b(n;y)$$

Bending moments

$$M_x(x;y) = q_0 \cdot \left(\frac{a}{\pi}\right)^2 \cdot \sum_{m=0}^{N} S_a(m;x) \cdot \sum_{n=0}^{N} B_1(m;n) \cdot S_b(n;y)$$

$$M_y(x;y) = q_0 \cdot \left(\frac{a}{\pi}\right)^2 \cdot \sum_{m=0}^{N} S_a(m;x) \cdot \sum_{n=0}^{N} C_1(m;n) \cdot S_b(n;y)$$

$$k_1(n) = 2 \cdot n$$

$$M_{xy}(x;y) = -q_0 \cdot \left(\frac{a}{\pi}\right)^2 \cdot (1-\nu) \cdot \alpha \cdot \sum_{m=0}^{N} \cos\left(\frac{k(m) \cdot \pi \cdot x}{a}\right) \cdot \sum_{n=0}^{N} A_1(m;n) \cdot \cos\left(\frac{k(n) \cdot \pi \cdot y}{b}\right)$$

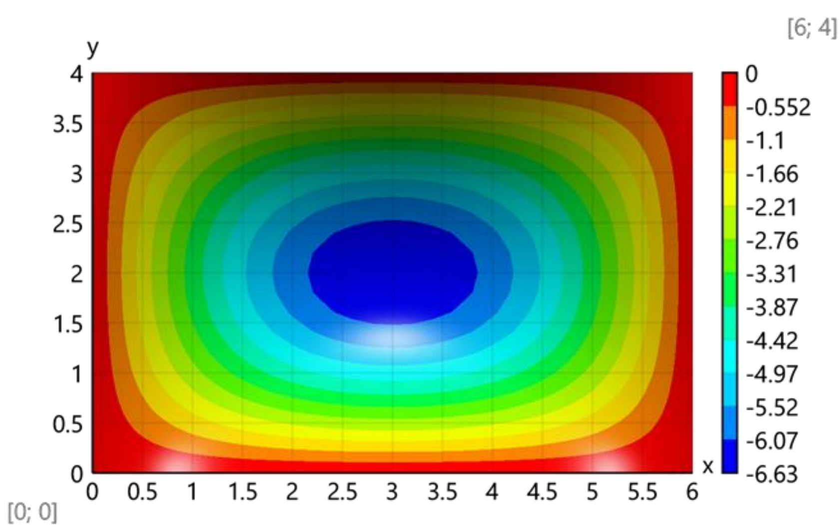
Principal bending moments

$$M_{max}(x;y) = \frac{M_x(x;y) + M_y(x;y)}{2} + \sqrt{\frac{\left(M_x(x;y) - M_y(x;y)\right)^2}{4} + M_{xy}(x;y)^2}$$

$$M_{min}(x;y) = \frac{M_x(x;y) + M_y(x;y)}{2} - \sqrt{\frac{\left(M_x(x;y) - M_y(x;y)\right)^2}{4} + M_{xy}(x;y)^2}$$

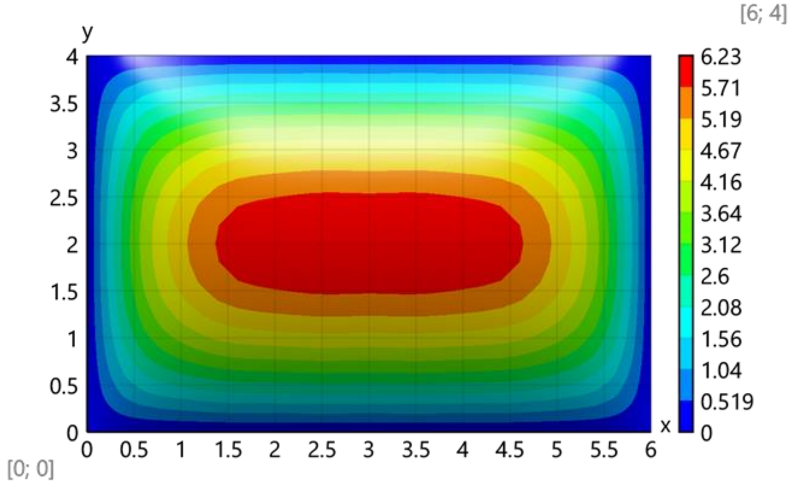
Results

Deflections, mm



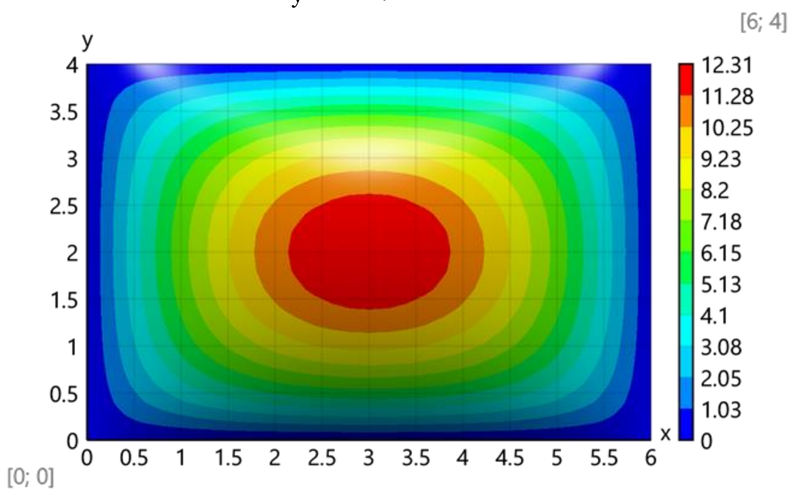
Maximum value - $w\left(\frac{a}{2};\frac{b}{2}\right) = w\left(\frac{6\,\mathrm{m}}{2};\frac{4\,\mathrm{m}}{2}\right) = 6.63\,\mathrm{mm}$

Bending moments - M_x, kNm/m



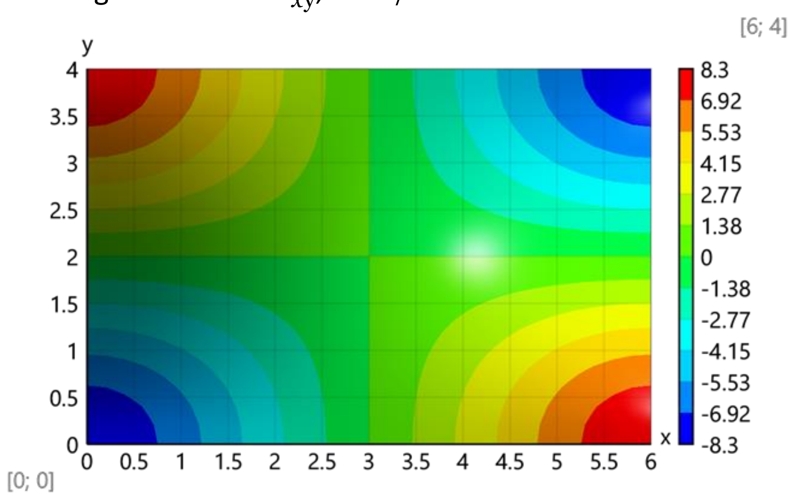
Maximum value -$M_x\left(\frac{a}{2};\frac{b}{2}\right) = M_x\left(\frac{6\ \mathrm{m}}{2};\frac{4\ \mathrm{m}}{2}\right) = 6.22\ \mathrm{kNm/\ m}$

Bending moments - M_y, kNm/m



Maximum value - $M_y\left(\frac{a}{2};\frac{b}{2}\right) = M_y\left(\frac{6\ \mathrm{m}}{2};\frac{4\ \mathrm{m}}{2}\right) = 12.31\ \mathrm{kNm/\ m}$

Bending moments - M_{xy}, kNm/m



Maximum value - $M_{xy}(0\ \mathrm{m}; 0\ \mathrm{m}) = \text{-}8.3\ \mathrm{kNm/\ m}$

Comparison of the results

	Analytical	FEA Calcpad	FEA SAP 2000
w, mm	6,627	6,629	6,529
M*x*, kNm/m	6,231	6,275	6,22
My, kNm/m	12,315	12,744	12,76
M*xy*, kNm/m	8,329	8,378	7,25

Difference, %

	Analytical	FEA Calcpad	FEA SAP 2000
w, mm	0,00%	0,03%	-1,48%
M*x*, kNm/m	0,00%	0,71%	-0,18%
My, kNm/m	0,00%	3,48%	3,61%
M*xy*, kNm/m	0,00%	0,59%	-12,95%