Analysis of continuous beam by force method

Input data

Span lengths - $\vec{l} = [4; 7; 3; 5] \cdot \mathrm{m} = [4\,\mathrm{m} \;\; 7\,\mathrm{m} \;\; 3\,\mathrm{m} \;\; 5\,\mathrm{m}]$

Number of spans - $n = \mathbf{len}(\vec{l}) = 4$

Coordinates of supports - $\vec{x} = [4\,\mathrm{m} \;\; 11\,\mathrm{m} \;\; 14\,\mathrm{m} \;\; 19\,\mathrm{m}]$

Total beam length - $L = \vec{x}_4 = 19\,\mathrm{m}$

Load - $q = 10\dfrac{\mathrm{kN}}{\mathrm{m}}$

Material

Elastic modulus - $E = 30\,\mathrm{GPa}$

Poisson's ratio - $\nu = 0.2$

Shear modulus - $G = \dfrac{E}{2 \cdot (1 + \nu)} = \dfrac{30\,\mathrm{GPa}}{2 \cdot (1 + 0.2)} = 12.5\,\mathrm{GPa}$

Cross section

Rectangular with dimensions: $b = 250\,\mathrm{mm}$, $h = 500\,\mathrm{mm}$

Area - $A = b \cdot h = 250\,\mathrm{mm} \cdot 500\,\mathrm{mm} = 125000\,\mathrm{mm}^2$

Moment of inertia - $I = \dfrac{b \cdot h^3}{12} = \dfrac{250\,\mathrm{mm} \cdot (500\,\mathrm{mm})^3}{12} = 2604166667\,\mathrm{mm}^4$

Shear area - $A_Q = \dfrac{5}{6} \cdot b \cdot h = \dfrac{5}{6} \cdot 250\,\mathrm{mm} \cdot 500\,\mathrm{mm} = 104167\,\mathrm{mm}^2$

Solution

The solution will be performed by the force method with a primary system - simply supported beam with internal supports removed and replaced by unknown forces X_i

Bending moments

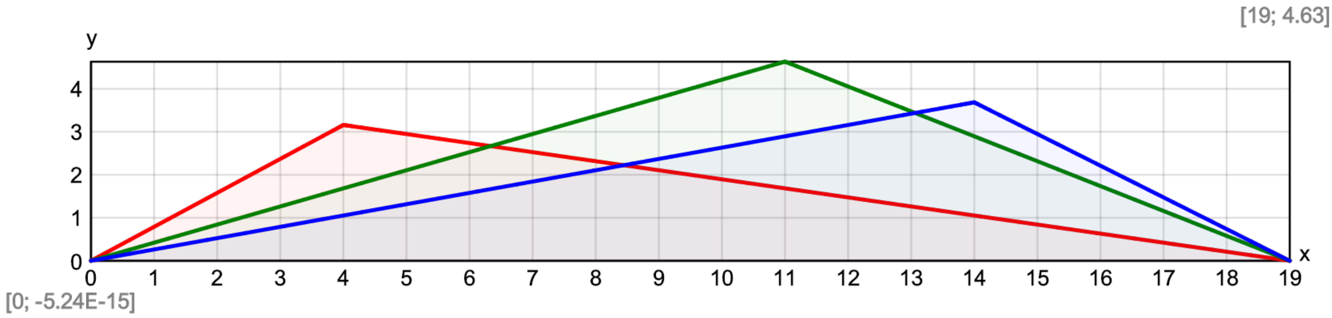
- in section *a* due to unit force at distance *x* from the beginning of the beam:

$$M_{1,\max}(x) = \left(\frac{x}{L} - 1\right) \cdot x$$

$$M_{1,a}(a; x) = M_{1,\max}(x) \cdot \begin{cases} \text{if } a < x: \dfrac{a}{x} \\ \text{else: } \dfrac{L - a}{L - x} \end{cases}$$

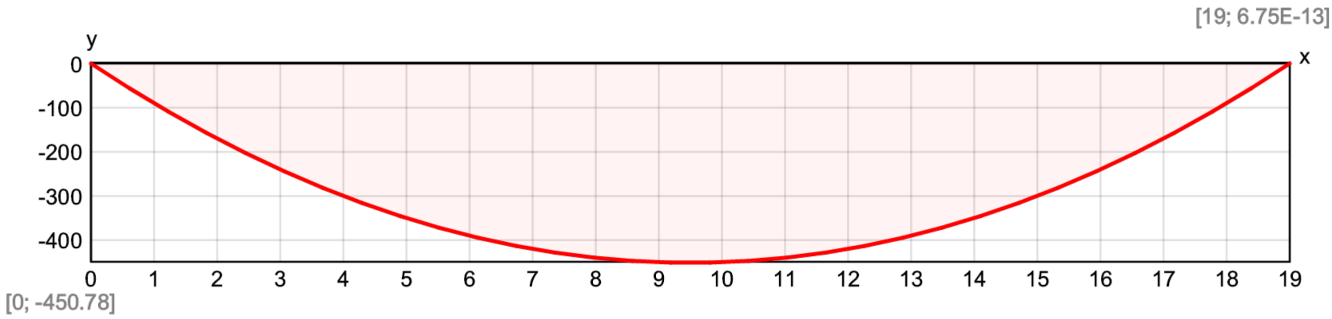
- in section *a*, due to unit force X_i:

$$M_1(a; i) = M_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$M_F(x) = \frac{q \cdot x}{2} \cdot (L - x)$$



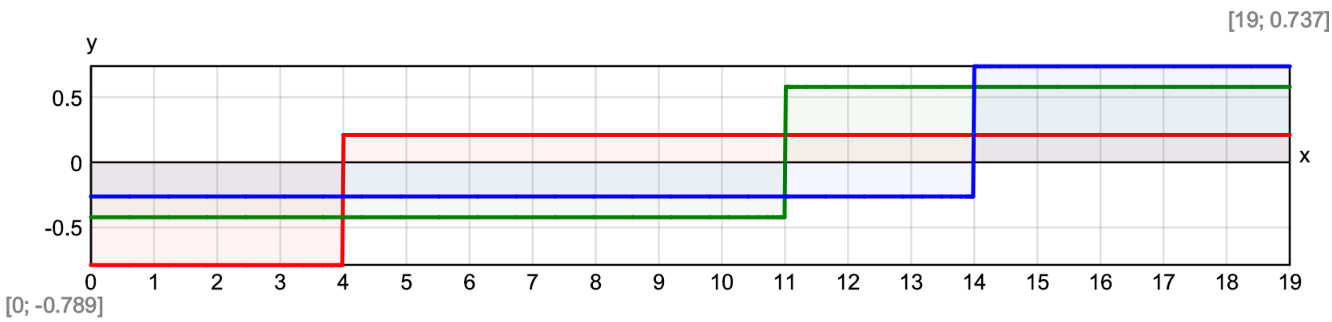
Shear forces

- in section *a* due to unit force at distance *x* from the beginning of the beam:

$$V_{1,a}(a; x) = \begin{cases} \text{if } a < x: \frac{x}{L} - 1 \\ \text{else: } \frac{x}{L} \end{cases}$$

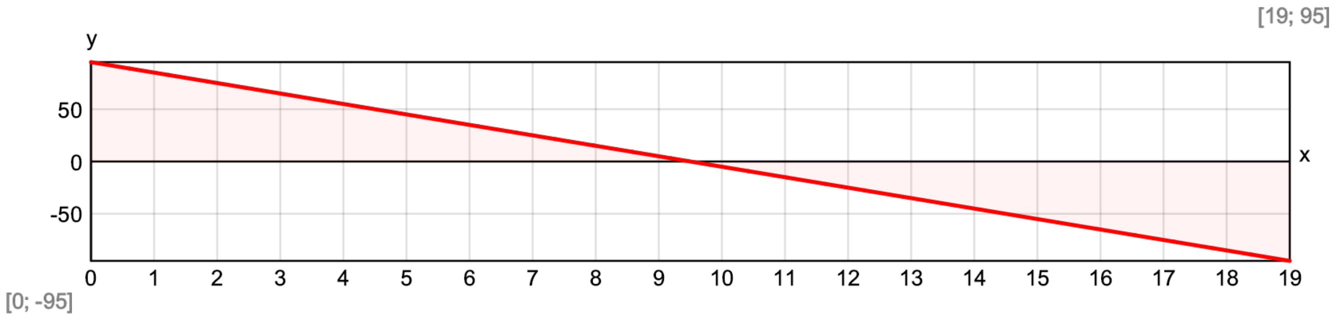
- in section *a*, due to unit force X_i:

$$V_1(a; i) = V_{1,a}(a; \vec{x}_i)$$



- due to external loads in primary system:

$$V_F(x) = q \cdot \left(\frac{L}{2} - x\right)$$



Number of unknowns by force method - $n_1 = n - 1 = 4 - 1 = 3$

Flexibility coefficients

$$\delta(i; j) = \int_{0\,\mathrm{m}}^{L} \frac{M_1(x; i) \cdot M_1(x; j)}{E \cdot I} \, \mathrm{d}x + \int_{0\,\mathrm{m}}^{L} \frac{V_1(x; i) \cdot V_1(x; j)}{G \cdot A_Q} \, \mathrm{d}x$$

$$\Delta_F(i) = \int_{0\,\mathrm{m}}^{L} \frac{M_1(x; i) \cdot M_F(x)}{E \cdot I} \, \mathrm{d}x + \int_{0\,\mathrm{m}}^{L} \frac{V_1(x; i) \cdot V_F(x)}{G \cdot A_Q} \, \mathrm{d}x$$

$$\delta = \mathbf{symmetric}(n_1) = \mathbf{symmetric}(3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$, \vec{\Delta}_F = \mathbf{vector}(n_1) = \mathbf{vector}(3) = [0 \;\; 0 \;\; 0]$

$\$\mathrm{Repeat}\{\$\mathrm{Repeat}\{\delta_{i,\,j} = \delta(i; j);\; i = 1 \ldots n_1\};\; j = 1 \ldots n_1\} = 0.0011\ \mathrm{m/kN}$

$\$\mathrm{Repeat}\{\vec{\Delta}_{F.i} = \Delta_F(i);\; i = 1 \ldots n_1\} = -0.161\ \mathrm{m}$

$$\delta = \begin{bmatrix} 0.000811\ \mathrm{m/kN} & 0.00101\ \mathrm{m/kN} & 0.000719\ \mathrm{m/kN} \\ 0.00101\ \mathrm{m/kN} & 0.00174\ \mathrm{m/kN} & 0.00133\ \mathrm{m/kN} \\ 0.000719\ \mathrm{m/kN} & 0.00133\ \mathrm{m/kN} & 0.0011\ \mathrm{m/kN} \end{bmatrix}$$

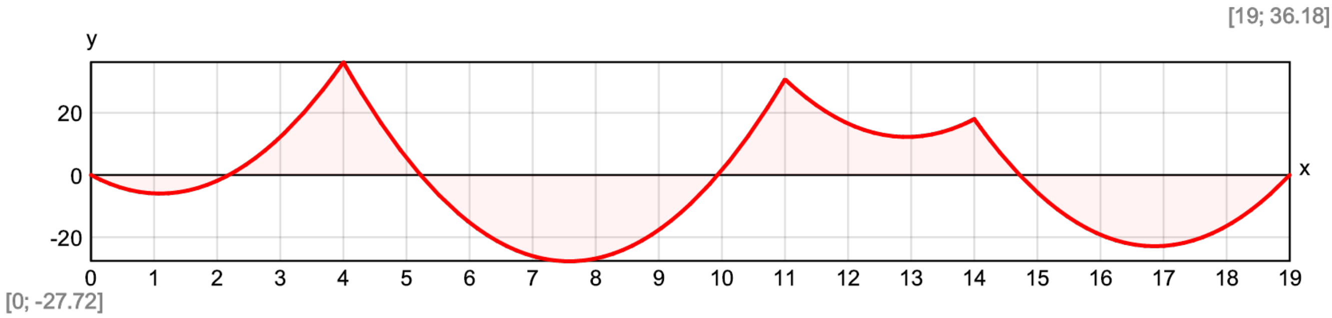
$, \vec{\Delta}_F = [-0.135\ \mathrm{m} \;\; -0.211\ \mathrm{m} \;\; -0.161\ \mathrm{m}]$

Calculation of the unknown forces X_i

$\vec{X} = -\mathbf{clsolve}(\delta; \vec{\Delta}_F) = [64.86\ \mathrm{kN} \;\; 53.48\ \mathrm{kN} \;\; 39.33\ \mathrm{kN}]$

Results

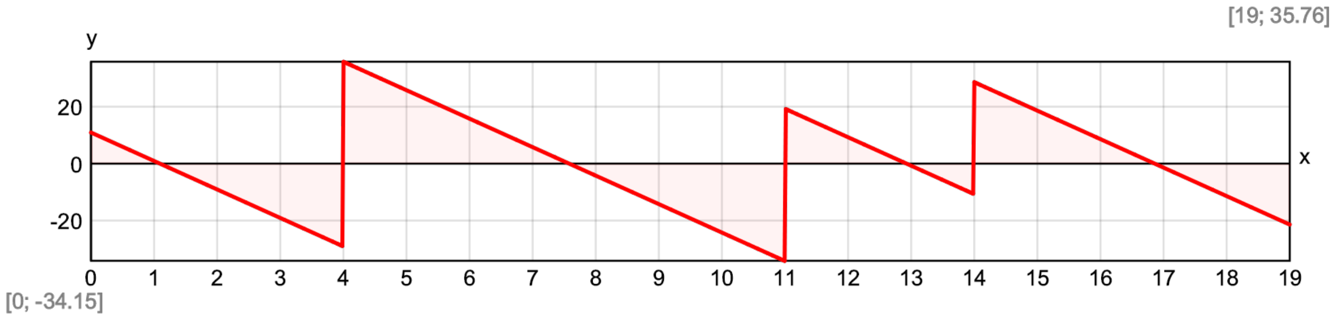
Bending moment diagram - $M(x) = M_F(x) + \sum_{i=1}^{n_1} M_1(\vec{x}; i) \cdot X_i$



$\vec{M}_{max} = [5.97\ \mathrm{kNm} \;\; 27.74\ \mathrm{kNm} \;\; -12.23\ \mathrm{kNm} \;\; 22.9\ \mathrm{kNm}]$

$\vec{M}_{min} = [-36.29\ \mathrm{kNm} \;\; -30.79\ \mathrm{kNm} \;\; -17.99\ \mathrm{kNm}]$

Shear force diagram - $V(x) = V_F(x) + \sum_{i=1}^{n_1} V_1(\vec{x}; i) \cdot \vec{X}_i$



$\vec{V}_{max} = [10.93\,\mathrm{kN}\;\; 35.79\,\mathrm{kN}\;\; 19.26\,\mathrm{kN}\;\; 28.6\,\mathrm{kN}]$

$\vec{V}_{min} = [-29.07\,\mathrm{kN}\;\; -34.21\,\mathrm{kN}\;\; -10.74\,\mathrm{kN}\;\; -21.4\,\mathrm{kN}]$

Deflections

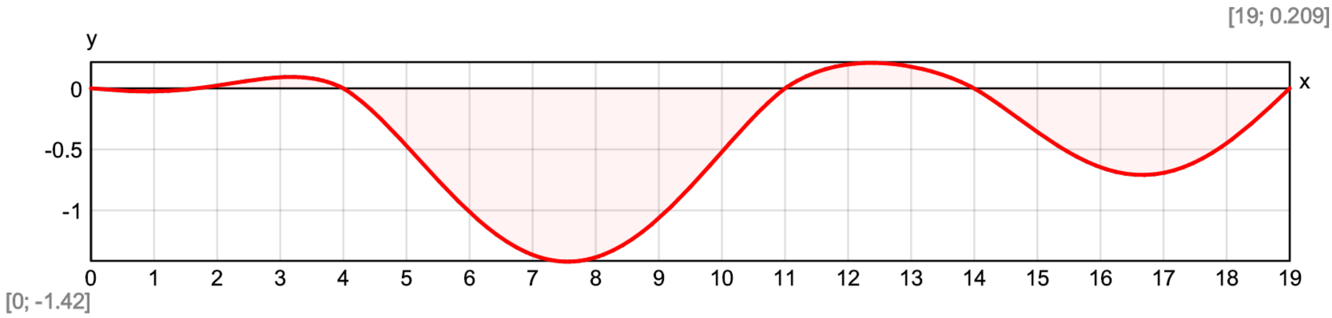
- in section *a*, due to unit force X_i:

$$d_1(a;\, i) = \int_{0\,\mathrm{m}}^{L} \frac{M_{1,a}(x;\, a) \cdot M_1(x;\, i)}{E \cdot I}\, \mathrm{d}x + \int_{0\,\mathrm{m}}^{L} \frac{V_{1,a}(x;\, a) \cdot V_1(x;\, i)}{G \cdot A_Q}\, \mathrm{d}x$$

- due to external loads in primary system:

$$d_F(a) = \int_{0\,\mathrm{m}}^{L} \frac{M_{1,a}(x;\, a) \cdot M_F(x)}{E \cdot I}\, \mathrm{d}x + \int_{0\,\mathrm{m}}^{L} \frac{V_{1,a}(x;\, a) \cdot V_F(x)}{G \cdot A_Q}\, \mathrm{d}x$$

$$d(x) = d_F(x) + \sum_{i=1}^{n_1} d_1(\vec{x};\, i) \cdot \vec{X}_i$$



Maximum deflection - $d_{max} = \$\mathrm{Inf}\{d(x);\, x \in [0\,\mathrm{m};\, L]\} = -1.42\,\mathrm{mm}$

At a distance from the origin - $x_{inf} = 7.56\,\mathrm{m}$