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Case Study: Linear Models in Economics

*Note: The MATLAB m-file **case1.m** accompanying this Case Study is not necessary to complete this project. If it is used, download the file and set the path browser in MATLAB to where the file is located.*

The Introductory Example for Chapter 1 used Wassily Leontief's work in economics as a starting point for studying systems of linear equations. Section 1.6 contains a simplified discussion of Leontief's "exchange model." This case study examines how Leontief actually created equations for his exchange model, and how some questions about the economy may be answered using this model.

The data in Table 1 was obtained directly from Leontief's original 1951 paper (Reference 1), although the 42 sectors Leontief considered have here been aggregated into just five: agriculture, manufacturing, services, government and other, and households. This collection of data is called an **input-output table** (or an **exchange table**) for an economy.

	Agriculture	Manufacturing	Services	Government and Other	Households	Total
Agriculture	34.69	5.28	10.45	7.92	26.22	84.56
Manufacturing	4.92	61.82	25.95	12.45	58.29	163.43
Services	5.62	22.99	42.03	42.53	105.86	219.03
Government and Other	6.27	15.32	16.17	38.99	30.91	107.66
Households	32.97	44.70	114.48	33.09	2.12	227.36
Totals	84.47	150.11	209.08	134.98	223.40	802.04

Table 1: Exchange of Goods and Services in the U.S. for 1947 (in billions of 1947 dollars)

The output of each sector is listed down each column; for example, in 1947 the agriculture sector produced 84.47 billion dollars worth of output, which was divided among the sectors as follows: 34.69 billion dollars of agricultural output was used by the agriculture sector itself, 4.92 billion dollars of agricultural output was used by the manufacturing sector, etc. The output of the household sector is **labor**: in 1947 26.22 billion dollars worth of labor was used by the agriculture industry, while 30.91 billion dollars of labor was used by the government sector.

In order for the analysis to proceed, the fraction of each sector's output which is used by each sector must be known. This is easily accomplished by dividing each column of the table by its total. The result is Table 2.

	Agriculture	Manufacturing	Services	Government and Other	Households
Agriculture	0.4107	0.0352	0.0500	0.0587	0.1174
Manufacturing	0.0582	0.4118	0.1241	0.0922	0.2609
Services	0.0665	0.1532	0.2010	0.3151	0.4739
Government and Other	0.0742	0.1021	0.0773	0.2889	0.1384
Households	0.3904	0.2977	0.5476	0.2451	0.0094

Table 2: 1947 Exchange Table -- Outputs Expressed as Fractions of Total Sector Output

The total dollar value of the output of a sector is the **price** of that sector. Thus the price of the agricultural sector in the analysis is 84.47 billion dollars. As discussed in Section 1.6 of the text, Leontief proved that there exist **equilibrium prices** for the output of each sector such that the income of each sector exactly balances its expenses. The goal is to find the equilibrium prices for the 1947 American economy. Let p_A, p_M, p_S, p_G, and p_H be the equilibrium prices of (respectively) the agriculture, manufacturing, services, government, and households sectors.

Consider the agriculture sector. At equilibrium, this sector's income will be p_A, that is, the price it charges the economy for its output. This must balance the sector's expenses. What are these expenses? First, the agriculture sector must "purchase" 41.07% of its own output; this will cost the sector .4107 p_A. Likewise the agriculture sector must purchase 3.52% of manufacturing output, and this will cost the agriculture sector .0352 p_M. Continuing in like fashion, the total expenses of the agriculture sector will be

$$.4107\,p_A + .0352\,p_M + .0500\,p_S + .0587\,p_G + .1174\,p_H\,.$$

For income to balance expenses it must be that

$$p_A = .4107\,p_A + .0352\,p_M + .0500\,p_S + .0587\,p_G + .1174\,p_H$$

or

$$.5893\,p_A - .0352\,p_M - .0500\,p_S - .0587\,p_G - .1174\,p_H = 0\,.$$

Thus each sector generates an income-expense equation, and the following homogeneous system of five equations in five unknowns results:

$$\begin{aligned}
.5893\,p_A - .0352\,p_M - .0500\,p_S - .0587\,p_G - .1174\,p_H &= 0\\
-.0582\,p_A + .5882\,p_M - .1241\,p_S - .0992\,p_G - .2609\,p_H &= 0\\
-.0665\,p_A - .1532\,p_M + .7990\,p_S - .3151\,p_G - .4739\,p_H &= 0\\
-.0742\,p_A - .1021\,p_M - .0773\,p_S + .7111\,p_G - .1384\,p_H &= 0\\
-.3904\,p_A - .2977\,p_M - .5476\,p_S - .2451\,p_G + .9906\,p_H &= 0
\end{aligned}$$

This homogeneous system will indeed have an infinite number of solutions.

Let's use a coefficient matrix made from the original table with entries written as rational numbers. (If your MATLAB working path is set to the m-file accompanying this case study, type case1 to get the matrix *A*.)

```
format rat;
A = eye(5) - [3469/8447 528/15011   1045/20908 792/13498 2622/22340;
            492/8447 6182/15011 2595/20908 1245/13498 5829/22340;
            562/8447 2299/15011 4203/20908 4253/13498 10586/22340;
            627/8447 1532/15011 1617/20908 3899/13498 3091/22340;
            3297/8447 4470/15011 11448/20908 3309/13498 212/22340]
```

You can row reduce *A* by using the MATLAB command

```
format long; rref(A)
```

to confirm that, with p_H as a free variable, the general solution of this system is

$$\rho = \begin{pmatrix} p_A \\ p_M \\ p_S \\ p_G \\ p_H \end{pmatrix} = \begin{pmatrix} .367763 \\ .748189 \\ .941724 \\ .442739 \\ 1 \end{pmatrix}.$$

The format long command was used to provide more decimal places.

Any choice for p_H will give a set of equilibrium prices, but the equilibrium prices should be able to be compared with the actual prices of the sectors from the data. Thus the entries in the equilibrium price vector should add up to the total output of the economy, which was 802.04 billion dollars. One may solve

$$p_A = .367763\, p_A + .748189\ p_M\ + .941724\, p_S + .442739\, p_G + 1\, p_H\ = 802.04$$

to find that p_H = 229.127, and thus p_A = 84.2646, p_M = 171.430, p_S = 215.774, and p_G = 101.444.

Questions:

1. What do the row and column totals in Table 1 signify?

2. Suppose that the amount that the Manufacturing sector pays for Household labor increases by 10% from 58.29 to 64.12 billion dollars, and that the amount that the Services sector pays for Household labor increases by 10% from 105.86 to 116.45 billion dollars. Construct a revised exchange table for this situation and compute the new equilibrium prices. How do they differ from the old equilibrium prices? (If you have the m-file case1.m accessible, type `case1; B` in MATLAB.)

More recent input-output tables may be found at the Bureau of Economic Analysis website (`www.bea.gov`). Exchange tables for several recent years (most recently 1998) may be examined at and downloaded from `www.bea.gov/bea/dn2/i-o.htm`. The 1998 exchange table accompanies this case study as an Excel worksheet. A good overview of the chart, entitled "Annual Input-Output Accounts of the U.S. Economy, 1998" may be found at the same website. Into how many sectors is the U.S. economy divided for these more recent tables?

References:

1. Leontief, Wassily W. "Input-Output Economics." *Scientific American*, October 1951, pp.15-21.

This article explains the author's input-output model, which is the subject of Section 2.7 of the text. The article also includes the complete 42-sector exchange table for 1947.

2. Leontief, Wassily W. *Input-Output Economics*. New York: Oxford University Press, 1966.

This book contains the full 42-sector exchange table for 1947, as well as an 81-sector table for 1958.

3. Leontief, Wassily W. "The Structure of the U.S. Economy." *Scientific American*, April 1965, pp. 25-35.

This article contains the 81-sector table mentioned above.