FIRST EXAM-C

Instructions: Begin each of the seven numbered problems on a new page in your answer book. Show your work.

1. [20 Pts] Find the general solution of the following homogeneous system of equations. Express your answer in parametric vector form. Use the method developed in class.

$$\begin{cases} x_1 - 2x_2 + 2x_3 - 6x_4 = 0 \\ x_3 - 5x_4 = 0 \\ 2x_1 - 4x_2 + x_3 + 3x_4 = 0 \end{cases}$$

2. [20] Let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix}$.

a. Give a geometric description of Span$\{\mathbf{u}, \mathbf{v}\}$. (What does it look like?)

b. Determine if $\mathbf{w}$ belongs to Span$\{\mathbf{u}, \mathbf{v}\}$.

c. Give a description of Span$\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. Justify your answer.
Hint: Consider the matrix A whose columns are $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.

3. [10] In (a) and (b), decide by inspection if the columns of the matrix A are linearly independent. Use **no** row operations. Justify your answers. (You may cite well-known facts or theorems.)

a. $A = \begin{bmatrix} -5 & 15 \\ 2 & -6 \\ -3 & 10 \end{bmatrix}$ **b.** $A = \begin{bmatrix} 7 & 0 & 4 \\ 3 & 0 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

4. [15] Complete the following definitions.

a. If A is an $m \times n$ matrix and $\mathbf{x}$ is a vector in $\mathbb{R}^n$, then $A\mathbf{x}$ ______. (Complete the statement in words, if possible.)

b. A set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ is linearly independent if ______. (Read carefully.)

c. A linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^m$ is a mapping with the properties: ______.

5. [5] How many rows and columns must a matrix A have in order to define a linear transformation $T\colon \mathbb{R}^5 \mapsto \mathbb{R}^7$ by the rule $T(\mathbf{x}) = A\mathbf{x}$?

6. [15] Suppose that A is a 4×3 matrix. Could the columns of A possibly span $\mathbb{R}^4$? Why or why not? (If appropriate, mention a theorem in your discussion.)

7. [15] A mining company has two mines. One day's operation of mine #1 produces ore that contains 20 metric tons of copper and 550 kilograms of silver, while one day's operation of mine #2 produces ore that contains 30 metric tons of copper and 500 kilograms of silver. Suppose the company has orders for 150 tons of copper and 2825 kilograms of silver.

Consider the problem: How many days should the company operate each mine in order to exactly fill these orders?

a. Set up (but do not solve) a *vector equation* that describes this problem. Include a statement about what the variables in the equation represent.

b. Write an equivalent *matrix equation* for this problem. (Do not solve it.)