Interpolating Polynomials

CHAPTER 1 - PROJECT A

The purpose of this project is to show how to use a system of linear equations to fit a polynomial through a set of points. This project expands ideas begun in Section 1.2, and applies these ideas to real data.

Recall from the text that an **interpolating polynomial** for a set of points in the plane is a polynomial whose graph passes through each of the points. In general, it can be shown that given n points with no two points having the same first coordinate, there exists an interpolating polynomial

$$p(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_{n-1} t^{n-1}$$

of degree n-1 or less. (See Exercise 25 in the Chapter 2 Supplementary Exercises for a proof of this fact.) Each of the n points determines a linear equation that the (unknown) coefficients $a_{0,}\, a_1, a_2, \ldots, a_n$ in the polynomial must satisfy.

Questions:

Note: The MATLAB M-file `interpoly.m` *contains data for the questions. Once the file is loaded on your machine, type* `interpoly` *in the MATLAB or Octave command window for the data. The M-file is downloadable from the same location as this PDF.*

1. Find the interpolating polynomial of degree 3 which passes through (1,29), (-1,-35), (2,31), and (-3,-19).

   In MATLAB, we can use the above information to find the coefficients for the interpolating polynomial. Given the x-values of 1, -1, 2, and -3, we can find the appropriate coefficient matrix by the following commands:

   ```
   xval = [1; -1; 2; -3]; b = [29; -35; 31; -19];
   B=[ones(4,1) xval xval.^2 xval.^3]
   ```

   The `ones(4,1)` command creates a 4×1 matrix of ones while `xval.^2` squares each entry of `xval`. You can then use appropriate row reduction techniques on the matrix `[B b]` to find the coefficients for the interpolating polynomial.

2. To construct a fourth degree polynomial which "looks like" the graph of $y = 2^t$, one could specify that the polynomial go through the following five points: (-1, 0.5), (0,1), (1,2), (2,4), and (3,8).

   a) Find the desired interpolating polynomial.

   b) Use your technology to graph the interpolating polynomial on the same graph with $y = 2^t$. How close are the values of the polynomial and the function for t values between -1 and 3? What happens when the polynomial is compared to the function at t values outside of this interval? If you needed to use the interpolating polynomial to approximate $y = 2^t$ what types of mathematical precautions would you take?

In MATLAB, the following commands would be one way to plot two functions on the same plot:

```
t=linspace(-5,5,30);
y=2.^t; z=1+17/24*t+11/48*t.^2;
plot (t, y, t, z)
```

The **`linspace`** command creates 30 points between –5 and 5. Since MATLAB stores t as a matrix, we must use the **`.^`** command to store the appropriate values for y. You can modify the function z for part b of this problem.

3. Interpolating polynomials allow us to estimate values of an unknown function between known data points. For example, consider the data in Table 1, which was taken from Car and Driver magazine[1]. Five sport utility vehicles were tested for their acceleration: the data reported were the time it took each vehicle to go from 0 to 30 m.p.h., from 0 to 60 m.p.h., and from 0 to 90 m.p.h. For each vehicle, write the third degree interpolating polynomial and use that polynomial to find an estimate for the time it would take the vehicle to accelerate from 0 m.p.h. to 50 m.p.h. Note: to obtain the polynomials, it may be easier to write the data in the form suggested by Table 2.

Vehicle	0 mph-30 mph	0 mph-60 mph	0 mph-90 mph
Honda CR - VEX	3.1	10.3	30.1
Jeep Cherokee SE	3.2	12	38.2
Kia Sportage	4.2	12.8	38.7
Subaru Forester L	2.8	9.5	22.7
Toyota RAV4	3.0	10.2	31.7

Table 1: Acceleration Times (seconds) for various models of automobiles

Honda CR - VEX	Time	0	3.1	10.3	30.1
	Velocity	0	30	60	90
Jeep Cherokee SE	Time	0	3.2	12	38.2
	Velocity	0	30	60	90
Kia Sportage	Time	0	4.2	12.8	38.7
	Velocity	0	30	60	90
Subaru Forester L	Time	0	2.8	9.5	22.7
	Velocity	0	30	60	90
Toyota RAV4	Time	0	3.0	10.2	31.7
	Velocity	0	30	60	90

Table 2: Elapsed Time (seconds) and Velocities (m.p.h.)
for various models of automobiles

As an aid to the solution of this problem, the MATLAB command **roots** computes the roots of the polynomial whose coefficients are the elements of a $1 \times n$ matrix with the highest degree coefficient first. For example, to solve the equation $11.7994t - 0.7295t^2 + 0.0145t^3 = 50$, consider the polynomial equation

$$0.0145t^3 - 0.7295t^2 + 11.7994t - 50 = 0.$$

```
>>roots([0.0145 -0.7295 11.7994 -50])
     ans =
       21.8879 + 6.9725i
       21.8879 - 6.9725i
        6.5346
```

The real solution to the polynomial is therefore approximately 6.5346.

4. In the problem above, consider the total **distance** each vehicle will travel while it accelerates from 0 to 90 m.p.h. That quantity can be calculated by integrating the velocity function *v(t)* for the vehicle. There is one small problem to overcome first. In this problem *v(t)* is measured in miles per hour and *t* is measured in seconds, so *v(t)* should be converted to miles per second before proceeding. The velocity function in miles per second is *v(t)*/3600. The distance traveled by the vehicle would be $\int_0^T v(t)/3600\,dt$, where T is the time at which the vehicle reaches 90 m.p.h. The real velocity function *v(t)* is not known in this case, but interpolating polynomials for *v(t)* are available which should approximate *v(t)*. Use the interpolating polynomials you generated in the previous exercise to approximate the distance needed for each vehicle to accelerate from 0 to 90 m.p.h.

5. One major issue in using interpolating polynomials can be called "goodness of fit." In fitting a polynomial through given data points, it is assumed that there ought to be a polynomial relationship between the independent and dependent variables. Question 2 showed that is not always the case. Use MATLAB to plot the interpolating polynomials you generated in Question 3. Do you believe there is a polynomial relationship between elapsed time and the velocity of the vehicles? What evidence do you have to support your conclusion?

Postscript: Another issue in using real data as in Question 3 is that measurement error is introduced into the data; these errors could obscure the true relationship between the dependent and independent variables. Fitting polynomials to data taking that issue into account may be addressed using methods from Chapter 6 of the text.

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[1] *Car and Driver*, May 1998, p.102