MATLAB Project: Exchange Economy and Homogeneous Systems Name________________

Purpose: To solve a homogeneous system to find equilibrium prices for an exchange model economy.
Prerequisite: Section 1.2
MATLAB built-in functions used: `-`, `/`, `eye`, `sum`
M-files used: `econdat` and `ref` from the `Laydata5` Toolbox or from pearsonhighered.com/lay

1. Let $T = \begin{bmatrix} .20 & .17 & .25 & .20 & .10 \\ .25 & .20 & .10 & .30 & 0 \\ .05 & .20 & .10 & .15 & .10 \\ .10 & .28 & .40 & .20 & 0 \\ .40 & .15 & .15 & .15 & .80 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$ and consider the system of linear equations $T\mathbf{x} = \mathbf{x}$.

(a) (hand) Write out the five equations in this system.

(b) Collect terms in your equations to get a homogenous linear system, and write out the five new equations.

2. (MATLAB) Let $B\mathbf{x} = \mathbf{0}$ denote the homogenous system you obtained in 1(b), and calculate the reduced echelon form of $[B \;\; \mathbf{0}]$. Record the reduced form below. These lines will get the matrix and do the calculation:

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econdat                 (get the matrix B)
ref([B zeros(5,1)])     (calculate the reduced echelon form of [B 0])
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Read about Leontief Economic Models in Section 1.6 of the text. Now consider an exchange model economy which has five sectors, Chemicals, Metals, Fuels, Power and Agriculture. Assume the matrix T in question 1 above gives an exchange table for this economy as follows:

$$\begin{array}{c} \begin{array}{ccccc} \mathbf{C} & \mathbf{M} & \mathbf{F} & \mathbf{P} & \mathbf{A} \end{array} \\ T = \begin{bmatrix} .20 & .17 & .25 & .20 & .10 \\ .25 & .20 & .10 & .30 & 0 \\ .05 & .20 & .10 & .15 & .10 \\ .10 & .28 & .40 & .20 & 0 \\ .40 & .15 & .15 & .15 & .80 \end{bmatrix} \end{array} \begin{array}{c} \mathbf{C} \\ \mathbf{M} \\ \mathbf{F} \\ \mathbf{P} \\ \mathbf{A} \end{array}$$

Notice that each column of T sums to one indicating that all output of each sector is distributed among the five sectors, as should be the case in an exchange economy. The system of equations $T\mathbf{x} = \mathbf{x}$ must be satisfied for the economy to be in equilibrium. As you saw above, this is equivalent to the system $B\mathbf{x} = \mathbf{0}$.

Let x_C represent the value of the output of Chemicals, x_M the value of the output of Metals, x_F the value of the output of Fuels, x_P the value of the output of Power, and x_A the value of the output of Agriculture.

3. (a) Using the reduced echelon form of $\begin{bmatrix} B & \mathbf{0} \end{bmatrix}$ from question 2, write the general solution for $T\mathbf{x} = \mathbf{x}$:

$$\begin{bmatrix} x_C \\ x_M \\ x_F \\ x_P \\ x_A \end{bmatrix} =$$

(b) Suppose the economy described above is in equilibrium and $x_A = 100$ million dollars. Calculate the values of the outputs of the other sectors and record this particular solution for the system $T\mathbf{x} = \mathbf{x}$:

$$\begin{bmatrix} x_C \\ x_M \\ x_F \\ x_P \\ x_A \end{bmatrix} =$$

4. (hand) Consider the matrices T and B created above. As already observed, each column of T sums to one. Consider how you obtained B from T and explain why each column of B must sum to zero.

5. (Extra credit) Let B be any matrix of any shape with the property that each column of B sums to zero. Explain why the reduced echelon form of B *must* have a row of zeros.