

$$\begin{cases} x^2 + xy + y^2 = a^2 \\ y^2 + yz + z^2 = b^2 \\ z^2 + zx + x^2 = c^2 \end{cases}$$

$$y = \pm \sqrt{\frac{1}{3} \left\{ 2(a^2 + b^2) - \frac{1}{2} \left[ (a^2 + b^2 + c^2) \pm \sqrt{-3(a^4 + b^4 + c^4) + 6(a^2b^2 + b^2c^2 + c^2a^2)} \right] - \frac{2(b^2 - a^2)^2}{(a^2 + b^2 + c^2) \pm \sqrt{-3(a^4 + b^4 + c^4) + 6(a^2b^2 + b^2c^2 + c^2a^2)}} \right\}}$$

记  $u = a^4 + b^4 + c^4$ ,  $v = a^2b^2 + b^2c^2 + c^2a^2$ ,  $w = a^2 + b^2 + c^2$ , 那么

$$\begin{cases} y = \pm \sqrt{\frac{1}{3} \left\{ 2(a^2 + b^2) - \frac{1}{2} \left[ w \pm \sqrt{3(-u + 2v)} \right] - \frac{2(b^2 - a^2)^2}{w \pm \sqrt{3(-u + 2v)}} \right\}} \\ x = \frac{1}{2} \left( -y \pm \sqrt{4a^2 - 3y^2} \right) \\ z = \frac{1}{2} \left( -y \pm \sqrt{4b^2 - 3y^2} \right) \end{cases}$$