

# Regression Discontinuity

EC 607, Set 10

Edward Rubin

# Prologue

# Schedule

## Last time

- Introduction to selection-on-unobservables designs
- Instrumental variables (IV) and two-stage least squares (2SLS)

## Today

Regression discontinuity<sup>†</sup>

<sup>†</sup> These notes largely follow notes from [Michael Anderson](#), [Imbens and Lemieux \(2008\)](#), and notes from [Teppei Yamamoto](#).

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**Regression discontinuity (RD)** offers a particularly clear/clean research design based upon an arbitrary threshold (the *discontinuity*).

That said, most RDs boil down to an implementation of IV.

In addition, while RD is all the rage in modern applied econometrics, **Thistlewaite and Campbell** wrote about it back in 1960.



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**New:** Suppose  $\mathbf{D}_i$  is determined<sup>†</sup> by whether some variable  $\mathbf{X}_i$  crosses a threshold  $c$  (the discontinuity).

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We will assume that  $Y_{0i}$  and  $Y_{1i}$  vary smoothly in  $X_i$ .

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- An election candidate wins if her vote share exceeds her competitors.
- Election runoffs are triggered if "winner" is below 50%.
- Antidiscrimination laws only apply to firms with >15 employees.
- Prisoners are eligible for early parole if some score exceeds a threshold.
- An individual is eligible for Medicare if her age is at least 65.
- You get a ticket if your speed exceeds the speed limit.
- Fifteen-percent discount at Sizzler if your age exceeds 60.
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In some cases, "treatment" is definite once we exceed the threshold.



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*E.g.*, crossing some GRE threshold discontinuously increases your chances of getting into some grad schools (but doesn't guarantee admittance).

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To estimate the causal effect of  $\mathbf{D}_i$  on  $\mathbf{Y}_i$ , we **compare the mean of  $\mathbf{Y}_i$  just above the threshold to the mean of  $\mathbf{Y}_i$  just below the threshold.**

# Sharp RDs

## More formally

We can write the comparison of means **at the threshold** as

$$\lim_{x \downarrow c} E[Y_i | X_i = x] - \lim_{x \uparrow c} E[Y_i | X_i = x]$$

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*i.e.*, Because we don't observe  $Y_{0i}$  for treated individuals, we extrapolate  $E[Y_{0i} | X_i = c - \varepsilon]$  to  $E[Y_{0i} | X_i = c + \varepsilon]$  for small  $\varepsilon$ .

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## Estimation

Thus, we estimate

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as the difference between two regression functions estimated "near"  $c$ .

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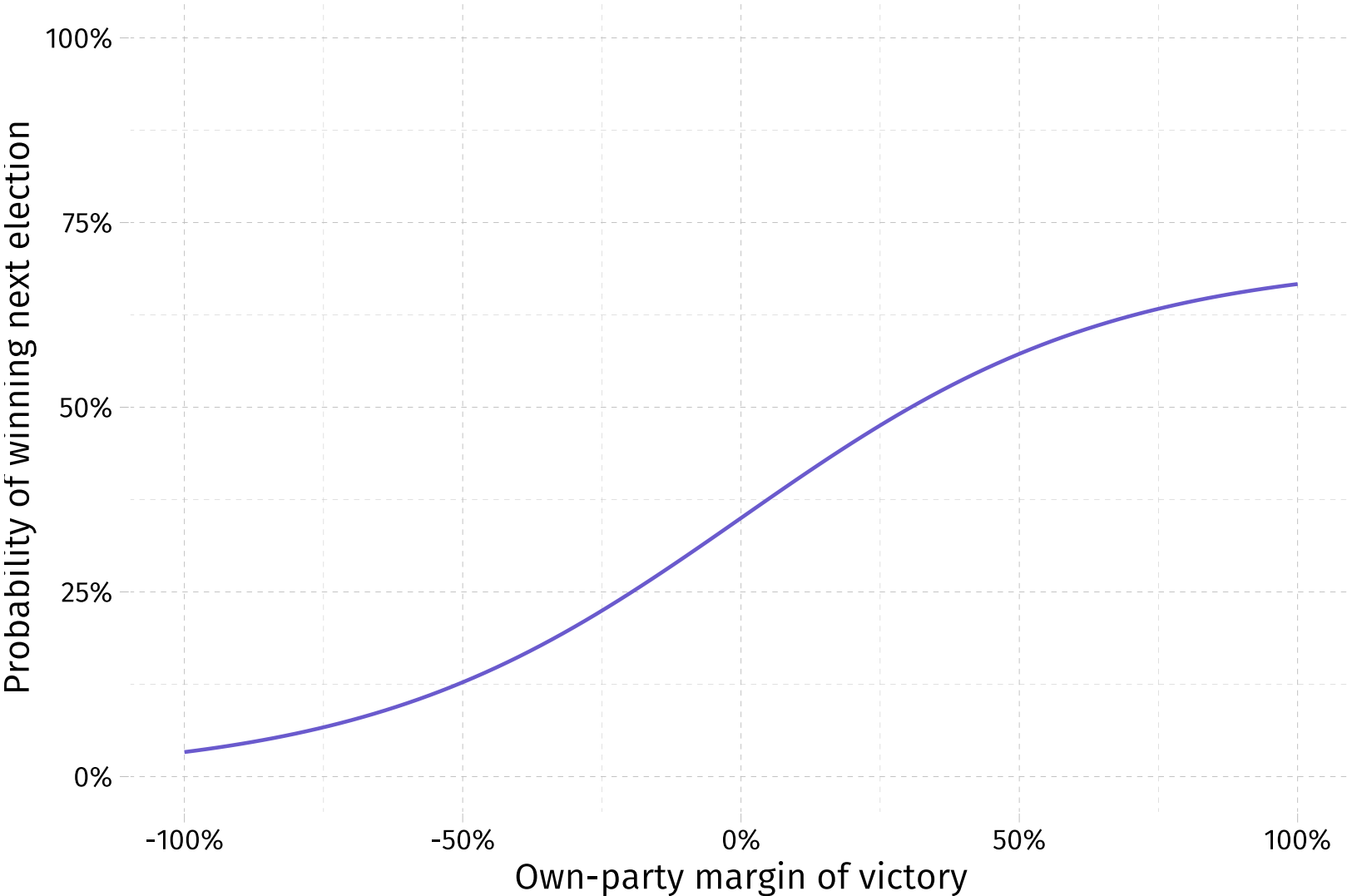
We must stay "near" to  $c$  to minimize the bias from extrapolating  $E[Y_{0i} | \mathbf{X}_i = c - \varepsilon]$  to  $E[Y_{0i} | \mathbf{X}_i = c + \varepsilon]$  (and assuming continuity).

*Ex.* Is there effect of a political party winning an election on that party's likelihood of winning the following election?

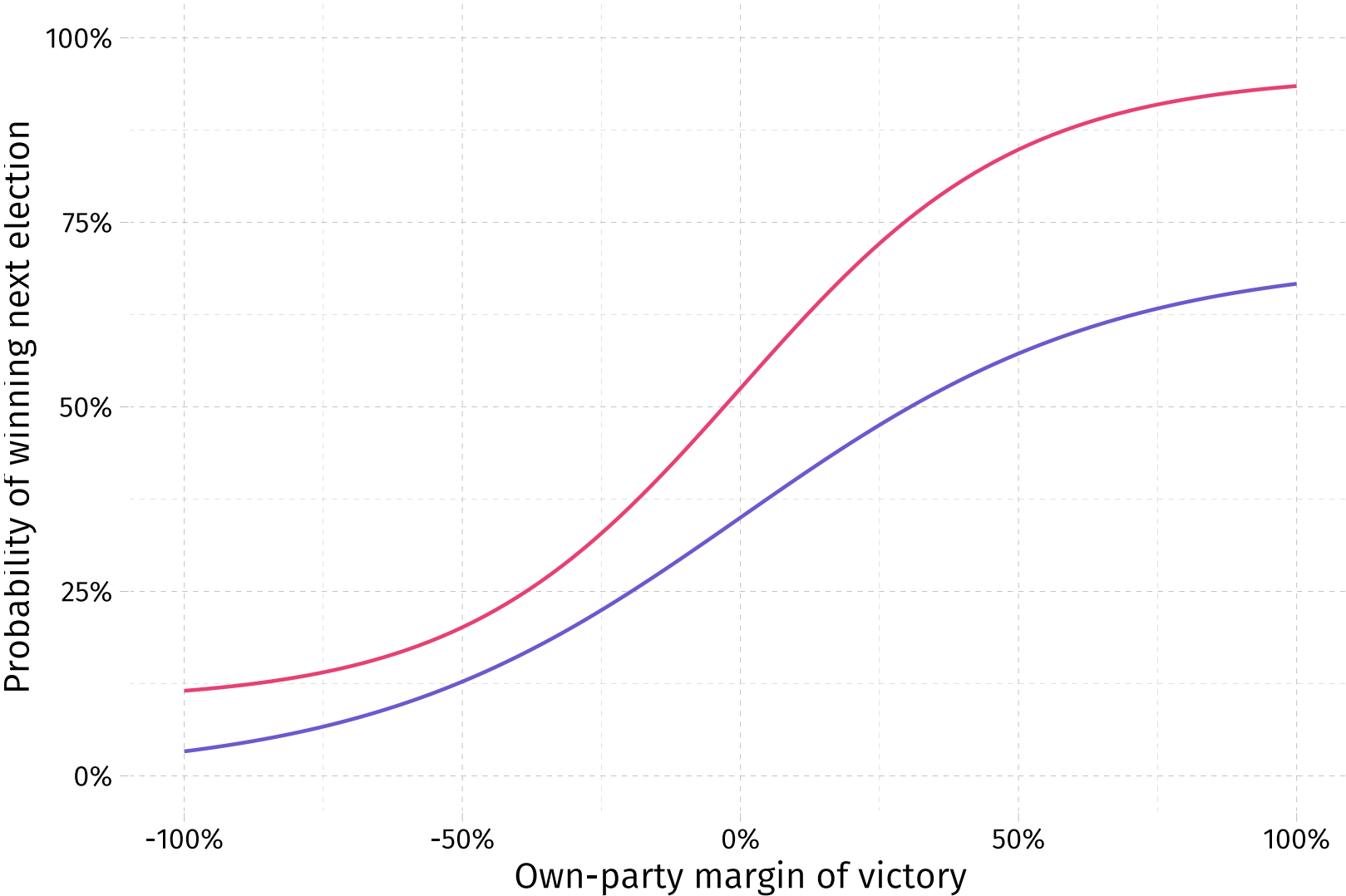
Is there a benefit of incumbency (at the party level)?<sup>†</sup>

<sup>†</sup> Lee (2008) addresses this question via RD. Caughey and Sekhon (2011) discuss RD in this setting.

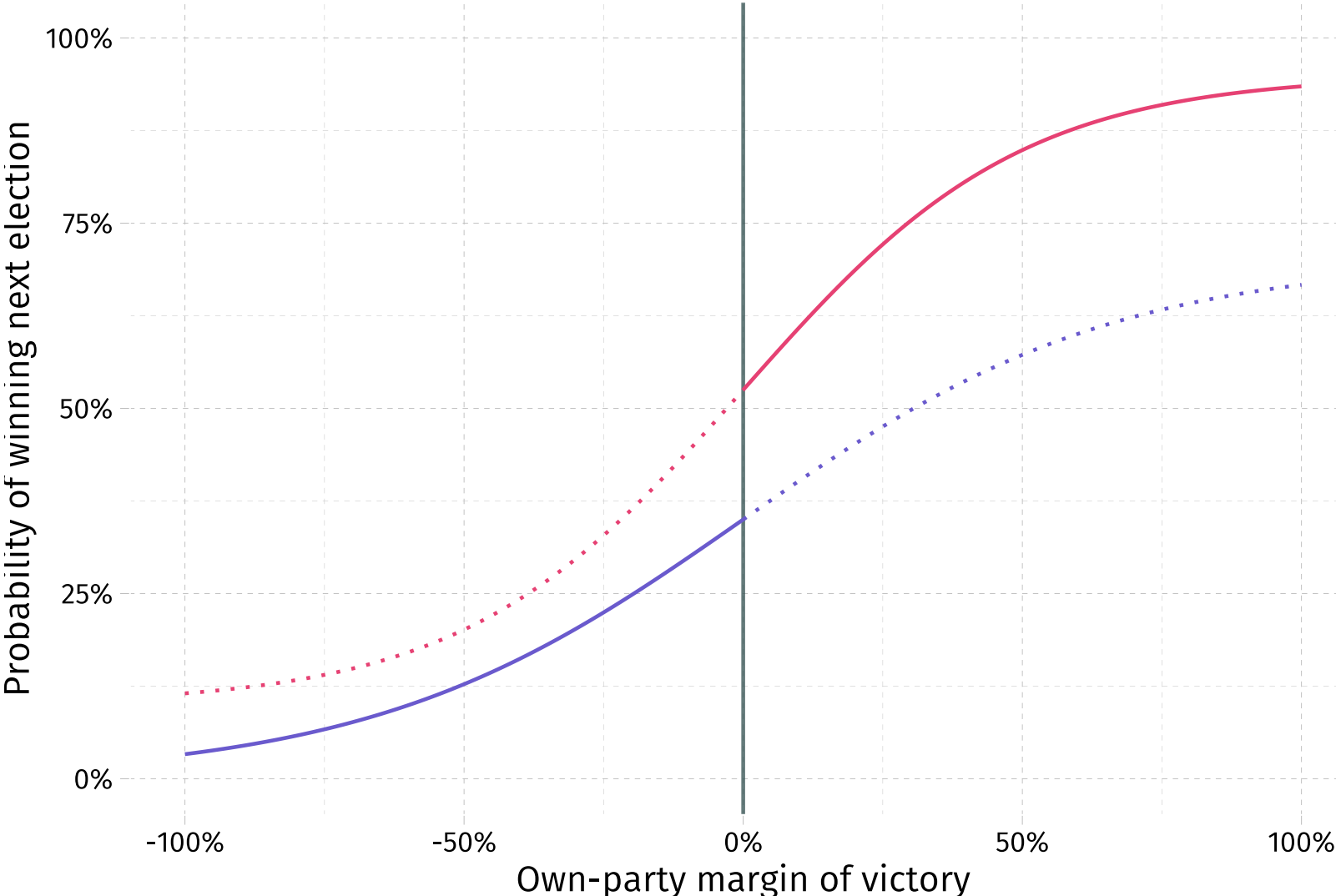
Let's start with  $E[Y_{0i} | X_i]$



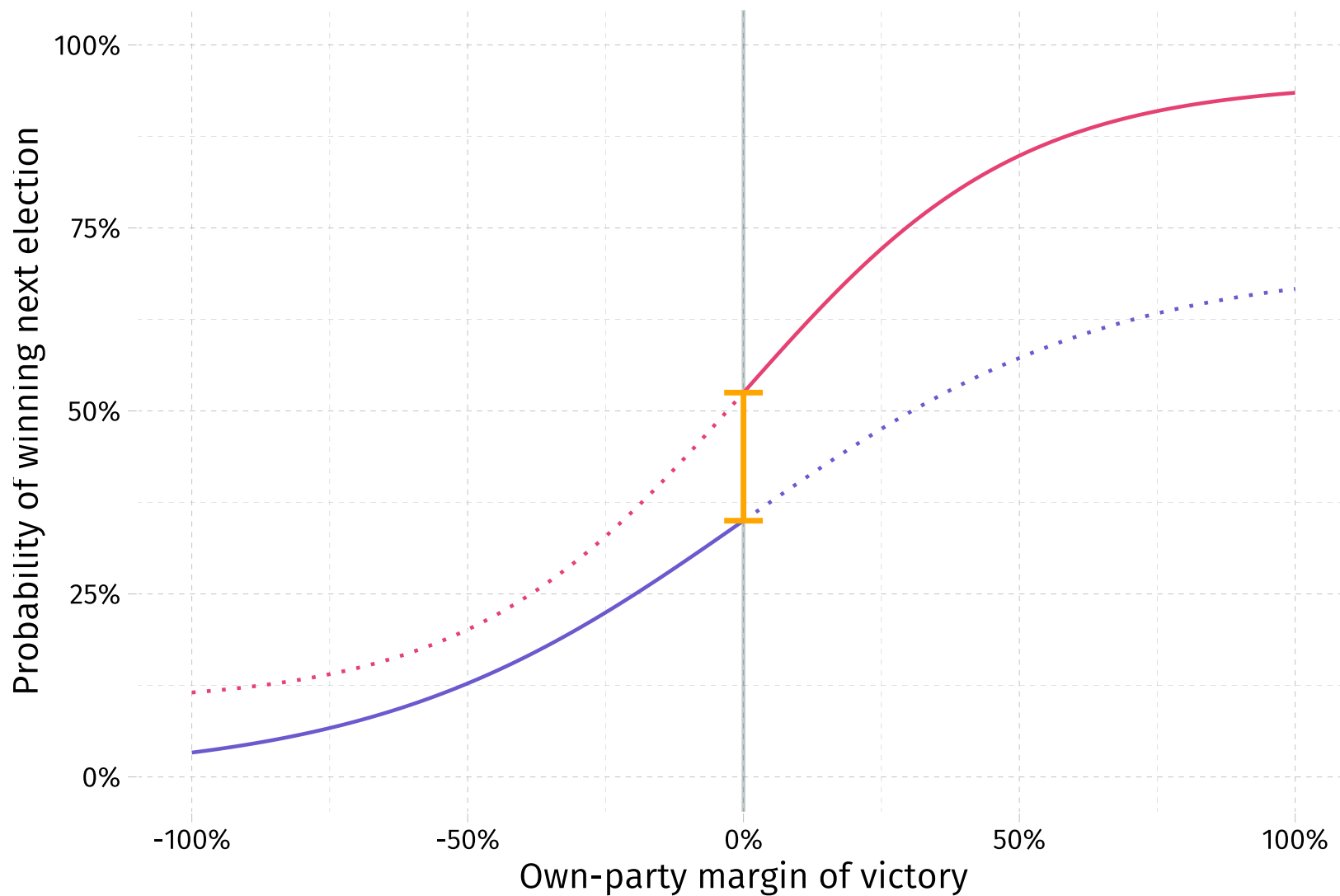
Let's start with  $E[Y_{0i} | X_i]$  and  $E[Y_{1i} | X_i]$ .



You only win an election if your **margin of victory exceeds zero**.

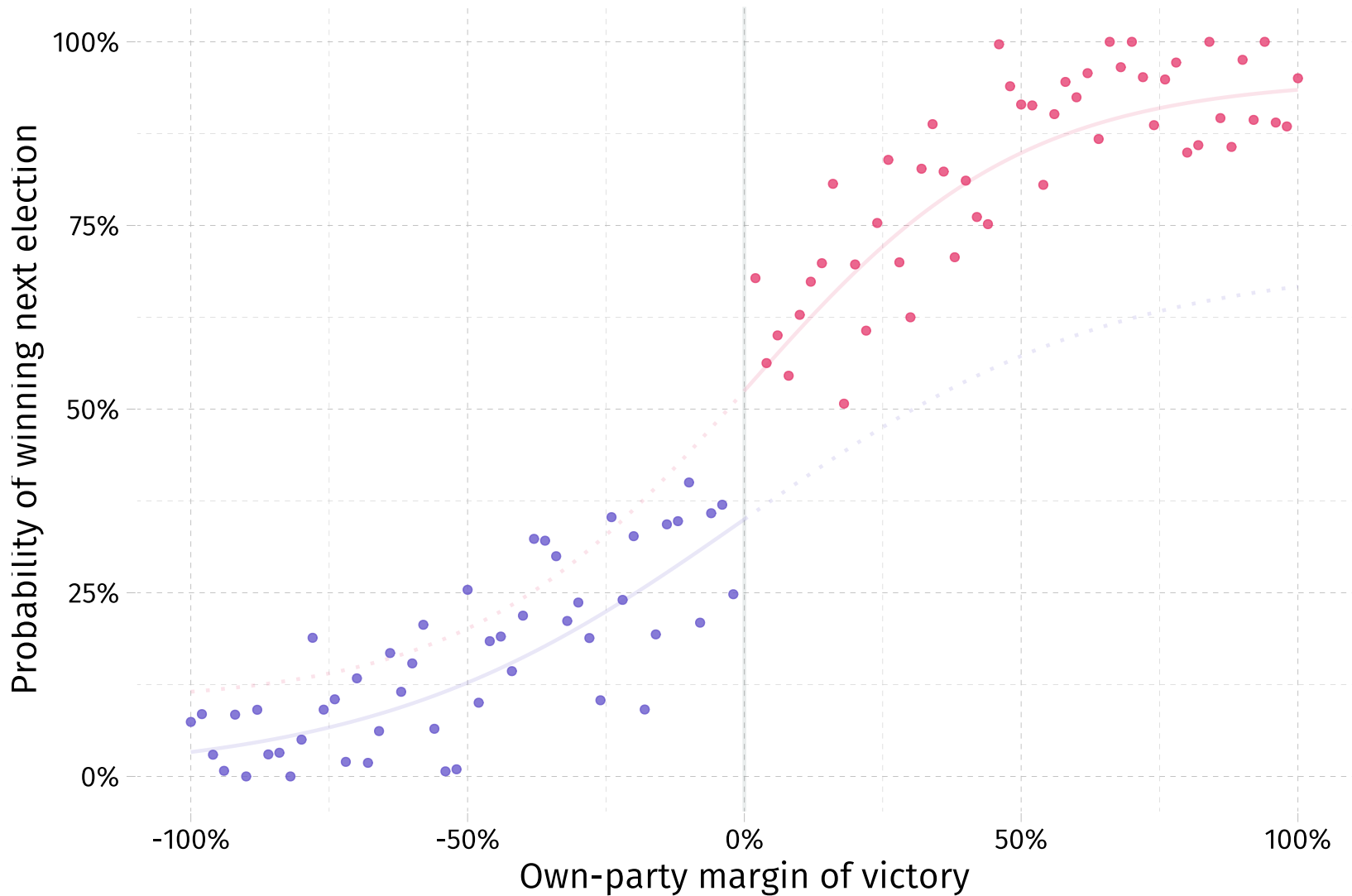


$E[Y_{1i} | X_i] - E[Y_{0i} | X_i]$  at the discontinuity gives  $\tau_{SRD}$ .





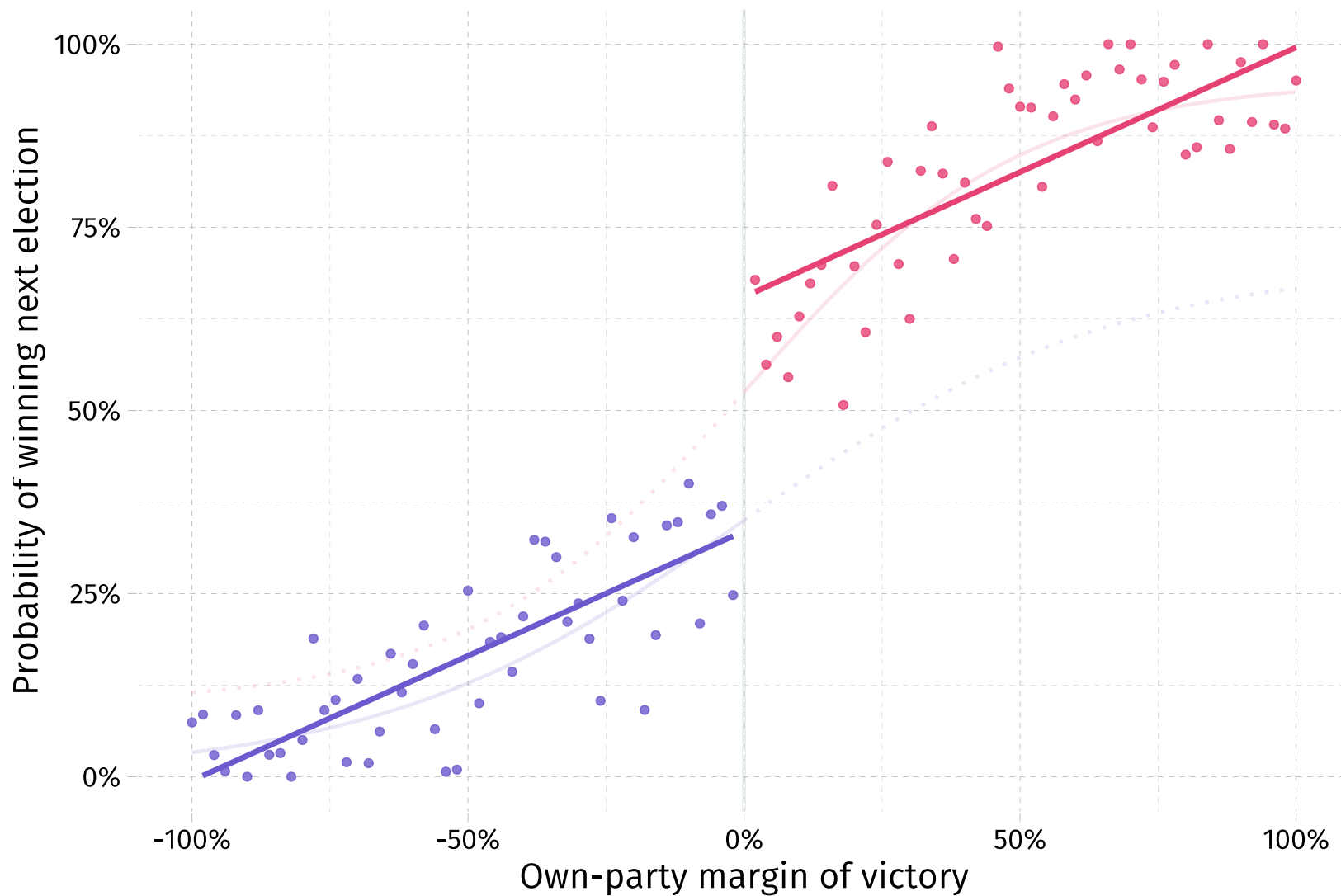
Real data are a bit trickier. We must estimate  $E[Y_{1i} | X_i]$  and  $E[Y_{0i} | X_i]$ .



## Questions

1. How should we estimate  $E[Y_{1i} | X_i]$  and  $E[Y_{0i} | X_i]$ ?
2. How much data should we use—*i.e.*, what is the right **bandwidth** size?

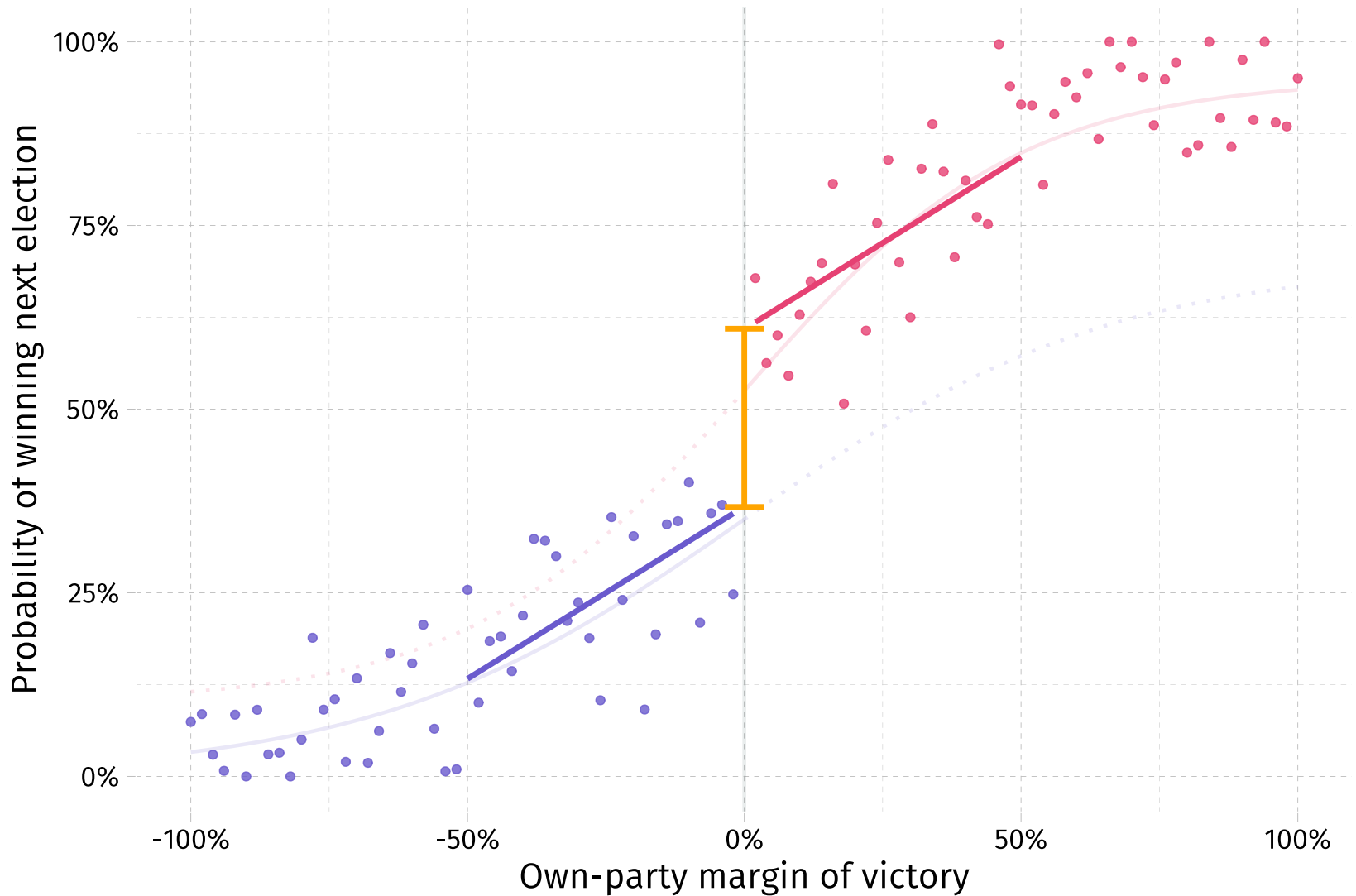
Option 1a Linear regression with constant slopes (and all data)



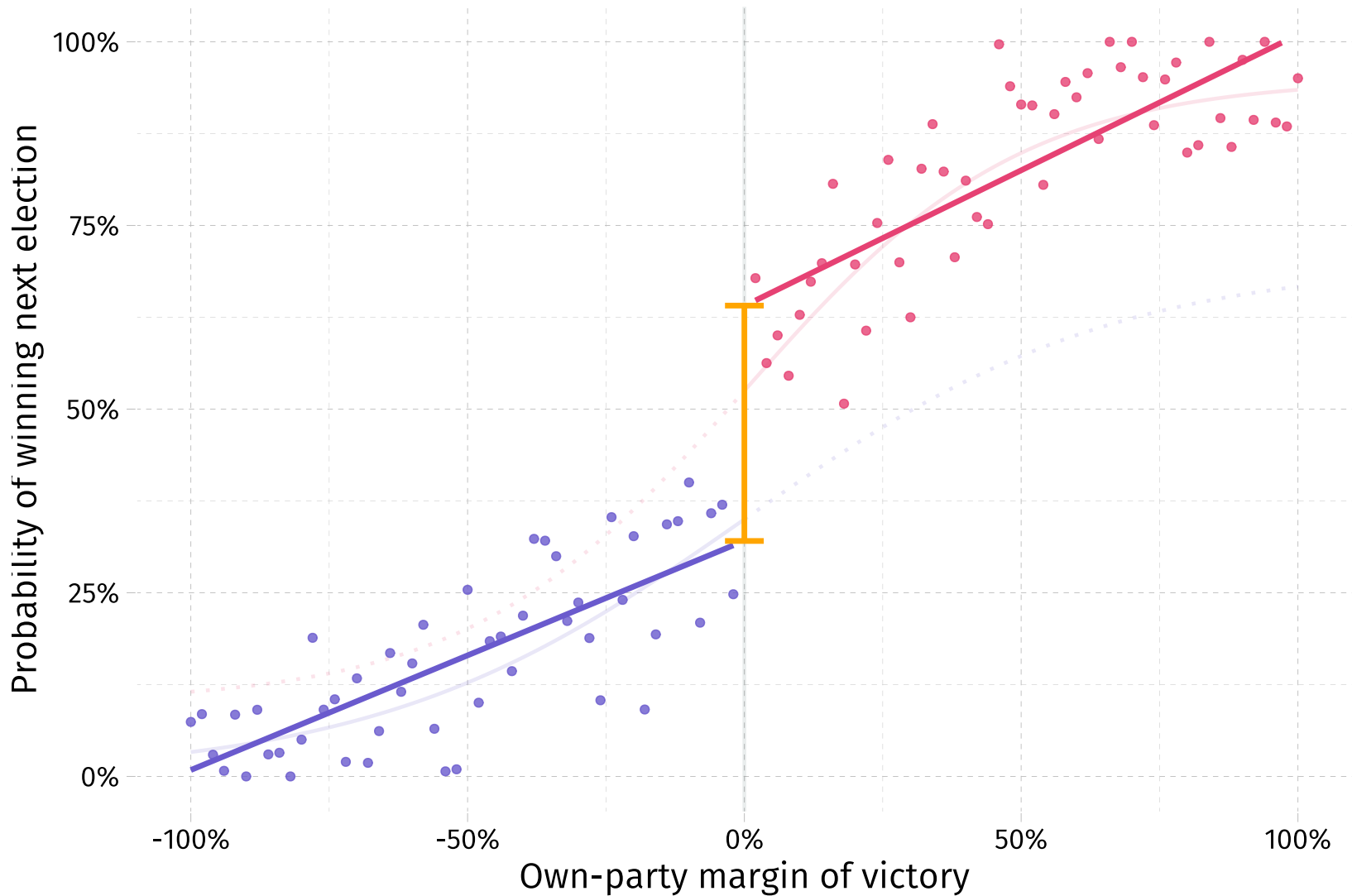
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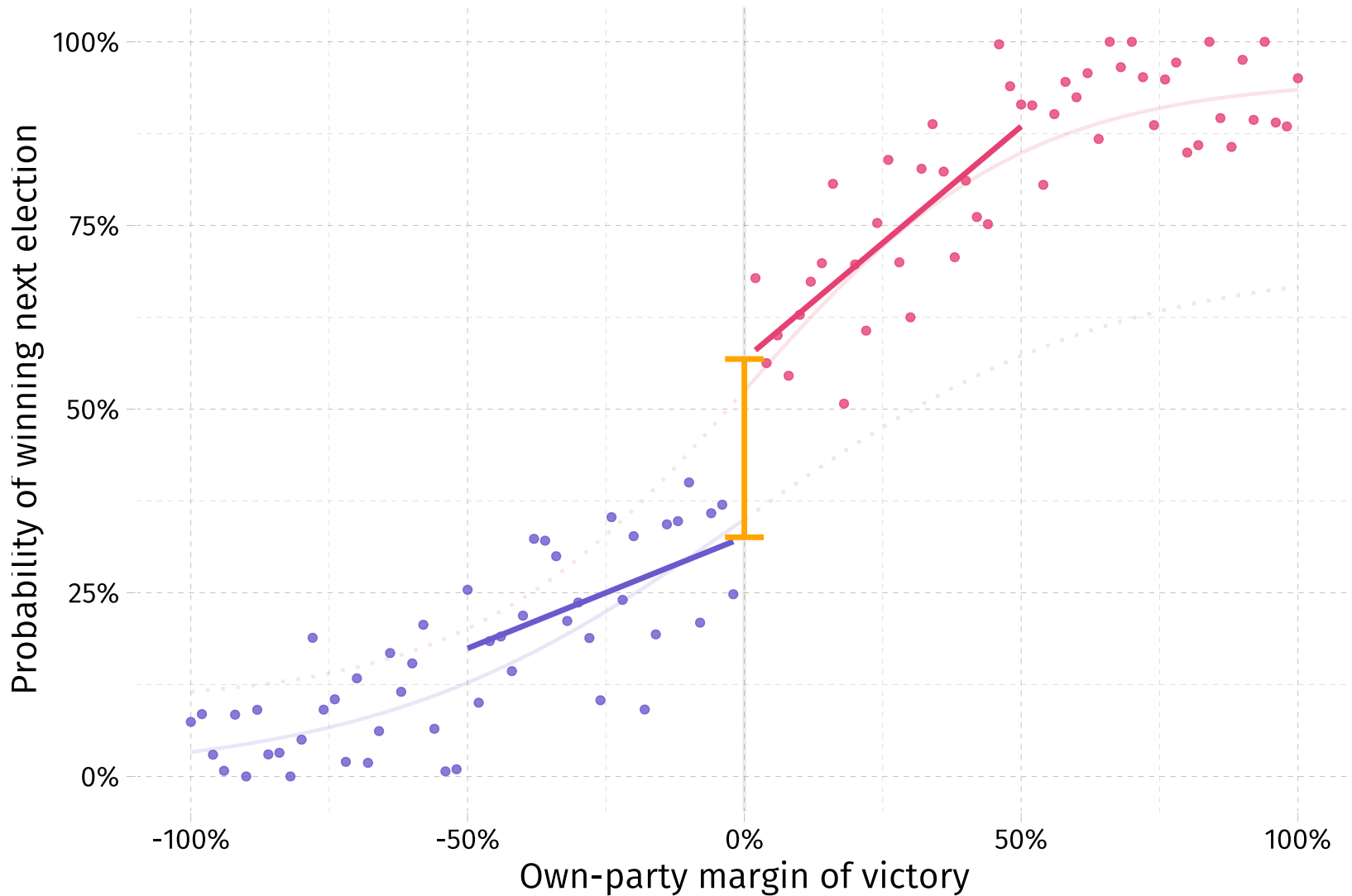
*Option 1b* Linear regression with constant slopes; limited to +/- 50%.



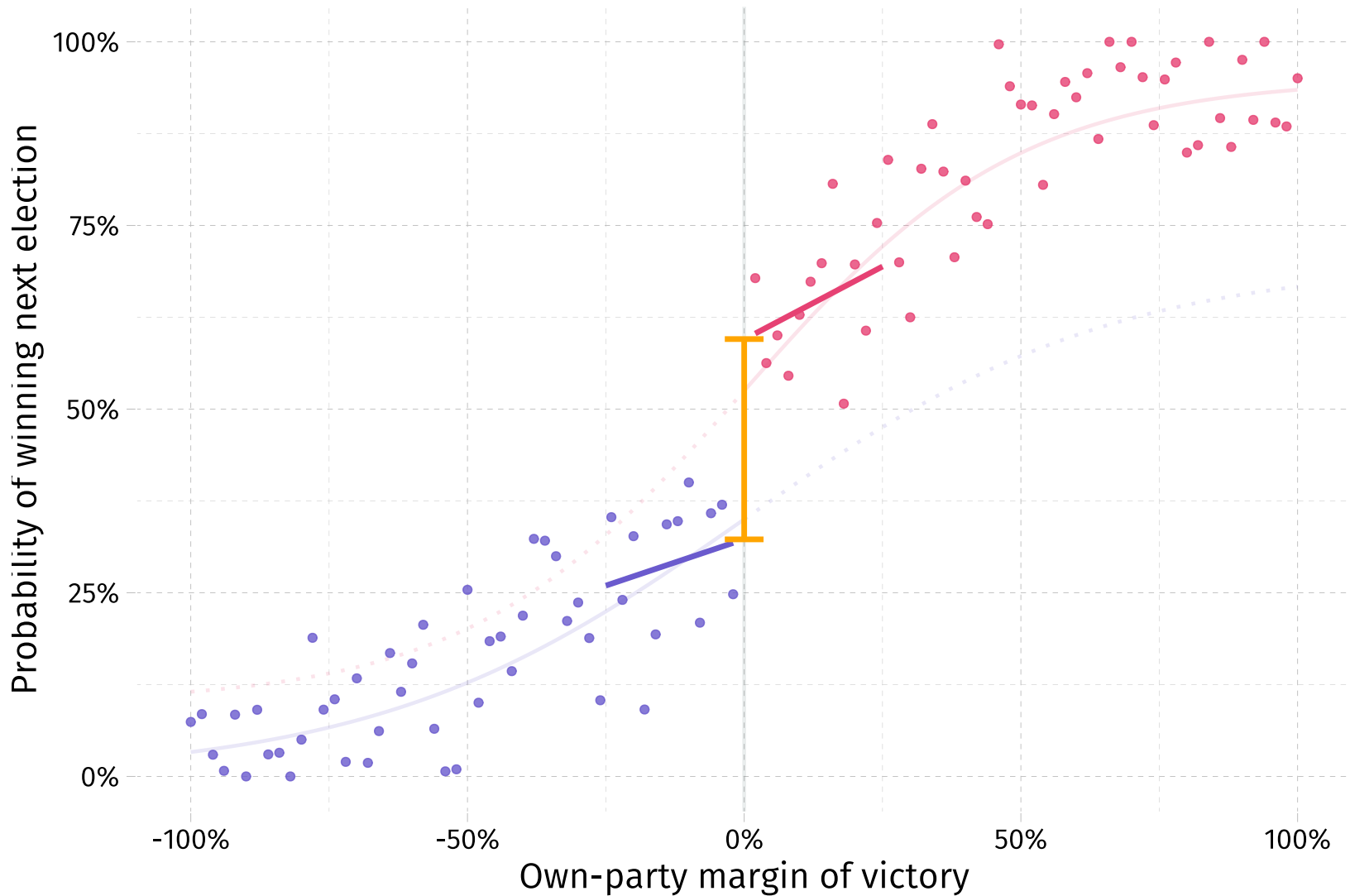
Option 2a Linear regression with differing slopes (and all data)



Option 2b Linear regression with differing slopes; limited to +/- 50%.

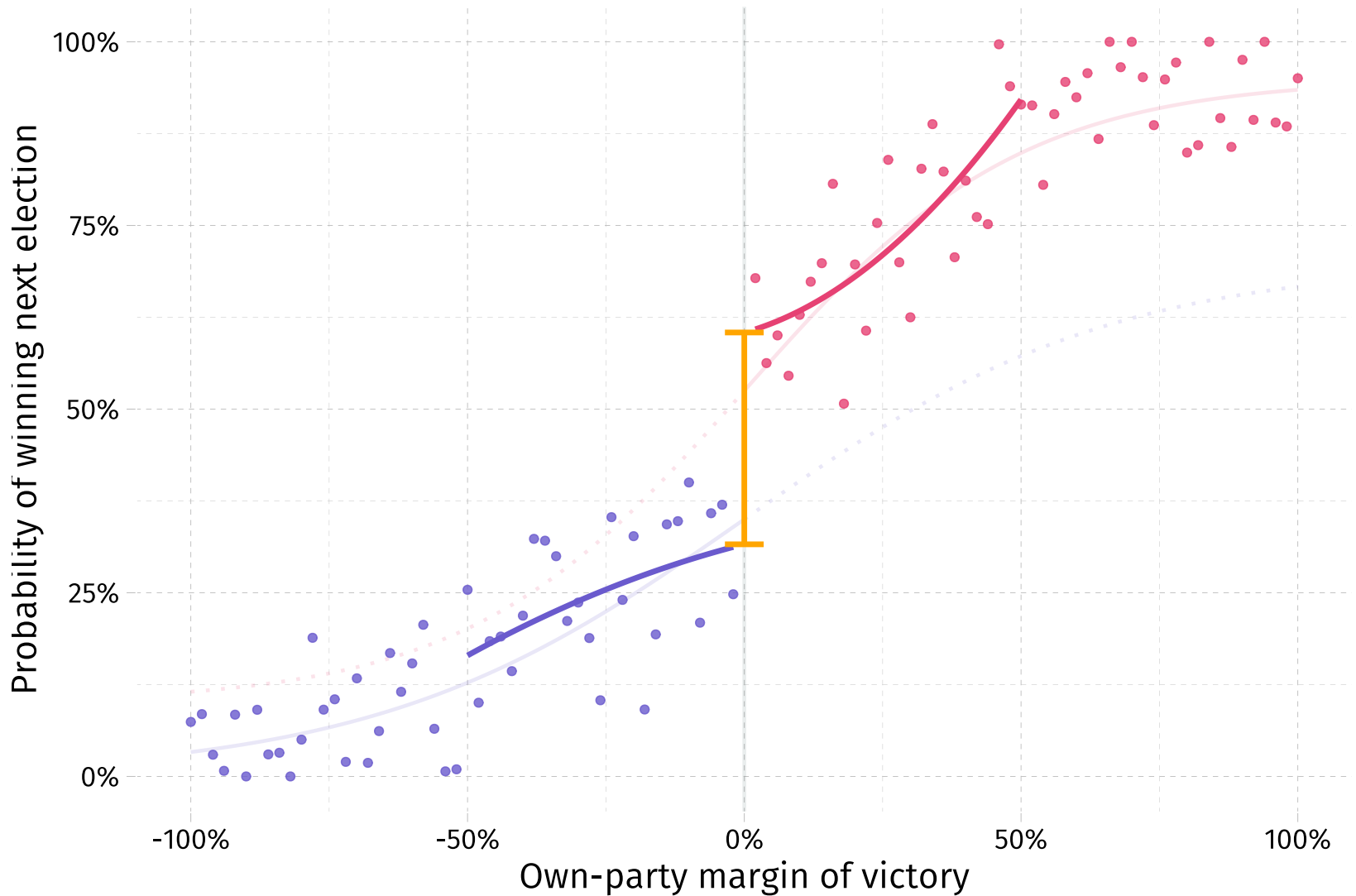


Option 2c Linear regression with differing slopes; limited to +/- 25%.

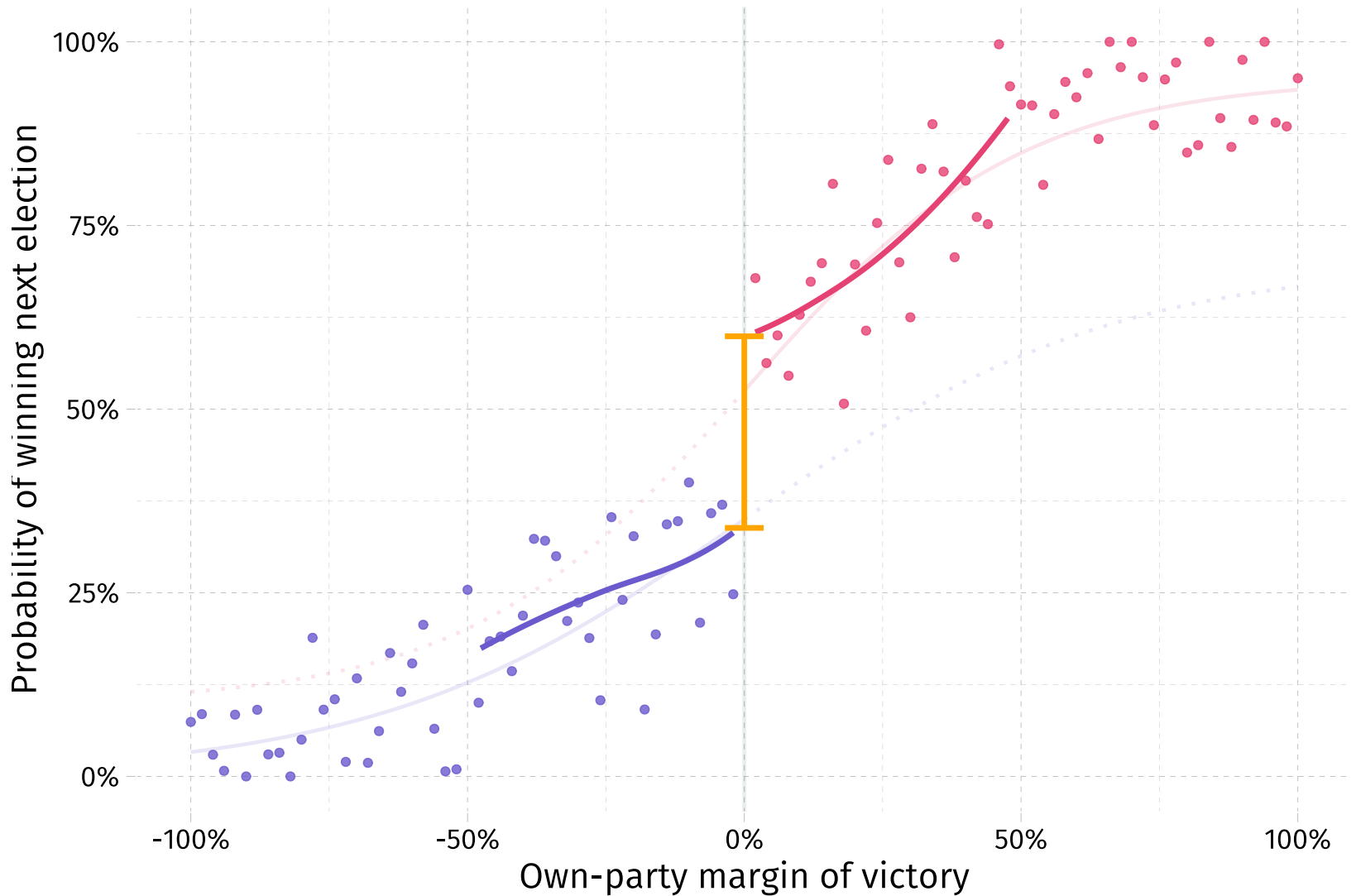




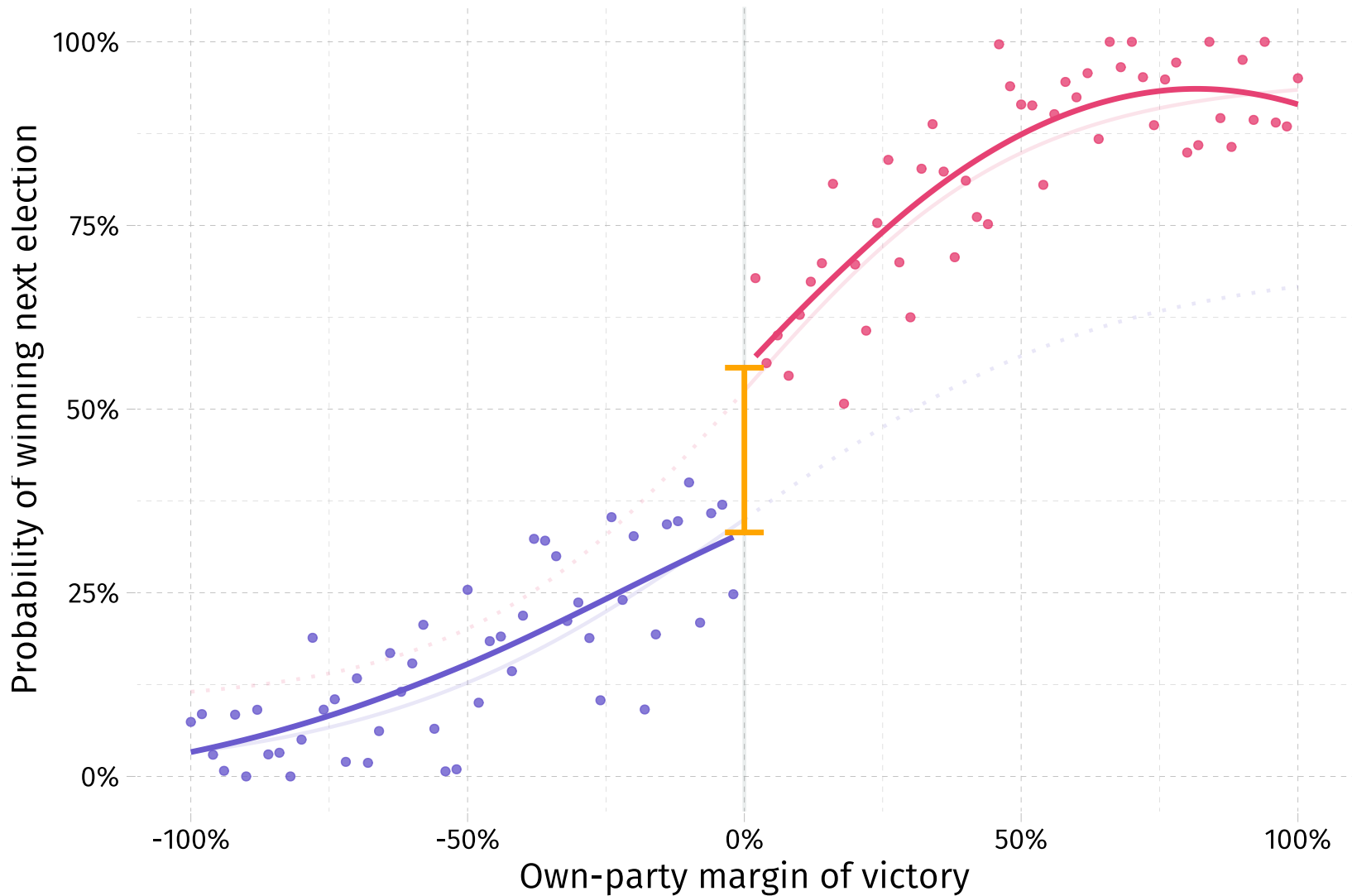
Option 3 Differing quadratic regressions (limited to +/- 50%).



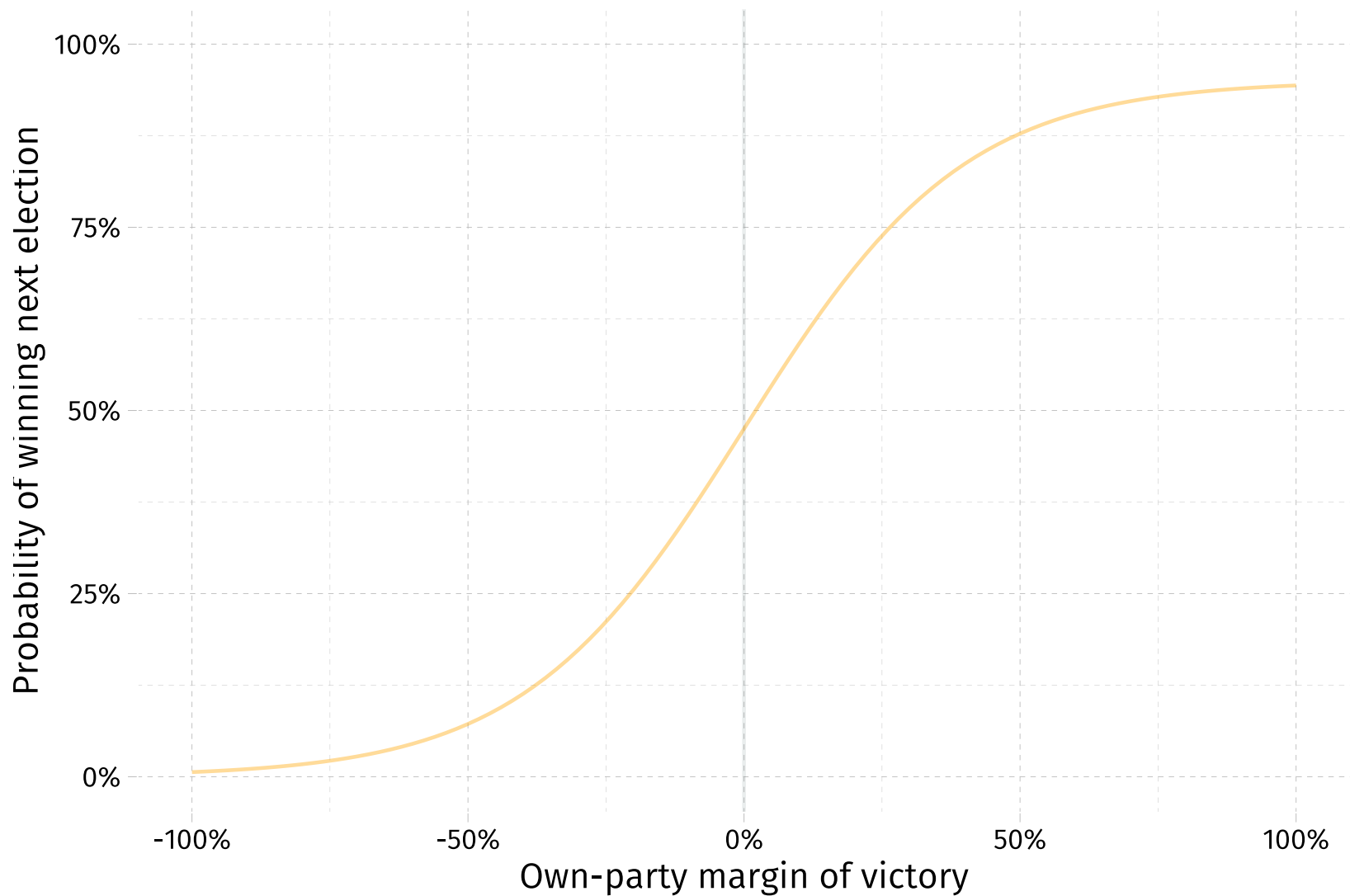
Option 4a Differing local (LOESS) regressions (limited to +/- 50%).



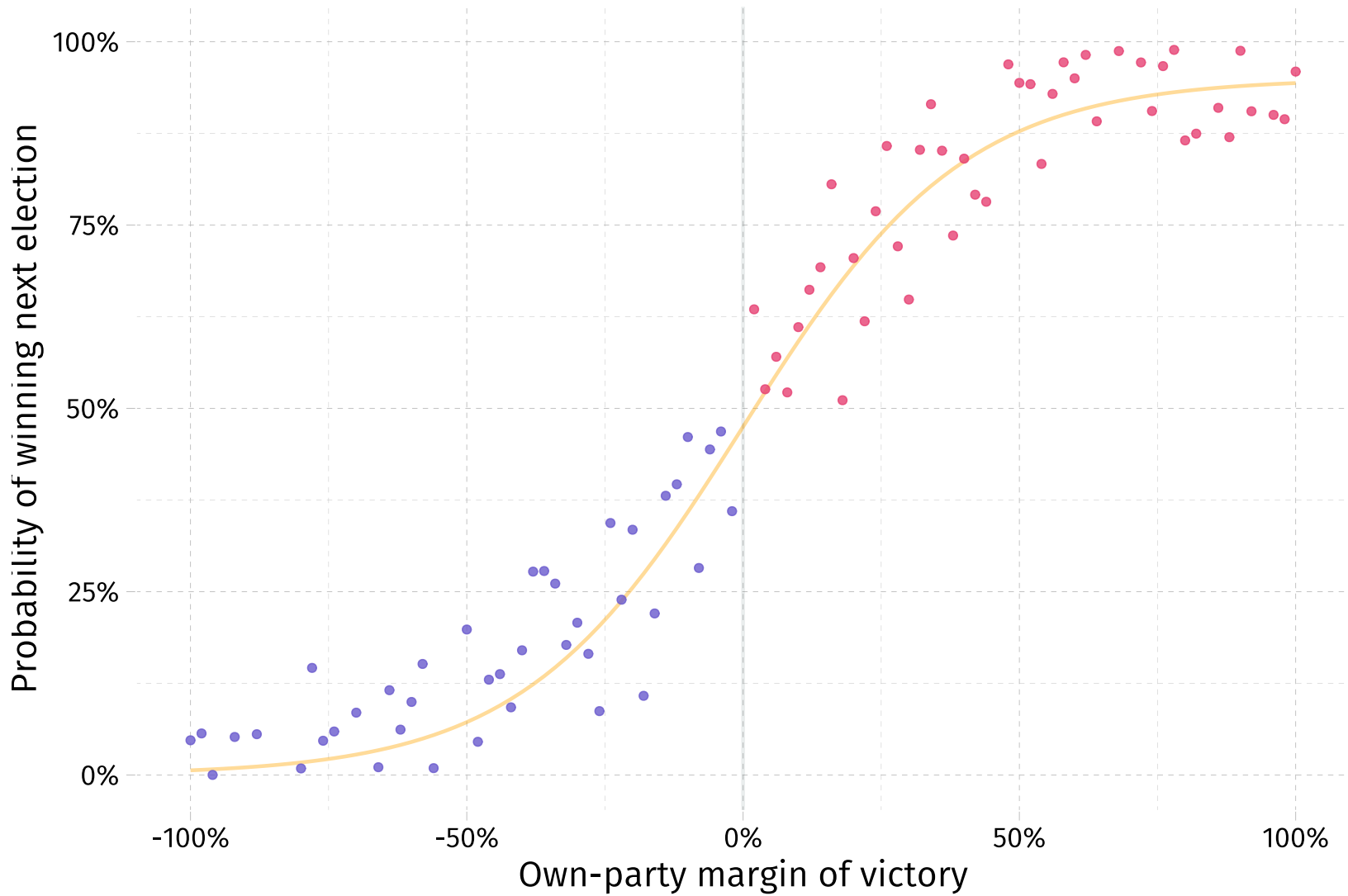
Option 4b Differing local (LOESS) regressions (all data).



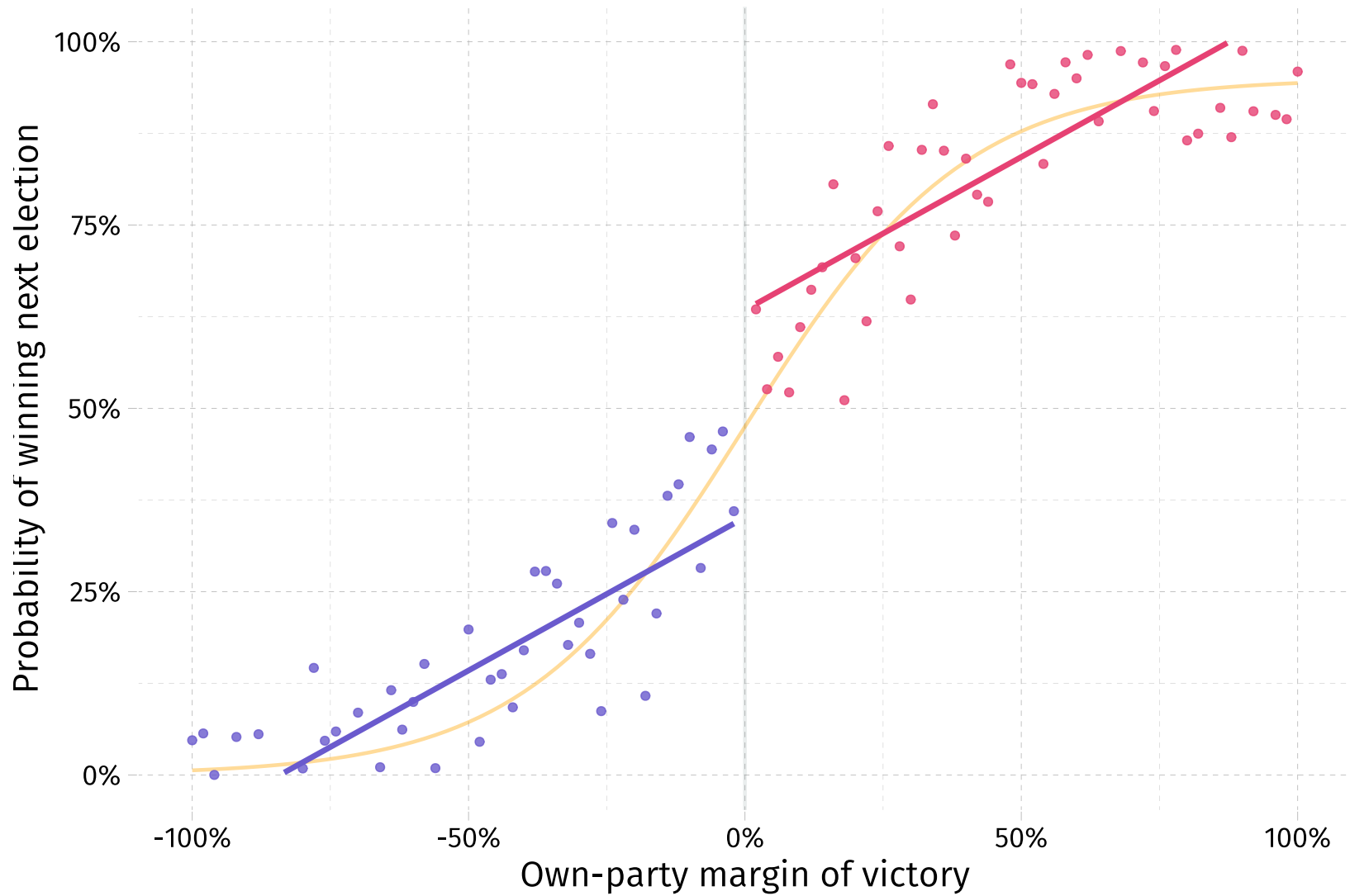
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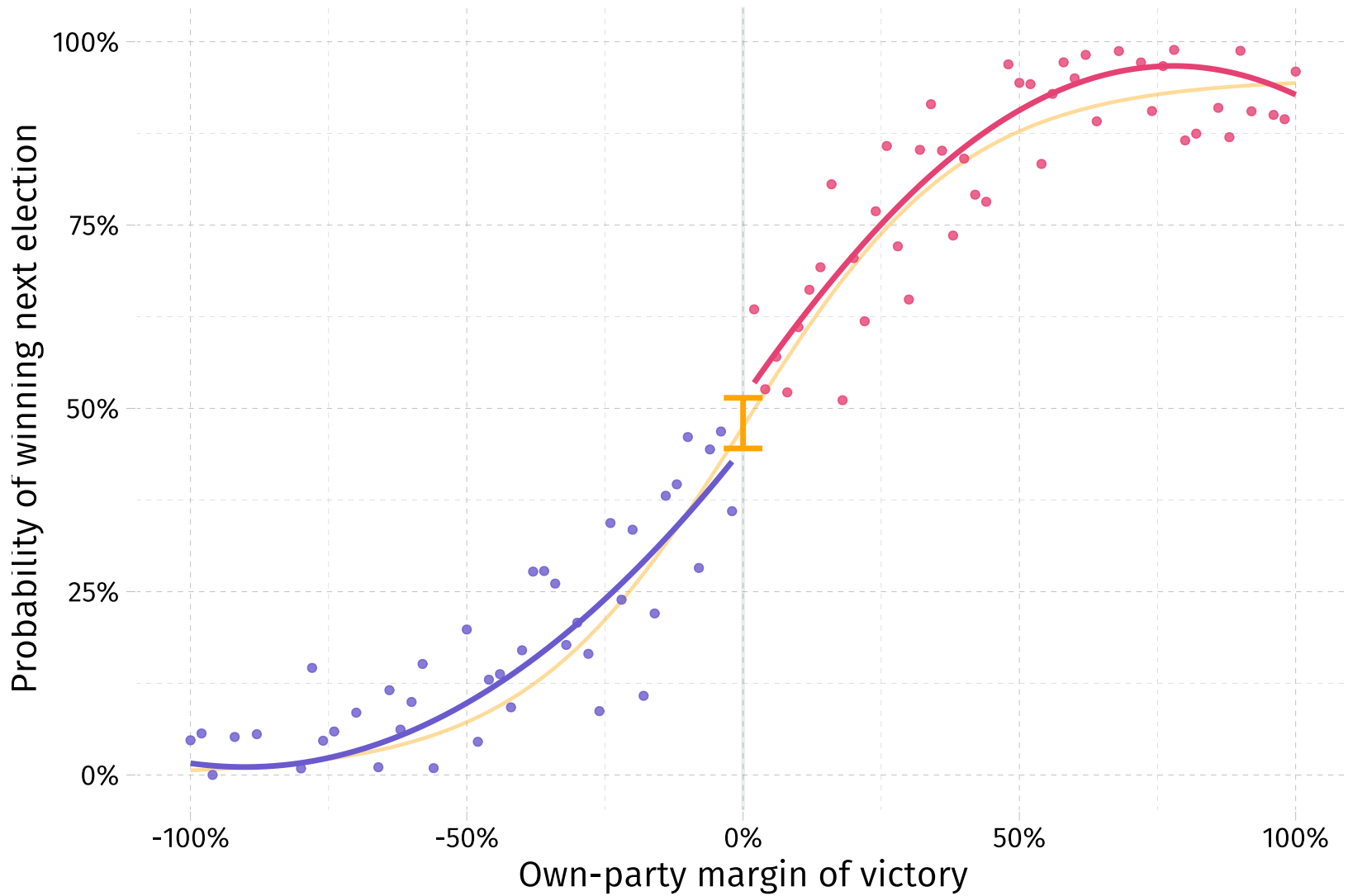
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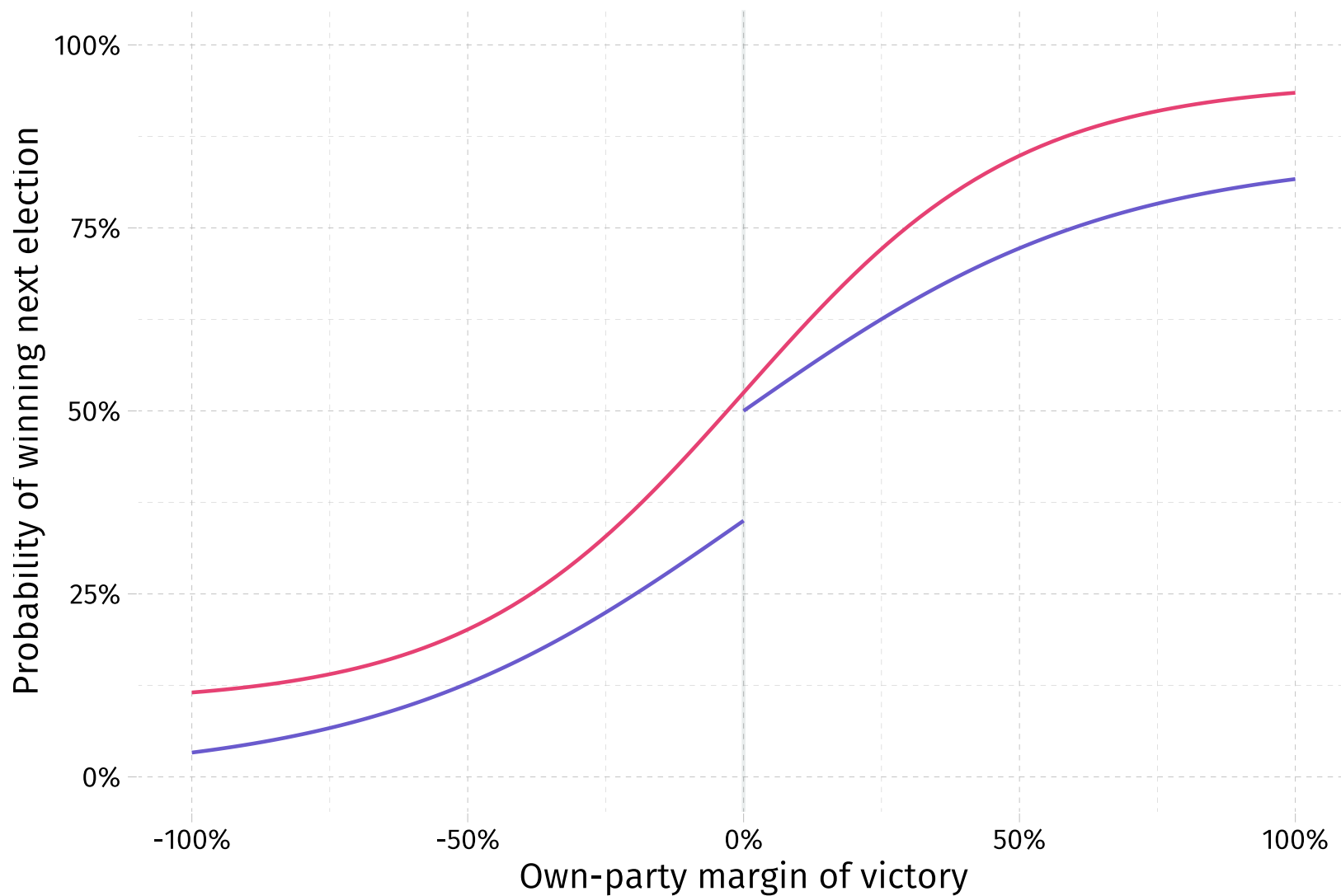


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The continuity of  $E[Y_{0i} | X_i = x]$  (in  $x$ ) is also very important. No sorting.



# Sharp RDs

## In practice

Gelman and Imbens (2018) on functional form:

We argue that **controlling for global high-order polynomials in regression discontinuity analysis is a flawed approach** with three major problems: it leads to noisy estimates, sensitivity to the degree of the polynomial, and poor coverage of confidence intervals. We recommend researchers instead use estimators based on local linear or quadratic polynomials or other smooth functions.

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See [Imbens and Kalyanaraman \(2012\)](#) for optimal bandwidth selection.

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3. Determine a model to **estimate**  $E[Y_i | \tilde{X}_i]$  for  $\tilde{X}_i$  above and below 0
  - Linear with common slopes for  $E[Y_i | \tilde{X}_i < 0]$  and  $E[Y_i | \tilde{X}_i > 0]$
  - Linear/quadratic/polynomial with differing slopes
  - LOESS, kernel regression, etc.

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## Estimation: Linear, common slope

### *Assumptions*

1.  $E[Y_{0i} | X_i = x]$  is linear in  $x$ , i.e.,  $E[Y_{0i} | X_i] = \alpha + \beta X_i$
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where (1) comes from linearity and (2) comes from common slopes.

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Recall our definition of  $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$ .

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# Sharp RDs

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$\tau$  is the LATE at  $\tilde{X}_i = 0$  ( $X_i = c$ ). Estimate: Regress  $Y_i$  in  $\tilde{X}_i$ ,  $D_i$ , and  $D_i \tilde{X}_i$ .<sup>†</sup>

<sup>†</sup> See [Appendix](#) for omitted steps.

Fuzzy RDs



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Formally,

$$0 < \lim_{x \downarrow c} \Pr(\mathbf{D}_i = 1 \mid \mathbf{X}_i = x) - \lim_{x \uparrow c} \Pr(\mathbf{D}_i = 1 \mid \mathbf{X}_i = x) < 1$$

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*Ex.*, Exceeding a minimum GRE requirement for graduate school.

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The treatment effect defined by a fuzzy RD is the ratio of (1) to (2)

$$\tau_{\text{FRD}} = \frac{\lim_{x \downarrow c} E[Y_i | X_i = x] - \lim_{x \uparrow c} E[Y_i | X_i = x]}{\lim_{x \downarrow c} E[D_i | X_i = x] - \lim_{x \uparrow c} E[D_i | X_i = x]}$$

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This definition of the fuzzy-RD treatment effect

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Accordingly, fuzzy RDs are going to have the **same requirements and interpretation as IV**.

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Let  $D_i(x^*)$  denote the **potential treatment status** of  $i$  **with threshold**  $x^*$ .

Why write potential treatment status  $D_i$  a function of the threshold?

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This is our monotonicity assumption for fuzzy RDs. If we raise  $x^*$  from  $c$  to  $c + \epsilon$ , no one joins treatment—no defiers.

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## Compliance

Our **compliers** in this setting are individuals such that

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Thus,  $\tau_{\text{FRD}}$  can be a *very local* LATE.

# Graphical analysis

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You're arguing you know that treatment assignment changes across the threshold. If your reader/viewer cannot see it, they're likely not going to believe your regression tables.<sup>†</sup>

<sup>†</sup> This skepticism may be well founded. We know RDs are sensitive to functional form—and researchers have been known to *p-hack*.

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## Three figures

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3. **Density** of running/forcing variable ( $\mathbf{X}_i$ )  
*Is there evidence of sorting into treatment (across the threshold)?*

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We then calculate summaries for each bin.

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## Outcomes by running variable

The bin's **number of observations**,  $N_k$

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## Outcomes by running variable

We then plot  $\bar{D}_k$  against the midpoint of each bin.

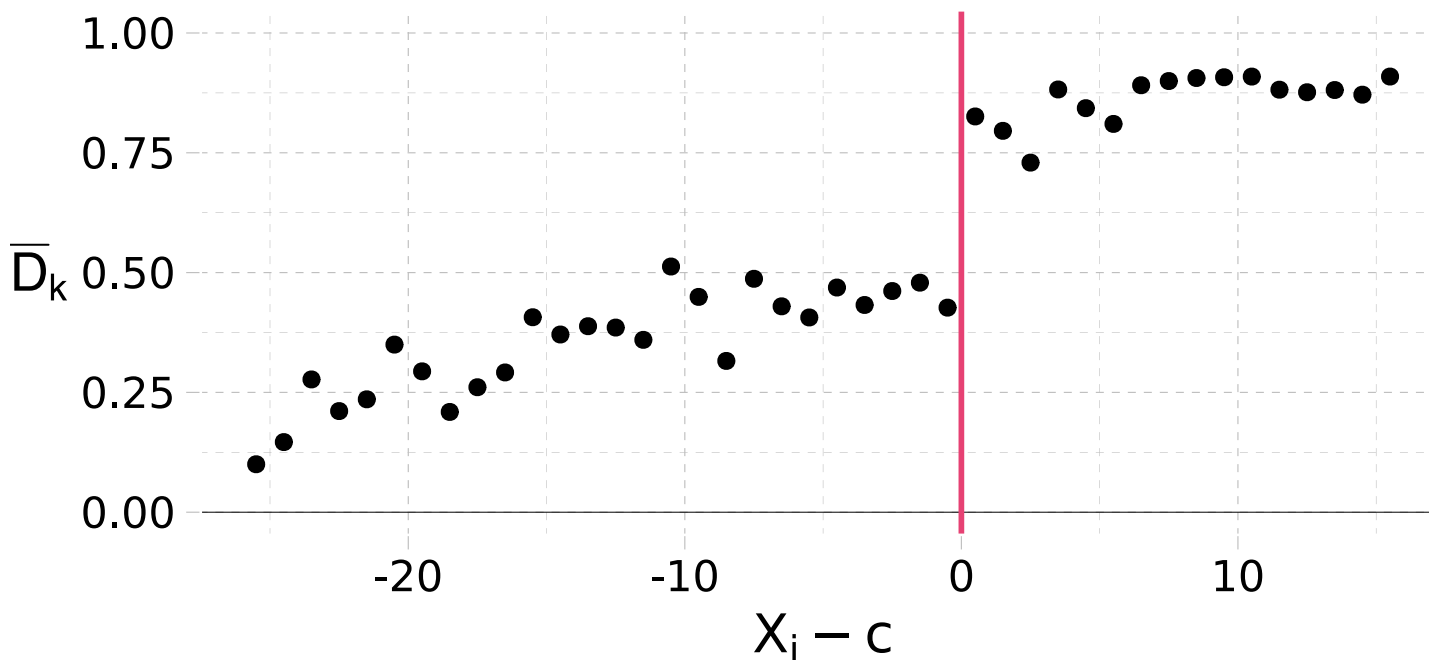
Q Does crossing  $c$  clearly affect  $\Pr(D_i = 1)$ ? (Fuzzy RD first stage)

# Graphical analysis

## Outcomes by running variable

We then plot  $\bar{D}_k$  against the midpoint of each bin.

Q Does crossing  $c$  clearly affect  $\Pr(D_i = 1)$ ? (Fuzzy RD first stage)



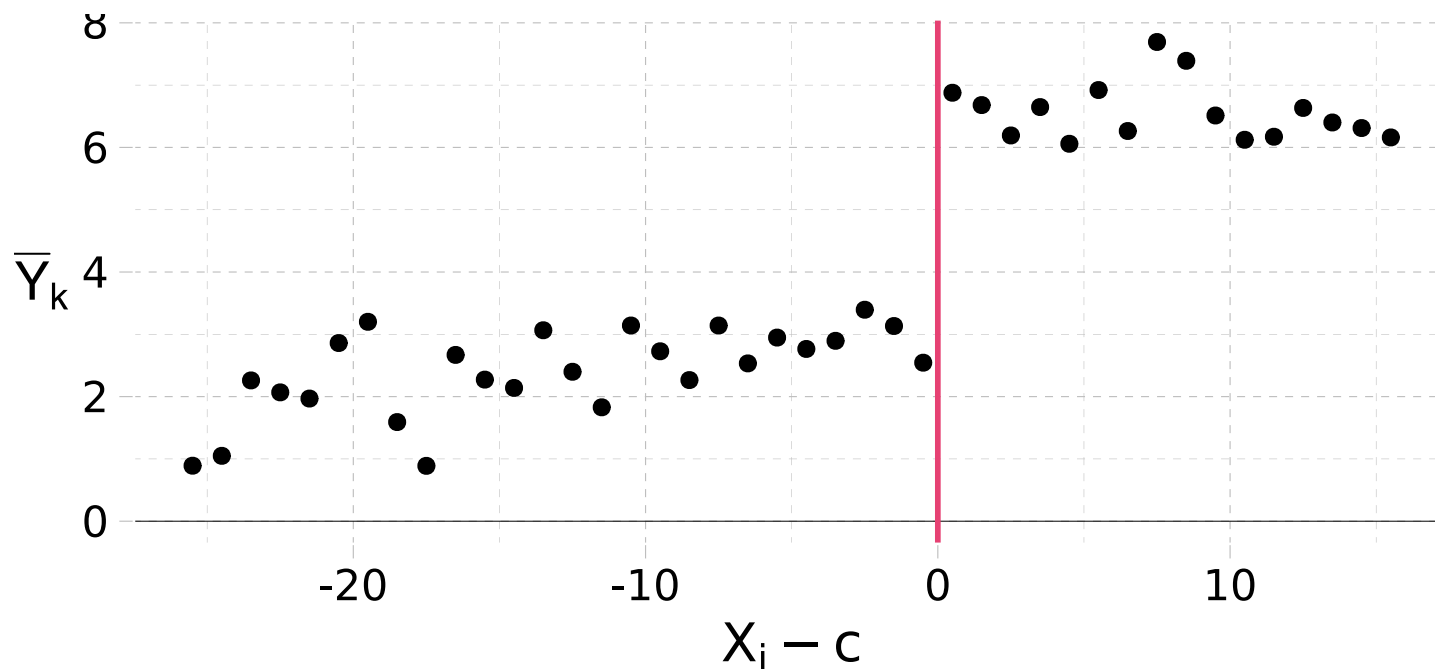


# Graphical analysis

## Outcomes by running variable

And then plot  $\bar{Y}_k$  against the midpoint of each bin.

Q Does crossing  $c$  clearly affect our outcome  $Y_i$ ? (Fuzzy RD reduced form)



# Graphical analysis

## Covariates by running variable

Now we apply the same approach to covariates ( $\mathbf{Z}_i$ ).

# Graphical analysis

## Covariates by running variable

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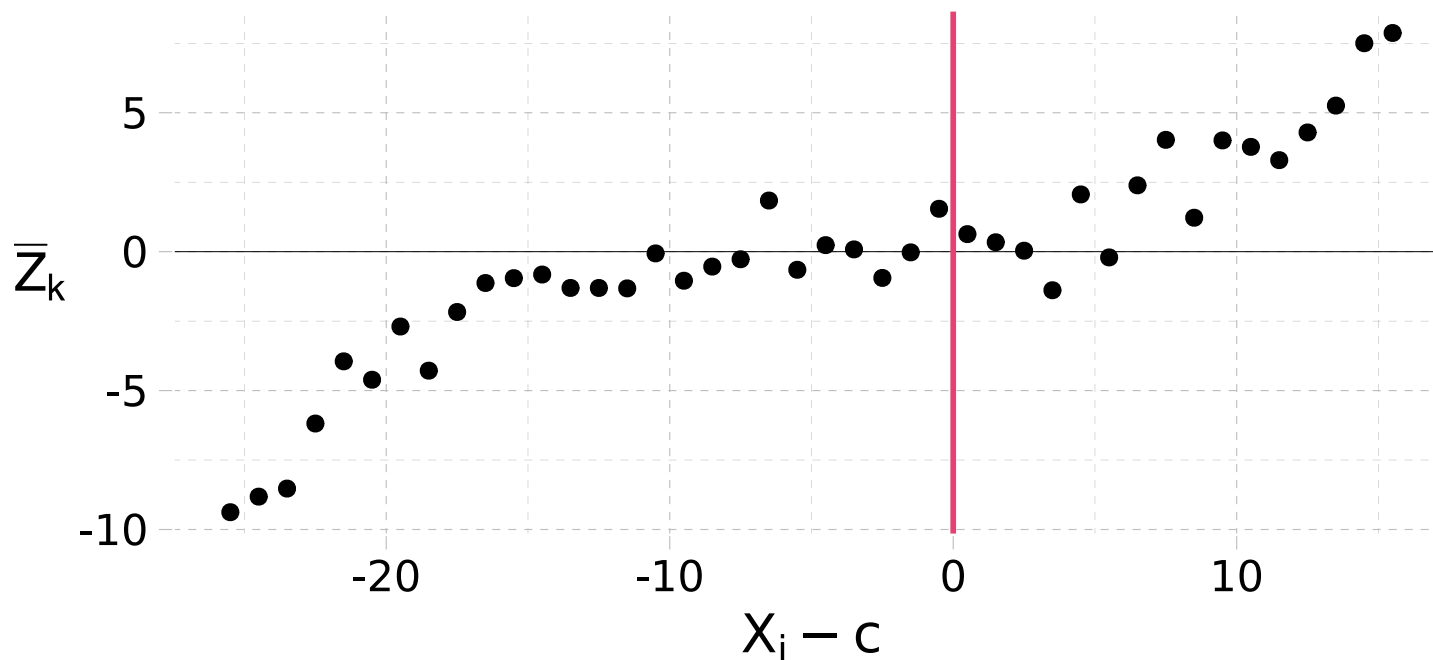
Q Are covariates **smooth** across  $c$ ? If not, your RD may be invalid.

# Graphical analysis

## Covariates by running variable

Now we apply the same approach to covariates ( $Z_i$ ).

Q Are covariates **smooth** across  $c$ ? If not, your RD may be invalid.



# Graphical analysis

## Density of running variable

Finally we looking for other violations of smoothness—particularly in form gaming the threshold.

In other words: Are individuals **bunching** just above or just below the threshold?

If so, folks just below the threshold don't give us the clean counterfactual that we want for the folks just above the threshold.

McCrary (2008) suggests testing the density of  $\mathbf{X}_i$  at  $c$ .

# Graphical analysis

## Density of running variable

Effectively, we can plot  $N_k$  at the midpoint of each bin.

# Graphical analysis

## Density of running variable

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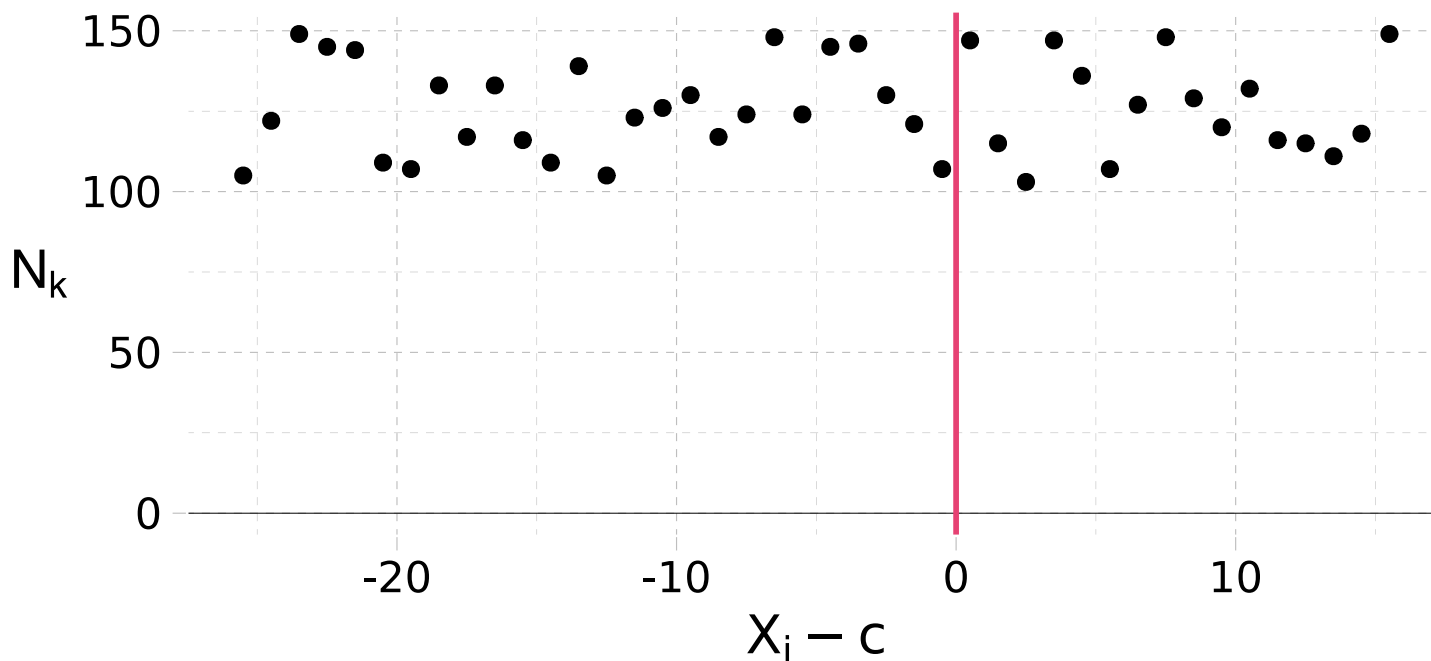
Q Is the distribution of  $\mathbf{X}_i$  smooth across  $c$ ?

# Graphical analysis

## Density of running variable

Effectively, we can plot  $N_k$  at the midpoint of each bin.

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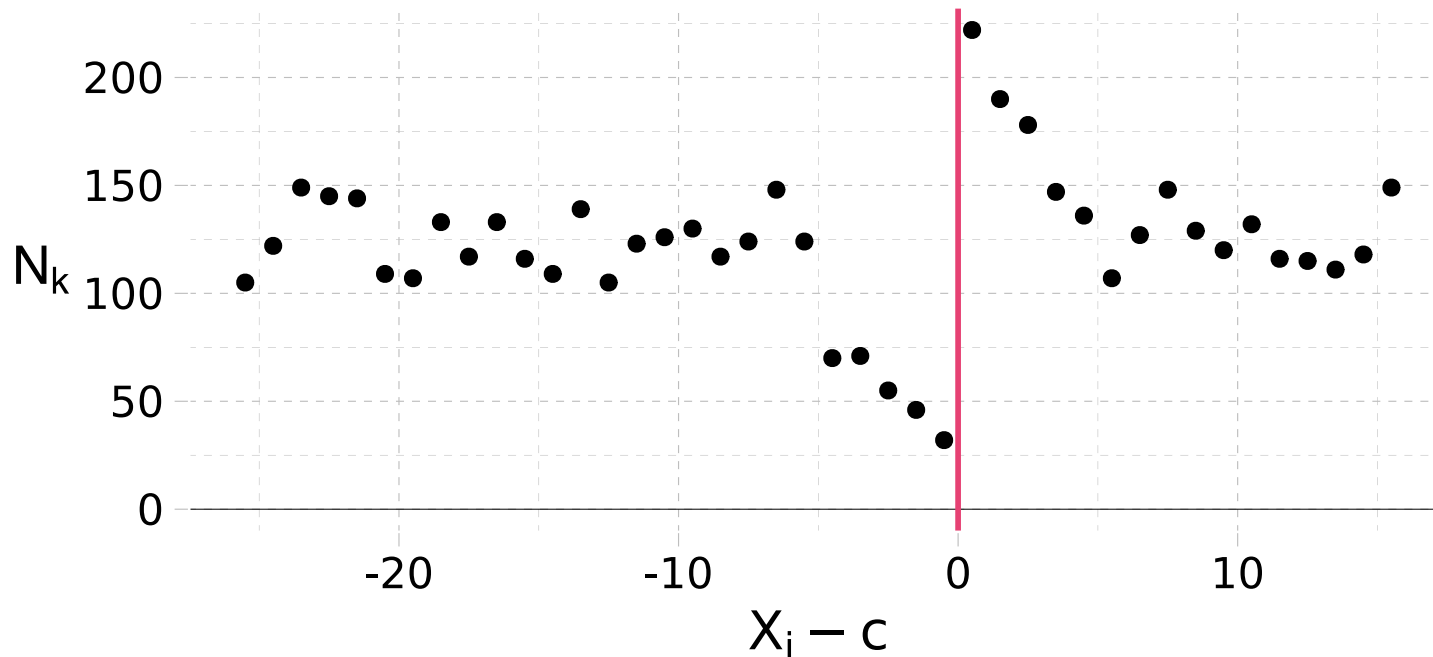


# Graphical analysis

## Density of running variable

**Likely bunching** (problem)

Q Is the distribution of  $X_i$  smooth across  $c$ ?



# Graphical analysis

## Additional points

1. No bin should cross the threshold.
2. Are there discontinuities other than  $c$ ? Should there be? Smoothness?

# Graphical analysis

## Additional points

1. No bin should cross the threshold.
2. Are there discontinuities other than  $c$ ? Should there be? Smoothness?

Again, if these graphs are not clear and convincing, it's going to be hard to make the case that you have a true/credible discontinuity.

# Appendix

## Estimation: Linear, differing slopes

Definitions of terms that **magically appear**

- $\tilde{\alpha} = \alpha_0 + \beta_0 c$
- $\tau = (\alpha_1 - \alpha_0) + (\beta_1 - \beta_0) c$
- $\tilde{\beta} = (\beta_1 - \beta_0)$

# Table of contents

## Admin

1. Schedule

## General RD

1. Setup
2. Framework
3. Examples
4. Sharp vs. fuzzy

## Graphical analysis

1. General
2. Outcomes by  $\mathbf{X}_i$
3. Covariates by  $\mathbf{X}_i$
4. Density of  $\mathbf{X}_i$

## Sharp RDs

1. Setup
2. (Semi) Formally
3. Estimation
4. Examples
5. In practice
6. More estimation

## Fuzzy RDs

1. Setup
2. As IV
3. Somewhat formally